Nonlinear Soil – Structure Interaction Analysis by Numerical Integration of Halfspace

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ABSTRACT

The original and universal method of contact stress solution by means of Jacobian transformation and numerical integration is derived. The stress state and settlement of halfspace can be calculated for arbitrary shape and plate contact stress. This is allowed by using 4- and 8- nodal points of isoparametric contact elements with a general nodal load. Numerical integration is executed for arbitrary number of integration points by means of Gauss quadrature formulas. The limit depth and settlement is defined for each nodal point of isoparametric plate element and the contact function of element subsoil is calculated. For the stress-strain plate analysis is used the finite element method (FEM) and Mindlin's plate theory with shear is taken into account. This non-linear interaction problem is solved by iteration method for desired precision of solution or number of iteration steps.

1. INTRODUCTION

The soil-structure interaction influences not only settlement and stress of foundation, but also the others bearing elements of building structures. The objective of this paper is also the results of improving and developing the numerical methods of soil-structure analysis based on finite element method (FEM). The base of proposal subsoil model is numerical integration of elastic halfspace, which is loaded by arbitrary shape of loaded area (Cajka, 2001). This is possible by using of Jacobean of transformation and isoparametric contact plate elements. The numerical integration and solving of large non-linear equations by means of iteration methods don't make any problem with sharp growth of computer techniques (Konecny, Brozovsky, Krivy, 2009). This proposal original solution was successfully used for various tasks in structure practice.

2. SURFACE SOIL – STRUCTURE MODELS

Dealing with tasks relating to interactions among various types of environments have been under development for many years. To provide a more realistic description
of the state of stress, this being in particular the case of foundation structures, it is essential to define in what way foundation structure stiffness influences the resulting settlement and vice versa, and in what ways stiffness or pliability of soil affects internal forces within the structure. Solutions presented by Gorbunov - Possadov, Winkler and Pasternak rank among first works published in this subject of study, see Kolář and Němec, (1989)

Numerical methods have become popular in praxis thanks to introduction and development of information technologies. The Finite Element Method (FEM) being an all-purpose variation method used when analysing civil structures has been studied in depth, as a matter of fact. Many authors have paid attention to surface models in the Czech Republic, for example best-known soil multi-parameter model prepared by Kolář and Němec (1989) and others modified soil – structure interaction approaches (Kralik and Jendzelovsky 1993, Fajman and Sejnoha 2007, Kuklik 2011).

The suggested solution of interaction aims to make use of advantages included in all models presented up until now, eliminating at the same time their deficiencies. What is entirely new and original is the presented solution used for calculation of contact stress and settlement of soil on the surface of a modified halfspace for an arbitrary shape and load course. This solution has made use of isoparametric elements and numeric integration (Cajka 2001, 2002, 2003 and 2005, Cajka and Manasek, 2005).

3. ISOPARAMETRIC ELEMENTS

In addition to the x,y Cartesian Co-ordinate Grid, dimensionless co-ordinates - $ξ, η$ have been introduced. Our requirement is that the mentioned co-ordinates should be of prescribed unit values in nodal points. Following formulae are used to describe a mutual dependency between the two co-ordinate grids (Ahmad, Irons and Zienkiewicz, 1970):

$$
x = x(ξ, η) = \sum_{i=1}^{r} N_i(ξ, η).x_i \quad y = y(ξ, η) = \sum_{i=1}^{r} N_i(ξ, η).y_i \tag{1}
$$

The sum of the $r$ upper limits depends on a number of nodal points in the element.

For instance, functions describing the shape of a 4-node element are as follows:

$$
N_1(ξ, η) = 0.25 \cdot (1 - ξ + η + ξ η) \\
N_2(ξ, η) = 0.25 \cdot (1 + ξ - η - ξ η) \\
N_3(ξ, η) = 0.25 \cdot (1 + ξ + η - ξ η) \\
N_4(ξ, η) = 0.25 \cdot (1 - ξ + η - ξ η) \tag{2}
$$

Functions describing the shape of an 8-node element can be formed in an identical way. An advantage of isoparametric elements consists in their ability of being transformed from a unit square element into any other shape such as a triangle, quadrangle or approximation of a circle (Cajka 2001, 2002 and 2003).
4. MODEL OF FOUNDATION PLATE

