

## **Numerical parametric study on liquid sloshing in a rectangular tank**

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### **ABSTRACT**

The present study is concerned with sloshing phenomena in a rectangular tank under a sway excitation. The study by Ha et al. (2012) made an intensive investigation on the validation of numerical technique based on the volume of fluid method implemented in a computational fluid dynamics code solving Reynolds-Averaged Navier-Stokes equations to deal with the free-surface of liquid sloshing in a rectangular tank. Main aim of the present study is to make an extensive parametric study using this numerical technique to analyze the effects of sway excitation magnitude and initial liquid filling level on the characteristics of liquid sloshing phenomena in a tank. These results are expected to be used as a basic reference data in designing effective tuned liquid dampers of the platform/substructure of a floating wind turbine.

### **1. INTRODUCTION**

The phenomenon related to fluid motion in a partially filled tank due to the tank motion is known as sloshing. The tank sloshing problems have been studied in ships or offshore structure mainly in view of safety, because the resonant condition in sloshing may cause large structural loads on the tank frame and the sloshing may create high impact loads on the wall of the tank at certain filling ratio (Rafiee 2011). However, in other applications, the same sloshing phenomenon has been used as a damping mechanism to reduce the vibration levels of structure subjected to external excitation sources. For example, tuned liquid dampers based on liquid sloshing have been used to suppress the reaction of high buildings subjected to seismic waves (Kareem 1999). A floating wind turbine is an offshore wind turbine mounted on a floating structure that enables the turbine to generate electricity in water depths where underside-mounted towers are not available. Since there are no topographic features that disturb wind flow, the wind is typically more consistent and stronger over the sea than mid land. However, one of key techniques in developing a successful floating wind turbine is to design

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effective floating platforms to support turbines by withstanding severe sea states. Tuned liquid damper (TLD) is one of potential methods to control motion of a floating structure.

As a first step toward developing TLD for floating platforms of off-shore wind turbines, Ha et al. (2012) made an intensive validation study on the numerical methods. They showed that the results predicted using the numerical method based on the volume of fluid method implemented in a computational fluid dynamics code solving Reynolds-Averaged Navier-Stokes equation to deal with the free-surface of liquid sloshing closely followed the measured data. In the present paper, extensive parametric study is performed to analyze the effects on the sloshing phenomena of excitation magnitudes and initial filling levels of liquid in a tank. The responses of sloshing liquid in the rectangular tank are analyzed in terms of the time-histories of pressure and force, peak pressure distribution on the tank side wall, and sloshing shape.

## 2. THEORETICAL MODEL FOR NATURAL FREQUENCY

Theoretical model for natural frequency of sloshing water was proposed by Dodge (2000). In this section, the model is briefly reviewed. Under the assumption that the liquid is inviscid and irrotational, the sloshing velocity can be obtained from a velocity potential satisfying the following Laplace equation,

$$\nabla^2\Phi = 0 \quad (1)$$

In addition, the velocity potential satisfies the equation of motion in the form,

$$\frac{\partial\Phi}{\partial t} + \frac{p}{\rho} + gz + \frac{1}{2}(u^2 + v^2 + w^2) = f(t) \quad (2)$$

where  $p$  is the fluid pressure,  $\rho$  is the fluid density, and  $g$  is the effective gravity directed in the negative  $z$  direction. The velocity vector  $(u, v, w)$  is assumed to be so small that squared and higher power terms of them can be neglected in comparison to linear terms. After  $\Phi$  is further redefined to include  $f(x)$ , Eq. (2) can be rewritten in the form,

$$\frac{\partial\Phi}{\partial t} + \frac{p}{\rho} + gz = 0 \quad (3)$$

Eq. (3) must satisfy the boundary conditions of the concerned problem; liquid inside a rectangular tank. Pressure at the free surface must equal to the ambient pressure  $p_0$ . Application of this boundary condition at the free surface leads to the following equation,

$$\frac{\partial\Phi(x, y, z, t)}{\partial t} + g\delta(x, y, t) = -\frac{p_0}{\rho} \quad \text{for } z = \frac{h}{2} \quad (4)$$

