

## Flutter Stability Analysis of Long Span Bridge Subjected to Wind Load by Non-White Noise Process

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### ABSTRACT

This paper describes a stochastic method of flutter stability analysis for long span bridges subjected to random wind load based on the theory of Markov processes. Then, wind load is modeled as the solution of Ito type of differential equation. Formulation for the flutter stability analysis is developed using this model. Numerical studies are carried out for rectangular cylinder as a bridge deck with  $B/D = 7.5$ . Also, in this analysis, the flutter derivatives measured in smooth and turbulent flows were applied. In this case, the torsional flutter is more stabilized in turbulent flows than in smooth flows as shown on the change sign of  $A_2^*$ . However, the flutter stability analysis shows that the flutter critical velocity (i.e., 2-nd moment) is smaller in turbulent flow by the turbulent characteristics (e.g.,  $I_u$ : Intensity of turbulence and that of scale) than in smooth flow.

### 1. INTRODUCTION

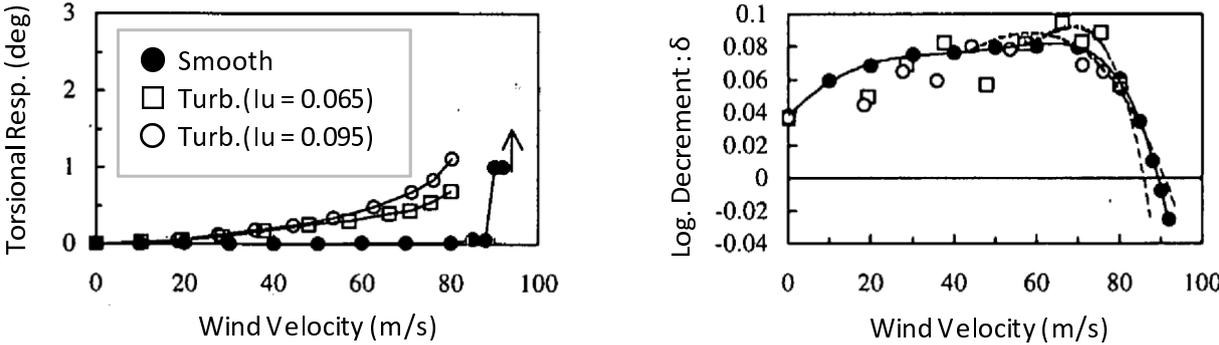
Natural gusty wind acting on a structure fluctuates randomly in time and space. The fluctuating wind pressure due to the turbulent wind (i.e. gusty wind) is a stochastic process. In general, a rigid structure has sufficient safety against the dynamic wind pressure. On the other hand, a flexible structure needs the checking of the structural safety of excitation by the gusty wind. For estimating the safety of flexible structures, dynamic response analysis considering the fluctuating aerodynamic forces is necessary.

To carry out dynamic response analysis for random vibration due to the fluctuating aerodynamic force, spectral analysis and Markov process have been applied. Davenport (1962) invented the former method. Nowadays it is called the classical spectral analysis

of random vibration theory. Scanlan (1978) exploited the dynamic equation with including not only buffeting loads but also self-excited loads. Lin (1979) developed the latter method by the assumption that random disturbances and dynamic responses can be approximated as the Markov process.

Especially, when considering complex engineering problems such as dynamic stability or nonlinear problems, the numerical method is very effective. In addition, although this numerical method is approximation algorithms, the numerical results can be obtained with good accuracy, or the same as the exact solution.

By the way, the dynamic stability of a flexible structure like the suspension bridge can be usually estimated by complex eigen-value analysis of the vibration model acting on motion induced aerodynamic forces. The motion induced aerodynamic forces are experimentally identified by wind tunnel tests in a smooth or a turbulent flow. The motion induced aerodynamic forces are formulated by Scanlan (1978) (e.g.  $H_i^*, A_i^*$ ). In this numerical analysis, the dynamic stability or instability (i.e. flutter onset velocity) can be uniquely estimated without considering the random disturbance by the turbulent wind gust. However, as shown in the experimental results (see Fig.1) by Katsuchi (1999), the comparative study by wind tunnel tests in smooth and turbulent flows have shown the difference of the flutter onset velocity between in a smooth flow and in a turbulent flow. The dynamic stability with considering the random disturbance by the turbulent wind gust has been studied. For convenience, in these studies, the random disturbance was modeled by white noise. For more practical purpose, the actual random disturbance should be modeled by non-white noise process.