A significant attention needs to be paid to a plate element used in the FEM for the soil-foundation system. Isometric plate elements are advisable to be used to depict a curvilinear edge. In case of massive foundation structures, the Mindlin’s theory of thick plates and shear impact (Mindlin, 1951) is preferred to the Kirchhoff’s thin plate theory. The detailed Mindlin’s theory assumes that points creating a normal to a central line plane shall be placed after deformation again on a line, not being however a normal to a bending plane. Therefore, three variables are typical of each node: w flexure and \( \vartheta_x \), \( \vartheta_y \) torsional displacements. Displacement deformations \( (u(\xi,\eta), v(\xi,\eta), w(\xi,\eta), \vartheta_x(\xi,\eta), \vartheta_y(\xi,\eta)) \) expressed as dimensionless co-ordinates \( (\xi,\eta) \) are functions of a similar type as geometric transformations. This means

\[
\begin{align*}
  u(\xi,\eta) &= \sum_{i=1}^{4} N_i(\xi,\eta) \cdot u_i, \quad v(\xi,\eta) = \sum_{i=1}^{4} N_i(\xi,\eta) \cdot v_i \\
  w(\xi,\eta) &= \sum_{i=1}^{4} N_i(\xi,\eta) \cdot w_i, \quad \vartheta_x(\xi,\eta) = \sum_{i=1}^{4} N_i(\xi,\eta) \cdot \vartheta_{xi}, \quad \vartheta_y(\xi,\eta) = \sum_{i=1}^{4} N_i(\xi,\eta) \cdot \vartheta_{yi} 
\end{align*}
\]

(3a)
ui, vi, wi, $\vartheta_x i$ and $\vartheta_y i$, being components in the general node displacement. Since a number of x (and, possibly, y) and w (or u, v, $\vartheta_x$, or $\vartheta_y$ as the case may be) is the same, the elements concerned are referred to as isoparametric elements.

To define an element stiffness matrix, it is essential to describe a deformation $\{\varepsilon\}$ array. Therefore, it is necessary to know partial derivation of displacement components, $u(\xi, \eta)$, $v(\xi, \eta)$, $w(\xi, \eta)$, $\vartheta_x(\xi, \eta)$ a $\vartheta_y(\xi, \eta)$, with respect to $\xi$ and $\eta$ Cartesian co-ordinates. Complying with rules applicable to derivatives of composite functions, for example $w(\xi, \eta)$ derivations can be described as follows:

$$
\left\{ \frac{\partial w}{\partial \xi} \right\} = \left[ \frac{\partial x}{\partial \xi} \frac{\partial y}{\partial \xi} \right] \cdot \left\{ \frac{\partial w}{\partial \xi} \right\} = [J] \cdot \left\{ \frac{\partial w}{\partial \xi} \right\}
$$

(4a)

Expressions for $\vartheta_x(\xi, \eta)$ and $\vartheta_y(\xi, \eta)$ derivations can be formed in an analogous way. $[J]$ stands for the Jacobi functional matrix. Since the determinant of the matrix – the transformation jacobian

$$
\det[J] = |\vartheta(x, y) / \vartheta(\xi, \eta)|
$$

(4b)

is not equal to zero, there must be such an inversion matrix, $[J]^{-1}$, that the formula below could hold good:

$$
\left\{ \frac{\partial w}{\partial \xi} \right\} = [J]^{-1} \cdot \left\{ \frac{\partial w}{\partial \xi} \right\} = [J]^{-1} \cdot \left\{ \sum_{i=1}^{N} \frac{\partial N_i}{\partial \xi} \cdot w_i \right\}
$$

(5)

Consequently, a stiffness matrix for the isoparametric elements has been derived step by step in accordance with the isoparametric element theory (Ahmad, Irons and Zienkiewicz, 1970):

$$
[K_c] = \int\int_{-1}^{1} [G]^{T} \cdot [D] \cdot [G] \cdot \det[J] \cdot d\xi \cdot d\eta
$$

(6)

In order to calculate the above mentioned integral, Gauss quadrature formulae are the best choice.

$$
[K_c] = \sum_{p=1}^{n} \sum_{q=1}^{n} \alpha_p \cdot \alpha_q \cdot [G(\xi_p, \eta_q)]^{T} \cdot [D] \cdot [G(\xi_p, \eta_q)] \cdot \det[J(\xi_p, \eta_q)]
$$

(7)
$\alpha_p$ and $\alpha_q$ being weight coefficients resulting from the n order of the Gauss quadrature.