where the origin is the center of water,  $h$  denotes water height, and  $\delta$  is the small displacement of the free surface above the undisturbed level  $z = h/2$ . Assuming that

the tank is excited into sway motion, the tank displacement can be expressed as  $X(t) = -iX_0 \exp(i\Omega t)$  in the horizontal motion parallel to the x axis. So the boundary condition at the wetted surfaces of the tank is expressed as

$$\mathbf{n} \cdot \nabla \Phi = iX_0 \Omega e^{i\Omega t} \quad \text{at wetted surfaces} \quad (5)$$

where  $\mathbf{n}$  is the unit vector normal to the wetted surface. Then, the equation of natural frequency of liquid in a rectangular tank is given by

$$\omega_m^2 = \pi \frac{mg}{a} \tanh\left(\pi \frac{mh}{a}\right) \quad (6)$$

This equation is used to determine the excitation frequency.

### 3. NUMERICAL METHOD

Numerical method is based on the volume of fluid method implemented in a computational fluid dynamics code solving Reynolds-Averaged Navier-Stokes equation to deal with the free-surface of liquid sloshing. The commercial software ANSYS CFX™ is used to realize this numerical scheme. The dimension of a target rectangular tank is 1000mm(L) × 250mm(B) × 600mm(H). Numerical computations are performed using two-dimensional mesh of the dimension, 1000mm × 600mm, which was shown to provide numerical results similar to those using the three-dimensional one (Ha et al., 2012). The inhomogeneous model is used as a multiphase model to simulate water and air, separately. Shear Stress Transport (SST) model is used as a turbulence model, which was revealed to show closer agreement to the measurement by Minho et al. (2012).

### 4. PARAMETRIC STUDY

Behavior of liquid sloshing is subjected to many factors such as excitation frequency and magnitude, initial filling ratio of liquid, shape of a tank, and so on. As described in Introduction, sloshing phenomena are involved in the safety problem in terms of two factors: high impact loads on the wall of the tank at certain filling ratio and the resonant condition in sloshing. In this respect, two factors, displacement magnitude of excitation and filling levels of liquid, are selected for a parametric study. The displacement magnitude is directly related to magnitude of sloshing impact loads on the wall of the tank, and filling level of liquid is one of parameters to determine the natural frequency of sloshing liquid. The case studied by Minho et al. (2012) is taken into account as the reference case: the filling ratio of liquid (water) was 20% of the tank height, and the tank was excited into sway motion with sway amplitude of 0.1m with frequency of 0.53Hz. In the following subsections, parametric studies are performed using these two parameters, respectively.

#### 4.1 Displacement Amplitude of Excitation

Displacement amplitude of external excitation is varied as  $A = 0.025:0.025:0.125$  m to investigate its influence on the liquid sloshing. Other conditions are kept the same as

the reference case.