(\*) The cross sectional shape is truss stiffening girder with center stabilizer.

Fig. 1 Wind tunnel results in smooth and turbulent flows by Katsuchi(1999)

Therefore, this paper describes a stochastic method of flutter stability analysis of long span bridges subjected to random wind load modeled by non white noise process.

Then, wind load is modeled as a solution of first-order linear stochastic differential equation. Formulations for the flutter stability analysis are developed using this model. Numerical studies are carried out using a rectangular deck with  $B/D = 7.5$ . The flutter derivatives of torsional motion were measured in smooth and turbulent flows by us. Using these data, flutter stability analysis in smooth and turbulent flows were carried out and compared with each other.

## 2. PROCEDURE OF DYNAMIC STABILITY ANALYSIS

### 2.1 Definition of Stability and Instability

Definition of stable or unstable for dynamic stability analysis will be shown in this section. A conventional way to solve the elastic stability problem is what is called a dynamic method. By the method, a disturbance gives the structural system with a stable equilibrium position in an initial state. The stability is estimated by studying the vibration around the equilibrium position. For the judgment of stability or instability, the transition from stability to instability can be classified into two types. One is based on the instability motion which was disturbed to increase monotonically as time goes by. It is called the static instability of divergent type (i.e. static divergence). The other one is based on the dynamic instability motion which vibrates with the divergent amplitude. It is called the dynamic instability of flutter type.

As phenomenon of dynamic instability occurred in a suspension bridge, galloping in bending mode, torsional flutter and coupled flutter in bending and torsional modes are listed. Now, to define mathematically the state of stability or instability is difficult in general. Among the judgment methods of stability or instability, the definition proposed by Lyapunov is excellent. His definition agrees to the intuitive concept based on experiences. And the definition can also be applied to many engineering problems. Therefore, in dynamic stability, the Lyapunov's definition on the structural system with a stable equilibrium position was extended to the definition on dynamic stability with stochastic process. Sunahara (1983) gives following three definitions on probabilistic convergence.

- a) stability on probability 1
- b) stochastic stability
- c) stability of m-th moment

In this paper, the third item: c) (i.e. m-th moment) was applied following Lin(1979) and Tsiatas (1986).

## 2.2 Equations of Motion

In this paper, we carry out dynamic stability analysis of torsional flutter of a long span bridge using a simple model of single degree of freedom for torsional motions.  $\alpha$  is the torsional motion due to random wind velocities of horizontal and vertical components acting on a bridge deck. Then, equations of motion of a bridge deck are given by using modal analysis as follows.

$$I_s(\ddot{\alpha} + 2\zeta_\alpha\omega_\alpha\dot{\alpha} + \omega_\alpha^2\alpha) = M_{\alpha s} + M_{\alpha B} \quad (1)$$

where  $I_s$  : Inertia moment per unit length

$M_{\alpha s}$  : Self excited aerodynamic force

$M_{\alpha B}$  : Buffeting force

$$M_{\alpha s} = \frac{1}{2}\rho u^2(2B^2)\left(KA_2^*\frac{B\dot{\alpha}}{u} + K^2A_3^*\alpha\right) \quad (2)$$

$$M_{\alpha B} = \frac{1}{2}\rho u^2B^2C_M + \frac{1}{2}\rho B^2\bar{U}\bar{\eta}\frac{dC_M}{d\alpha} \quad (3)$$

$$\left. \begin{aligned} u(x, t) &= \bar{U}[1 + \bar{\xi}(x, t)] \\ v(x, t) &= \bar{U}\bar{\eta}(x, t) \end{aligned} \right\} \quad (4)$$

where  $\bar{\xi}(x, t), \bar{\eta}(x, t)$ : Non-dimensional random function (wide band weak steady-process),  $u(x, t)$  and  $v(x, t)$  are respectively horizontal and vertical gust components.