5. CONTACT SUBSOIL MODEL

A relation describing the size of the vertical component of the state-of-stress sensor in the elastic halfspace \{\sigma\} depending on impacts from the general continuous load, \(p_z(x,y)\), can be expressed similarly as for remaining five components as follows

\[
\sigma_z = \int \frac{p_z}{2 \cdot \pi} \frac{3 \cdot z^3}{r^5} \cdot dA 
\]  

(8a)

\[
r^2 = x^2 + y^2 + z^2 
\]  

(8b)

The similar equation of the state-of-stress sensor in the elastic halfspace \{\sigma\} are wellknown for horizontal continuos load \(p_x(x,y)\) and \(p_y(x,y)\), see (Florin 1959, Poulos and Davis 1974).

When integrating in practice, mathematical difficulties are faced even in simplest cases. We succeeded in solving the integral above for certain verticals only (verticals passing through the centre, edge and corner point of the rectangular areas and verticals loaded with the constant/linear load at most). When dealing with the verticals in general positions, stress in rectangular areas can be defined only by superposing simple load patterns (positive as well as negative ones). Some authors mention expressions derived for other load courses, for instance (Florin 1959, Poulos and Davis 1974, Kolář and Němec 1989). No resources available mention an explicit expression of stress components applicable to the general load defined by four different intensities in rectangle corners.

Fig. 2 Vertical Contact Stress Course in Element
Another possibility to calculate the integration is to use a numerical method. Taking into account accuracy and a number of necessary integration points, Gauss quadrature formulae seem to be the best solution. If we know sizes of contact stress in element nodal points, the stress course within the element can be approximated by following shape functions (Cajka 2004a, 2004b).

\[
p_z(x, y) \equiv \tau_{ci}(x, y) = \sum_{i=1}^{r} N_i(x, y) \cdot \tau_{ci}, \quad p_y(x, y) \equiv \tau_{cy}(x, y) = \sum_{i=1}^{r} N_i(x, y) \cdot \tau_{cyi}
\]

\[
p_z(x, y) \equiv \sigma_z(x, y) = \sum_{i=1}^{r} N_i(x, y) \cdot \sigma_{zi}
\]

(9a)

(9b)

The surface integral can be calculated by means of the mentioned numerical calculation without any major difficulties just for a rectangular loading surface. Should other shapes appear (as a general triangle or quadrangle being widely used in FEM), the calculation entails difficulties resulting from variable integration limits.

A solution eliminating the mentioned difficulties is the transformation jacobian describing transformation relations. In such a way it is possible to determine any state-of-stress component in a homogenous elastic halfspace for any load surface and any load course. The solution suggested eliminates complications arisen up until now when trying to apply a soil standard model in FEM interaction tasks (Cajka 2003a, 2003b).

\[
\sigma_z = \int\int_{-1}^{1} p_z(\xi, \eta) \cdot \frac{3 \cdot z^3}{r^5} \cdot \det[J] \cdot d\xi \cdot d\eta
\]

(10)

When calculating the stress components by integrating the Gauss quadrature formulae numerically (Davis and Rabinowitz 1956), the integral is transferred into a double summation, \( \xi \) and \( \eta \) points being replaced with \( \xi_p \) and \( \eta_q \) integration points (Cajka 2001, 2002, 2003b):

\[
\sigma_z = \sum_{p=1}^{n} \sum_{q=1}^{n} \alpha_p \cdot \alpha_q \cdot p_z(\xi_p, \eta_q) \cdot \frac{3 \cdot z^3}{2 \cdot r^5} \cdot \det[J(\xi_p, \eta_q)]
\]

(11)