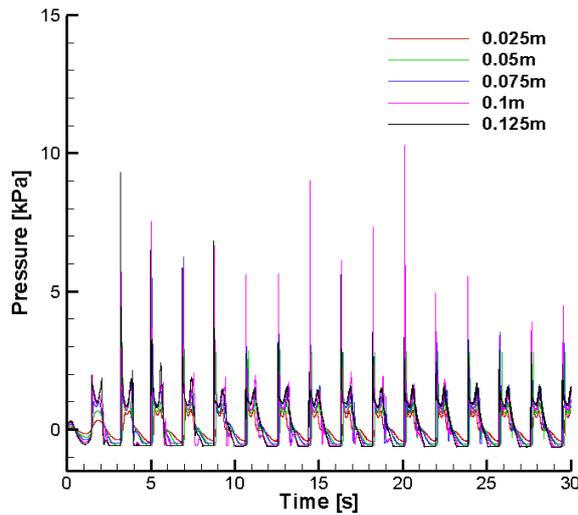


Fig. 1 Predicted time-variations of pressure

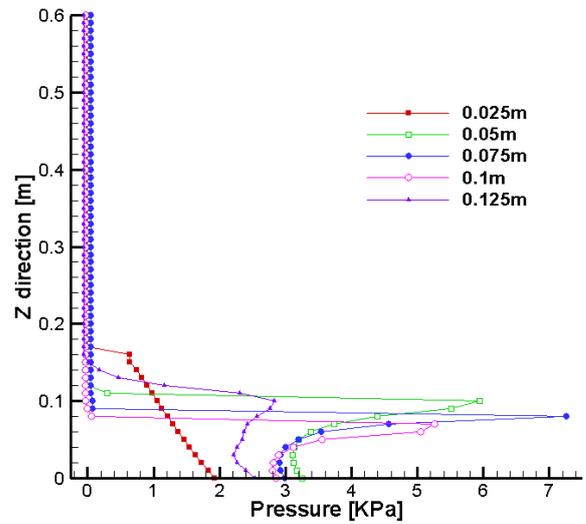


Fig. 2 Pressure distribution on side wall

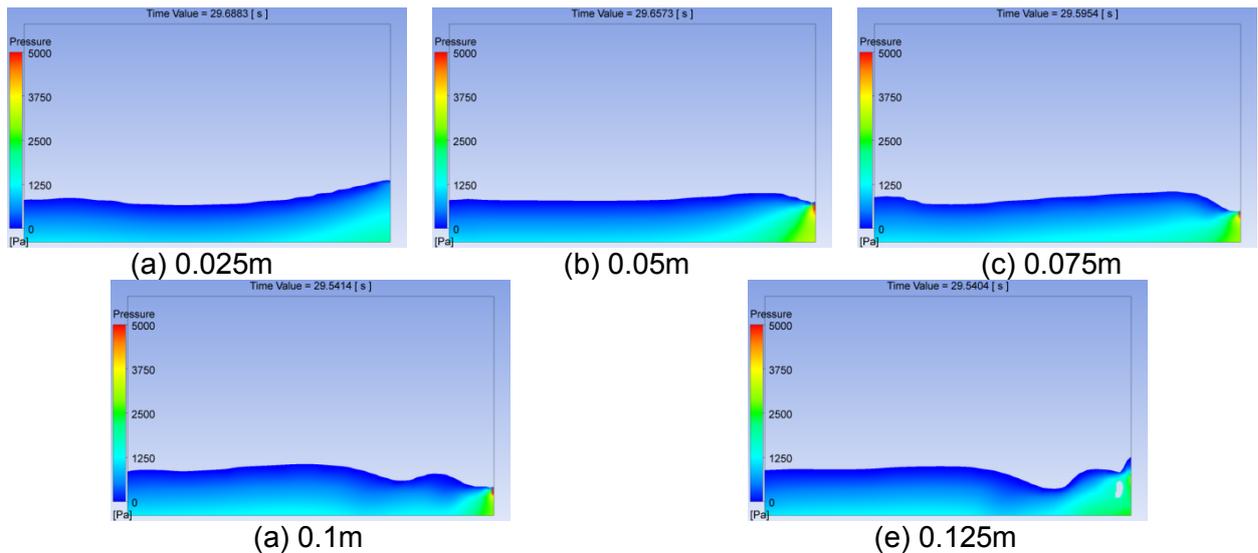


Fig. 3 Snap-shot of free surface and iso-contours of pressure in sloshing water at the time when maximum pressure occurs

Fig. 1 shows time-histories of pressure predicted at the point of  $z=0.06\text{m}$ . The high impact pressure can be identified in the case of  $A = 0.1\text{ m}$ . However, the peak pressure position may vary according to the shape of sloshing motion. Therefore, peak pressure distributions are computed for each of cases, and the result is shown in Fig. 2. It is seen that the peak pressure distribution varies according to the applied displacement amplitudes and the highest pressure occurs in the case of  $A = 0.075\text{m}$ . Fig. 3 shows the snap-shot of sloshing motion with iso-contours of pressure in sloshing water at the instant when the maximum pressure is observed. The reason that the peak pressure decreased above  $A = 0.1\text{ m}$  can be inferred from these figures. The high amplitude

displacement induces more violent sloshing. Therefore, the free surface is distorted and the contact area is increased. For the use of sloshing phenomena as a TLD, the total force exerted by liquid on the tank is critical.