$\bar{U}$  : Mean wind velocity along bridge span

$$u^2(x, t) \doteq \bar{U}^2[1 + 2\bar{\xi}(x, t)] \quad (5)$$

Then, modal analysis method is applied by the following expressions:

$$\alpha(x, t) = \phi(x)q(t) \quad (6)$$

where  $\phi(x)$  : eigen-mode function

$q(t)$  : generalized coordinates

$$\begin{aligned} I(\ddot{q} + 2\zeta_\alpha\omega_\alpha\dot{q} + \omega_\alpha^2q) &= \rho B^2KA_2^*\dot{q}\int_0^L u(x, t)\phi^2(x)dx + \rho B^2K^2A_3^*q\int_0^L u^2(x, t)\phi^2(x)dx \\ &+ \frac{1}{2}\rho B^2C_M\int_0^L u^2(x, t)\phi^2(x)dx + \frac{1}{2}\rho B^2\bar{U}^2\frac{dC_M}{d\alpha}\int_0^L \bar{\eta}(x, t)\phi(x)dx \end{aligned} \quad (7)$$

where  $I = \int_0^L I_x(x)\phi^2(x)dx$

Also, for simplicity of the above expression, the following expressions are introduced.

$$\left. \begin{aligned}
 a_1 &= \frac{\rho B^2 K A_2^*}{I} \bar{U} \int_0^L \phi^2(x) dx \\
 a_2 &= \frac{\rho B^2 K^2 A_3^*}{I} \bar{U}^2 \int_0^L \phi^2(x) dx \\
 a_3 &= \frac{\rho B^2 C_M}{2I} \bar{U}^2 \int_0^L \phi(x) dx \\
 a_4 &= \frac{\rho B^2 \bar{U}^2}{2I} \frac{dC_M}{d\alpha} \\
 a_5 &= \frac{\int_0^L \phi(x) dx}{\int_0^L \phi^2(x) dx} \\
 \xi_1(t) &= \frac{\int_0^L \bar{\xi}(x,t)\phi^2(x) dx}{\int_0^L \phi^2(x) dx} \doteq \tilde{\xi}(t) \\
 \xi_2(t) &= \frac{\int_0^L \bar{\xi}(x,t)\phi(x) dx}{\int_0^L \phi^2(x) dx} \doteq a_5 \tilde{\xi}(t) \\
 \eta(t) &= \frac{\int_0^L \bar{\eta}(x,t)\phi(x) dx}{\int_0^L \phi^2(x) dx} \doteq a_5 \tilde{\eta}(t)
 \end{aligned} \right\} \quad (8a)-(8h)$$

By using Eq. (8a)–(8h), Eq. (7) is given as following expressions:

$$\ddot{q} + (2\zeta_\alpha \omega_\alpha - a_1)\dot{q} + (\omega_\alpha^2 - a_2)q = a_3 + a_1 \dot{q}\tilde{\xi} + 2a_2 q\tilde{\xi} + 2a_3 a_5 \tilde{\xi} + a_4 a_5 \tilde{\eta} \quad (9)$$

Moreover, by introducing the following definition (see Eqs.(10)), Eq. (11) is given by Eq. (9):

$$\left. \begin{aligned} b_1 &= 2\zeta_s \omega_s - a_1 \\ b_2 &= \omega_s^2 - a_2 \\ b_3 &= \frac{2a_2 a_3}{b_2} \\ p &= q - \frac{a_3}{b_2} \end{aligned} \right\} \quad (10)$$

$$\left. \begin{aligned} \ddot{p} + b_1 \dot{p} + b_2 p &= a_1 \dot{p} \tilde{\xi} + 2a_2 p \tilde{\xi} + 2a_3 a_5 \tilde{\xi} + a_4 a_5 \tilde{\eta} + b_3 \tilde{\xi} \\ &= (a_1 \dot{p} + 2a_2 p + 2a_3 a_5 + b_3) \tilde{\xi} + a_4 a_5 \tilde{\eta} \end{aligned} \right\} \quad (11)$$

Then, by introducing the state vector  $x_1 = p$ ,  $x_2 = \dot{p}$ , Eq. (11) becomes as follows.

$$\left. \begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -b_1 \dot{p} - b_2 p + (a_1 \dot{p} + 2a_2 p + 2a_3 a_5 + b_3) \tilde{\xi} + a_4 a_5 \tilde{\eta} \end{aligned} \right\} \quad (12)$$

If you carry out only flutter stability analysis, buffeting load (i.e. non-self-induced forces) in Eq. (12) can be discarded.

$$\left. \begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -b_1 \dot{p} - b_2 p + (a_1 \dot{p} + 2a_2 p) \tilde{\xi} \end{aligned} \right\} \quad (13)$$