In order to verify suitability of numeric integration for practical calculation of stress components caused by general loads from the halfspace surface, check examples have been calculated and results obtained have been compared with the known solution. Accuracy of \( \sigma_z \) stress as calculated has been checked for a various number of integration points and various \( z \) depths under the corner of a square, triangle and circular area of an elastic halfspace subject to an even load. Furthermore, the jacobian transformation has been verified (Cajka 2001, 2002, 2003b).
When calculating settlement with respect to structure strength of earth pursuant to the ČSN 73 1001 (1988) or ČSN EC 1997-1 (2006) standard, a definite integral in the \(<0,z_z>\) interval can be used as follows:

\[
\int z_0 z_{oz} \frac{\sigma_z(x,y,z)}{E_{eod}(x,y,z)} \cdot dz = \int z_0 z_{oz} \frac{m(x,y,z) \cdot \sigma_{or}(x,y,z)}{E_{eod}(x,y,z)} \cdot dz = s_{z,al} - s_{z,or} \tag{12}
\]

\(z_z(x,y)\) deformation zone depths need to be defined considering the condition that the \(\sigma_z(x,y,z_z)\) resultant vertical stress on the lower edge of the deformation zone is zero. This means

\[
\sigma_z(x,y,z_z) = \sigma_{al}(x,y,z_z) - m(x,y) \cdot \sigma_{or}(x,y,z_z) = 0 \tag{13}
\]

The non-lineal equation above is to be solved numerically. The interval halving method seems to be appropriate in this case.
The structural strength of the soil, similarly as the vertical stress (Cajka 2004a) reduces also the horizontal deformation caused by the horizontal stress. According to (Kos 1988, Reiser and Zeman 2001) the influence of the structural strength is solved using the soil pressure theory where the strength equals to the passive rest pressure, this means to the passive soil pressure that is proportional to a very small u,v-displacement that can be neglected:

$$\sigma_{str} = \sigma_{or} \cdot \frac{k_{po}}{2}$$

(14)

Where $\sigma_{str}$ is the structural strength of the soil in the horizontal X or Y direction (axis) $\sigma_{or}$ geostatic stress $k_{po}$ coefficient of the passive pressure at rest

The coefficient of the passive pressure at rest is the difference between the coefficient of the passive soil pressure at rest and soil pressure at rest (Reiser and Zeman 2001), this means:

$$k_{po} = k_{op} - k_o$$

(15)

where the coefficient of the passive soil pressure at rest, see ČSN 73 0037 (1990), is:

$$k_{op} = \frac{1 + \sin \phi}{1 - \sin \phi}$$

(16)

and the coefficient of the soil pressure at rest is:
\[ k_o = 1 - \sin \phi \quad (17) \]

In order to calculate horizontal components of deformations, it is possible (similarly as for the vertical deformation, \textit{Cajka 2011, 2002}) to consider the effective component of the horizontal stress that results in the deformation. That component is determined on the basis of the original horizontal stress in an elastic half space caused by effects of the horizontal forces \( \sigma_{x,ol} \) decreased by a corresponding stress component caused by the structural strength of the soil \( \sigma_{x,str} \) (Kos 1988, Reiser and Zeman 2001).

The horizontal deformation can be calculated with a better accuracy using the modified relation:

\[
s_x = \frac{1}{E_{oed}} \int_{0}^{y} (\sigma_{x,ol} - \sigma_{x,str}) \, dx, \quad s_y = \frac{1}{E_{oed}} \int_{0}^{y} (\sigma_{y,ol} - \sigma_{y,str}) \, dx \quad (18)
\]

![SETTLEMENT OF HALFSPACE \( s(x,y) \) ITERATION=0 INTPPOINTS=12](image)

**Fig. 5** Settlement Course \( s(x,y) \) of Halfspace under Circular Plate with 72 Elements, 0th Iteration with 12 Integration Points

The isoparametric element soil stiffness matrix has been derived by using an identical approach as in the case of the element stiffness matrix.

\[
[K_p] = \sum_{p=1}^{n} \sum_{q=1}^{m} \alpha_p \cdot \alpha_q \cdot \left[ N(\xi_p, \eta_q) \right]^T \cdot \left[ C(\xi_p, \eta_q) \right] \cdot \left[ N(\xi_p, \eta_q) \right] \cdot \det[J(\xi_p, \eta_q)] \quad (19)
\]
For the plane element, the matrix of contact functions \([C]\) can be expressed pursuant to (Cajka 2004a, Cajka and Sekanina 2007b) as:

\[
[C] = \begin{bmatrix}
C_{ix} & 0 & 0 \\
0 & C_{iy} & 0 \\
0 & 0 & C_{iz}
\end{bmatrix}
\]  