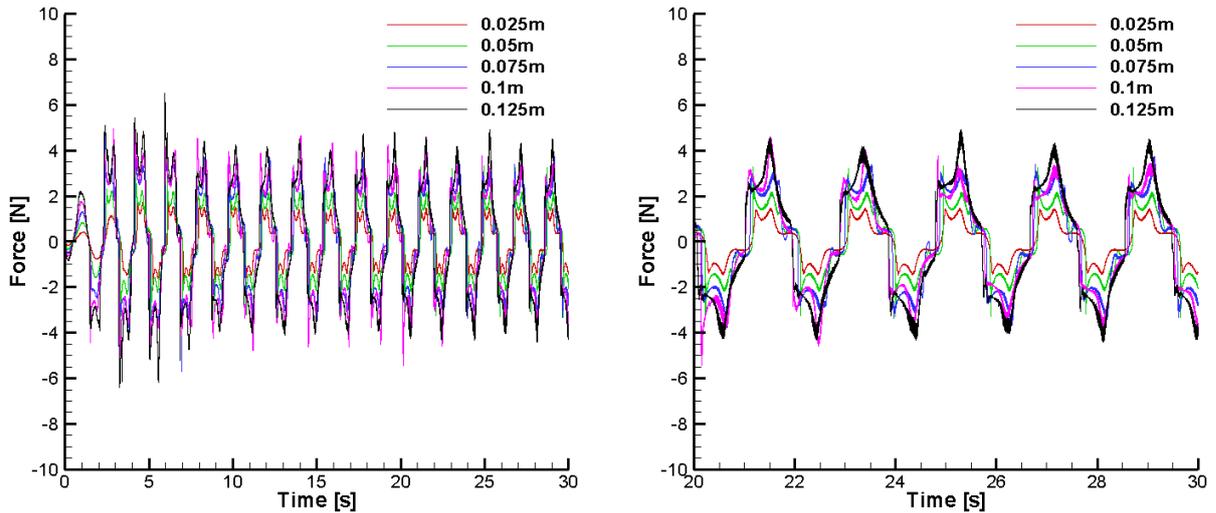


Fig. 4 Predicted time-variations of force

Fig. 4 shows the time-variation of force in sway direction for each case. As expected, the high amplitude of displacement induces the high amplitude of force. In addition, as the applied displacement amplitude increases, the 2<sup>nd</sup> peak in the time-variation of force increases and become greater than the first peak force.

#### 4.2 Filling Level of Water in a Rectangular Tank

Height of water in a rectangular tank affects the natural frequency of the tank as shown in Eq. (6). In reality, the water in a TLD evaporates continuously after the TLD installed. The changing height of water affects the performance of the TLD so that the height of water in TLD is checked periodically.

The filling ratio of water in the tank is varied as FR= 10:5:30 % to investigate the effect of the difference filling ratio on the characteristics of sloshing water. Excitation frequency is kept as 0.53Hz that is the first natural frequency of sloshing water for the case of filling ratio, FR=20%. That is, the excitation frequency in the cases of FR=10% and 15% is higher than the first natural frequency, while the excitation frequency in the cases of FR=25% and 30% lower than the first natural frequency. The other conditions are kept the same as the reference case.