### 2.3 Probabilistic Model of Wind Load

In this section, we describe the modeling of an actual wind load to apply to stochastic analysis method. To focus on flutter stability problem, only the wind load due to horizontal wind gust will be considered. As the matrix form of Eq.(13) is Itô stochastic differential equation,  $\tilde{\xi}$  will be thought to be white noise. However, the fluctuations of actual natural wind gust are not white noise. The power spectral density (i.e. PSD) of natural wind gust was modeled by Davenport and von Karman etc. In this study, von Karman's formula is applied. The method needs mean wind velocity, turbulent intensity and turbulent scale as the input parameters as shown below:

$$S_u(f) = \frac{\sigma_u^2}{f} \frac{4 \left( \frac{f L_x^u}{U} \right)}{\left[ 1 + 70.8 \left( \frac{f L_x^u}{U} \right)^2 \right]^{\frac{5}{6}}} \quad (14)$$

where  $\sigma_u$  : standard deviation of wind fluctuation of horizontal component (m/s)

$L_x^u$  : scale of turbulence for horizontal component (m)

$f$  : frequency (Hz)

The above PSD can be approximated by following the two sided PSD function  $S_Z(f)$  of the response of 1-st order linear filter.

$$\dot{Z} + \beta Z = \xi_Z(t) \quad (15)$$

$$S_Z(f) = \frac{S_0}{\lambda^2 + \beta^2} \quad (16)$$

where  $S_0$ : spectral density of white noise process and constant value

$\beta$ : constant value

$\lambda (= 2\pi f)$  : circular frequency (rad/s)

When  $S_u(0) \doteq S_0$ ,

$$S_0 = 0.226\pi^2 k \sigma_u^2 \frac{U}{L_u^x}$$

$$\beta_1 = \frac{2\pi \bar{U}}{8.414 L_u^x}$$

(17a & b)

Where  $k (= 1.34)$  : modification factor to make each variances of two PSD of Eqs.(14) and (16). Fig.2 shows the result of the approximation.

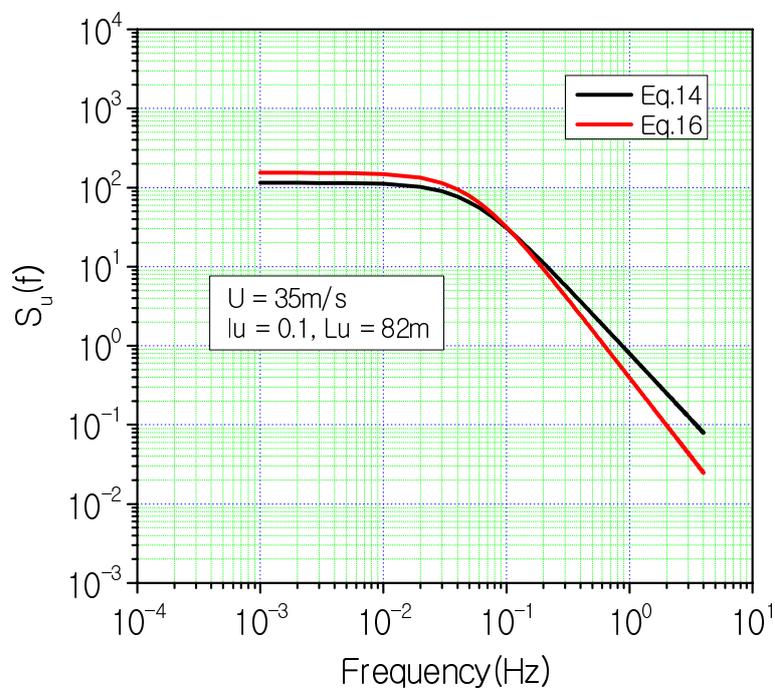


Fig. 2 Comparison of PSD by eqs. (14) and (16)

## 2.4 Flutter Stability with Proposed Probabilistic Model of Wind Load

As above mentioned, because the fluctuation of wind gust is wide band weak steady process, the dynamic responses of suspension bridge due to the fluctuation wind pressure can be approximated by Markov process. This study carries out the dynamic stability analysis of dynamic system given by Eqs. (13) and (15). The theory is based on Itô stochastic differential equation. Then, the dynamic stability analysis will be executed by moment equation defined in this chapter.