(20)

Where

\[
C_{ix} = \sum_{i=1}^{r} N_i(\xi, \eta) C_{ix,i}, \quad C_{iy} = \sum_{i=1}^{r} N_i(\xi, \eta) C_{iy,i}, \quad C_{iz} = \sum_{i=1}^{r} N_i(\xi, \eta) C_{iz,i}
\]  

(21)

![Fig. 6 Settlement Course \(s(x,y)\) of Halfspace under Circular Plate with 72 Elements, 12th Iteration with 12 Integration Points](image)

The friction parameters \(C_{1x}, C_{1y}\), and stress parameter \(C_{1z}\), represent the contact functions that can be, similarly as the contact stress and shear stresses in the isoparametric plane element, approximated using the shape functions \(N_i\) and functional values \(C_{1z,i}, C_{1x,i}\) or \(C_{1y,i}\) in individual nodes (Cajka 2001, 2002, 2004a, 2004b).
The values of the contact functions can be determined for the individual node elements in FEM as the stress or share in the footing bottom and deformation of the node.

\[
C_{1x,j} = \frac{\tau_{xj}}{s_{xi}}, \quad C_{1y,j} = \frac{\tau_{yj}}{s_{yi}}, \quad C_{1z,j} = \frac{\sigma_{zj}}{s_{zi}}
\]  

When calculating an interaction task, it is essential to define such course of the contact stress that could result in the same deformation of the soil and the plate. Since the relation between the modified elastic halfspace load and modified elastic halfspace settlement is that of non-linearity, FEM non-linear methods need to be employed to solve the mentioned task. The iteration method seems to be the best choice in this case, since it converges towards a technically accurate solution as early as after approximately 7-8 iteration steps (Cajka, 2001, 2002). An advantage of the suggested
iteration solution used to solve the non-linear task consists in a possibility of checking the plate structure stiffness with respect of the cracking limit (Pukl et al. 2006, Sucharda and Brozovsky 2009). The method also enables the limit contact stress to be checked in the footing bottom, eliminating thus tensile stress between the plate and soil (so-called one-sided bonds).

6. CREATION OF PLASTIC AREAS IN SUBSOIL

Should contract stress under foundation corners exceed the plastic area creation limit in the soil, stress in the footing bottom shall redistribute, deformation in the foundation shall change and internal forces in the foundation structure shall redistribute too.

\[
R_m = R_{ecr} = \gamma d \left( \frac{1 + \sin \phi_{ef}}{1 - \sin \phi_{ef}} \right) \left( 1 - \sin \phi_{ef} \right) - \left( 1 - \sin \phi_{ef} \right) \left( 1 + \sin \phi_{ef} \right)
\]

where \( \gamma \) being volume mass, \( \phi_{ef} \) being an internal friction angle, \( c_{ef} \) being soil cohesion in the soil and \( d \) being a foundation depth.

Fig. 8 Contract stress under foundation and plastic area creation

To define a critical stress at which plastic areas arise, an approach suggested by (Gorbunov – Possadov 1953) can be employed.
7. BOTTOM PLATE OF SPREAD FOOTING – EXAMPLE

In order to illustrate the influence of the ground water level on the changes in the subsidence and distribution of internal force, let’s deal with a segment of the foundation slab of the spread box-line foundation (dimensions: 8.0 x 2.0 m, slab height: 0.5 m). The foundation slab made from the C16/20 concrete is founded in the depth of 1.0 m under the original ground on the loamy subsoil from the F5 class soil. The foundation slab is loaded at edges by two single loads 2,000 kN representing the response of the foundation piers and upper structure.

The interaction task is solved pursuant to (Cajka 2005, Cajka and Manasek (2005) using the iteration for the average contact pressure under foundations \(\sigma_{ol} = 250\) kPa with \(E_{\text{def}} = 7.0\) MPa and \(v = 0.4, \gamma = 20\) kN.m\(^{-3}\), and structural strength coefficient \(m = 0.2\). For average internal friction angle \(\varphi_{\text{ef}} = 22^\circ\) and \(c_{\text{ef}} = 16\) kPa. The limit critical pressure where plastic areas start (Gorbunov – Possadov 1953 ) appearing is \(R_{\text{ccr}} = 426.5\) kPa.