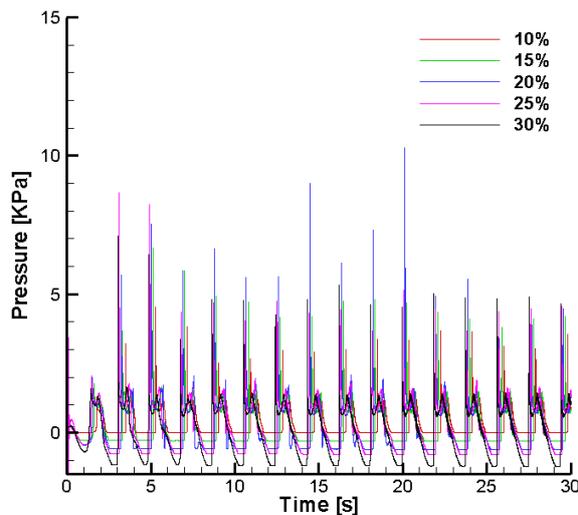


Fig. 5 Predicted time-variations of pressure

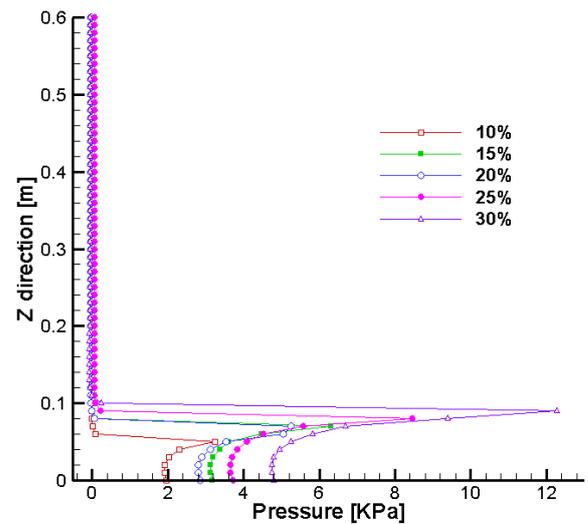


Fig. 6 Pressure distribution on side wall

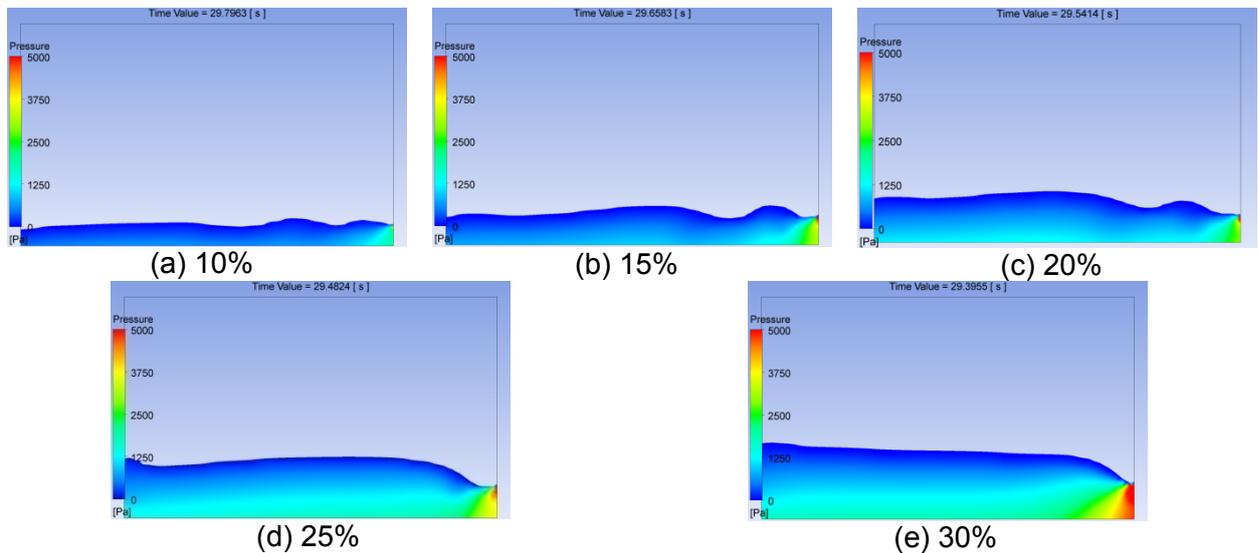


Fig. 7 Snap-shot of free surface motion and iso-contours of pressure of sloshing water at maximum pressure time

Fig. 5 show the time variation of pressure at the point of  $z=0.06\text{m}$  for each case. Because of different filling ratio, the time-position of high pressure peak occurring is different for each case. Fig. 6 shows the pressure distribution along the side wall of the tank. The magnitude of highest peak pressure and its position ( $z$ -coordinate) increases as the filling level increases. However, the position of highest peak pressure is lower than the initial free surface location for all of cases. Fig. 7 shows the snap-shot of free surface motion and the iso-contours of pressure in water at the time when the maximum pressure occurs.