### a) Itô Stochastic Differential Equation and the Wang-Zakai Conversion Rule

The dynamic system given by differential equation of Eq.(13) and Eq.(15) can be rewritten by the matrix form with state vector  $\mathbf{X} = \{x_1, x_2, x_3\}^T$  ( $x_3 = Z_1$ ) as follows.

$$\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -b_2 & -b_1 & 0 \\ 0 & 0 & \beta_1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} + \begin{bmatrix} 0 \\ a_1 x_2 + 2a_2 x_1 \\ 1 \end{bmatrix} \tilde{\xi} \quad (18a)$$

$$\dot{\mathbf{X}} = \tilde{\mathbf{f}}(t, \mathbf{X}) + \tilde{\mathbf{G}}(t, \mathbf{X})\tilde{\xi}(t) \quad (18b)$$

Where  $\tilde{\xi}(t)$  is a white noise process with intensity  $Q_{\tilde{\xi}\tilde{\xi}}$ . Tsiasas (1986) gives  $Q_{\tilde{\xi}\tilde{\xi}}$  as the expression with the cross correlation between the two points ( $x_1, x_2$ ) along the bridge axis. For simplicity, in this paper,  $Q_{\tilde{\xi}\tilde{\xi}}$  is assumed to be the gusty wind with full correlation (i.e.  $c=0$ ).

$$Q_{\tilde{\xi}\tilde{\xi}} = \iint_0^L (S_0 \exp^{-c|x_1-x_2|/u}) \psi(x_1)\psi(x_2) dx_1 dx_2 \quad (19)$$

$$\text{where } \psi(x) = \frac{\phi^2(x)}{\int_0^L \phi^2(x) dx} \quad (20)$$

The solution of Eq. (18) is a Markov process whose behavior is studied using Itô stochastic differential equation.

$$d\mathbf{X} = \mathbf{f}(t, \mathbf{X})dt + \mathbf{G}(t, \mathbf{X})dw(t) \quad (21a)$$

$$\mathbf{G}(t, \mathbf{X}) = \sigma\{\tilde{\mathbf{G}}(t, \mathbf{X})\mathbf{X} + (0,0,1)^T\} \quad (21b)$$

Also,  $w(t)$  are called Brownian motion process. The following relationship is given as a formal notation.

$$\sigma dw(t) = \tilde{\xi}(t)dt, \quad \sigma^2 = S_0 \quad (22)$$

The transformation is made using the Wang-Zakai conversion rule. For the transformation, the correction factors  $0.5 \sum_{k=1}^3 G_k \left( \frac{\partial G_j}{\partial X_k} \right)$  are added to the  $j$  element of the vector  $\tilde{f}(t, \mathbf{X})$ . Then, the element of the vector  $\mathbf{f}$  are given as;

$$\left. \begin{aligned} f_1(t, \mathbf{X}) &= x_2 \\ f_2(t, \mathbf{X}) &= (-b_2 + a_1 a_2 Q_{\xi\xi})x_1 + \left(-b_1 + \frac{1}{2} a_1^2 Q_{\xi\xi}\right)x_2 \\ f_3(t, \mathbf{X}) &= \beta_1 \end{aligned} \right\} \quad (23a-c)$$

$$\begin{Bmatrix} f_1 \\ f_2 \\ f_3 \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -b_2 + a_1 a_2 Q_{\xi\xi} & -b_1 + \frac{1}{2} a_1^2 Q_{\xi\xi} & 0 \\ 0 & 0 & \beta_1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} \quad (23d)$$

#### b) Moment Equation

Using Itô's lemma, Tsiatas G. (1986) gives the following ordinary differential equation for the expectation of  $g(t, X)$ .

$$\frac{dE(g(t, X))}{dt} = E \left[ \frac{\partial g(t, X)}{\partial t} \right] + \sum_{i=1}^n E \left[ \frac{\partial g(t, X)}{\partial X_i} f_i(t, X) \right] + \frac{1}{2} \sum_{i,j=1}^n E \left[ \frac{\partial^2 g(t, X)}{\partial X_i \partial X_j} (G Q_{\xi\xi} G^T)_{i,j} \right] \quad (24)$$