![Deformation of Foundation Slab Corner](image)

Fig. 9 Influence of Ground Water Level and Plastification of Subsoil on Deformations

Fig. 9 and 10 show the results of the deformation and flexural moments of the strip foundation and influence of different assumptions of the calculation (the ground water level – GWL, plastification of the subsoil). Below are design situations which have been solved:

- solution without the influence of creation of plastic areas and ground water level
- solution without the influence of creation of plastic areas and with the influence of the ground water level in the foundation
- solution with the influence of creation of plastic areas under foundations and without the influence of the ground water level
- solution with the influence of creation of plastic areas and with the influence of the ground water level in the foundation

The influence of individual factors (plastification of the subsoil and ground water level) on the state of stress and deformation of the foundation structure depends on the cross-section and quantity under study. While the deformation and lower rigidity of the subsoil under the foundation wall is considerably influenced by the higher ground water level (see Fig 9), the increase in the flexural moments in the middle of the slab is mostly influenced by impacts of the plastic areas in the foundation under the slab edge (see Fig 10).

![Bending Moments inside Slab](image)

**Fig. 10 Influence of Ground Water Level and Plastification of Subsoil on Bending Moments**

It follows from this task that the flooding of the area results in lower reliability of the load-carrying capacity of the upper structure of the carrying construction (the edges of the foundation subside under the wall, resulting in the additional flexural moments). The flooding also decreases the reliability of the foundation structure (increase in flexural moments by ca. 16 %) because of the creation of plastic areas in the foundation soil. In order to determine the influence of the factors above on the general reliability of the interaction system (subsoil / foundation / carrying structure), it would be essential to solve the general reliability tasks including the influence of other random variable parameters.
8. CONCLUSION

The suggested calculation approach enables a basic condition of soil-foundation interaction to be fulfilled, choosing at the same time accuracy required. This means, a technically acceptable equality can be achieved for (w) plate deformation and (s) modified elastic halfspace settlement in accordance with ČSN 73 1001 (1988) standard and ČSN EN 1992-1-1 (2006) resp. ČSN EN 1997-1 (2006). The calculation model does not require any extra input data about the soil. Just data available within a usual geology survey are needed.

An original solution is also represented by transformation jacobian and numerical integration used for purposes of calculation of an arbitrary state-of-stress component in an homogenous elastic halfspace subject to any load surface and any load course represent also. The mentioned solution eliminates difficulties encountered up until now, when trying to apply a soil standard model ČSN 73 1001 (1988) resp. ČSN EN 1997-1 (2006) in FEM interaction tasks. While the deformation and lower rigidity of the subsoil under the foundation is considerably influenced by the higher ground water level, the change in the flexural moments in the middle of the slab is mostly influenced by impacts of the plastic areas in the foundation under the slab edge.

This article analyses certain problems connected with the solution of interaction of the buildings and subsoil on the flooded area (Cajka 2005, Cajka and Manasek 2005). Reasons are analysed for different subsidence of the buildings because of the changes in the mechanical and physical properties of the soil, ground water level, and washing of fine grain particles from the subsoil. This enhances the importance of the solution of the interaction methods with the influence of the unilateral links, loss of contacts with the subsoil, lower rigidity of the structures, and creation of plastic areas under the edge of the foundation, when calculating the contact pressures depending on the mechanical and physical properties of the subsoil.

In near future there are plans about including isoparametric semispace contact element into computer system, see (Sucharda and Brozovsky 2009) with taking concrete nonlinearity into account. Next potential step for utilization in application could be analysis of interaction between prestressed foundations and subsoil, see (Sekanina 2008), soil-structure interaction on undermined territory (ČSN 73 0039 1989, Kalab and Lednicka 2012) with sliding joints taking into account (Cajka and Manasek 2006, 2007, Cajka et. al 2011) and simulation based reliability assessment method (Janas and Krejsa 2009) together with using parallel computing (Konecny et. al 2009, 2010).

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