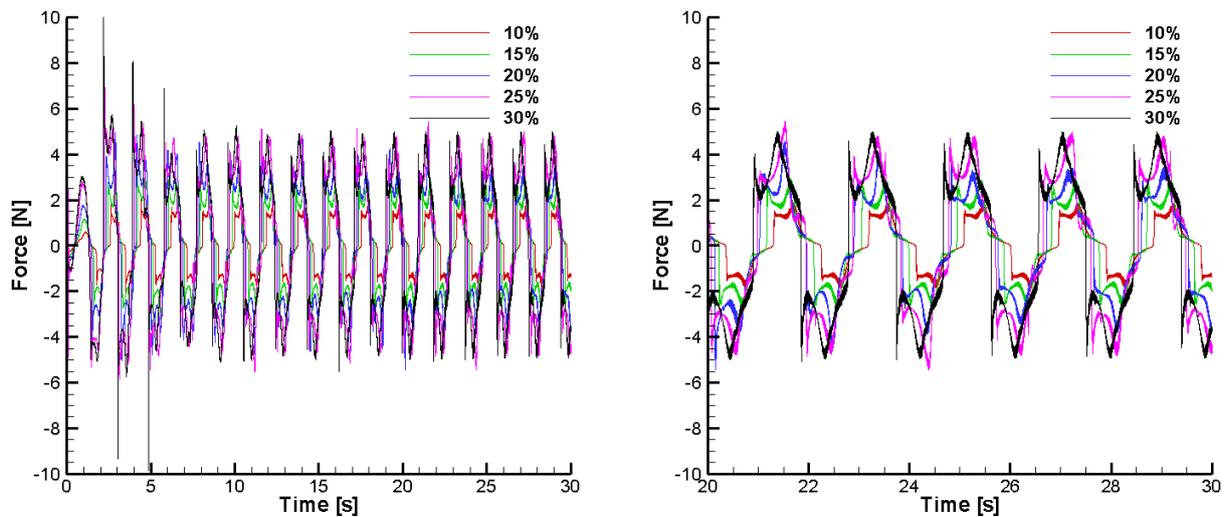


Fig. 8 Predicted time-variations of force

Fig. 8 compares the force variations between the cases. The high filling ratio of water induces high reaction force. As the filling ratio increases, the time at which the high peak occurs becomes earlier, though the period is the same as in each case. This means that the phase as well as the magnitude of response force changes as the filling ratio increase. Interestingly, there is no observed specific phenomenon related to the resonance.

## 5. CONCLUSION

Numerical parametric study was performed to investigate the effect of the displacement amplitude of excitation and filling ratio of water on sloshing phenomena of liquid inside a rectangular tank. The numerical method was based on volume of fluid method implemented in a CFD code solving RANS equation. The amplitude of displacement was varied from 0.025 m to 0.125 m by the step 0.025 m. The predicted peak pressure distribution showed that the highest peak value of pressure occurred in the case of the amplitude, 0.075 m. However, the force exerted by water on the tank increased as the excitation amplitude increase and was maximum in the case of the excitation amplitude 0.125 m. However, the pattern of time-variation of force had similarity between the cases of different excitation amplitudes. The filling ratio of water was varied from 10 % to 30 % of the tank height by the step size 5 %. There seemed to be positive correlation between the filling ratio, and the peak pressure/force of sloshing water. Although the resonance is theoretically predicted to occur in the case of 20 % filling ratio, there are no observed phenomena related to the resonance.

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