We exploited following equations by letting  $g(t, X) = X_1, X_2, X_3$ , the governing equations for 1-st moment can be expressed as,

$$\dot{M}_1 = F M_1 \quad (25)$$

$$\left. \begin{aligned} M_1 &= \{m_{100}, m_{010}, m_{001}\}^T \\ F_{1,2} &= 1.0 \\ F_{2,1} &= -b_1 + a_1 a_2 Q_{\xi\xi} \quad , \quad F_{2,2} = -b_1 + \frac{1}{2} a_1^2 Q_{\xi\xi} \\ F_{3,3} &= -\beta_1 \end{aligned} \right\} \quad (26)$$

Equations for the second moments by using Eq. (24) with  $g(t, X) = X_1^2, X_2^2, X_3^2, X_1 X_2, X_1 X_3$  and  $X_2 X_3$  are given as;

$$\dot{M}_2 = A M_2 \quad (27)$$

$$\left. \begin{aligned}
M_2 &= \{m_{200}, m_{020}, m_{002}, m_{110}, m_{101}, m_{011}\}^T \\
A_{1,4} &= 2.0 \\
A_{2,1} &= 4a_2^2 Q_{\xi\xi} \quad , \quad A_{2,2} = -2(b_1 + a_1^2 Q_{\xi\xi}) \quad , \quad A_{2,4} = -4(b_2 + a_1 a_2 Q_{\xi\xi}) \\
A_{3,3} &= -2\beta_1 \\
A_{4,1} &= -b_2 + a_1 a_2 Q_{\xi\xi} \quad , \quad A_{4,2} = 1.0 \quad , \quad A_{4,4} = -b_1 + \frac{1}{2} a_1^2 Q_{\xi\xi} \\
A_{5,5} &= -\beta_1 \quad , \quad A_{5,6} = 1.0 \\
A_{6,5} &= -b_2 + a_1 a_2 Q_{\xi\xi} \quad , \quad A_{6,6} = -b_1 + \frac{1}{2} a_1^2 Q_{\xi\xi} - \beta_1
\end{aligned} \right\} (28)$$

### c) Judgments of Stable or Unstable

Based on the above preparation, the eigen-values of the matrix **F** (i.e. Eq. (26)) are solved. Then, when the eigen-values have one positive real part at least, the suspension bridge is judged to be unstable with respect to 1-st moment. However the eigen-values have all negative real part, the bridge is judged to be stable. Similar to it, when the eigen-values of matrix **A** (i.e. Eq. (28)) have one at least positive real part; the bridge becomes unstable with respect to the 2-nd moment criterion.

## 3. NUMERICAL SIMULATION

### 3.1 Simulation Model

#### a) Specification of Suspension Bridge

In this study, numerical calculation was carried out using the specifications of the suspension bridge by Beliveau (1977) but its deck shape is changed from their truss to our box deck in Fig.3. Dimensions are as follows;

- Span length :  $L = 490\text{m}$
- Box deck width  $B = 16.4\text{m}$ ,  $D = 2.2\text{m}$  (i.e.  $B/D = 7.5$ )  
(N.B. Middle class suspension bridge is assumed)
- Inertia moment of stiffening girder :  $I = 8.98 \times 10^5 \text{kg} \cdot \text{m}^2 / \text{m}$
- Natural frequency of torsional motion :  $f_\alpha = 0.37\text{Hz}$
- Damping ration of torsional motion :  $\zeta_\alpha = 0.01$
- Air density :  $\rho = 1.226 \text{kg}/\text{m}^3$

## b) Flutter Derivatives

Numerical studies are carried out by the data of rectangular cylinder with  $B/D = 7.5$  ( $B = 0.3\text{m}$ , see Fig.3). The model scale is about  $1/55$  ( $\approx 0.3/16.4$ ). The flutter derivatives (i.e. F.D.) measured by us in smooth and turbulent flow were applied. The turbulent flow was generated using our active gust generator as shown in Fig.4.

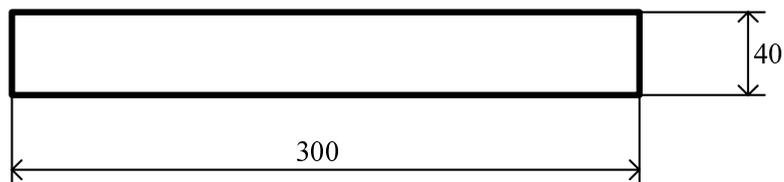


Fig. 3 Wind tunnel model of rectangular cylinder with  $B/D = 7.5$

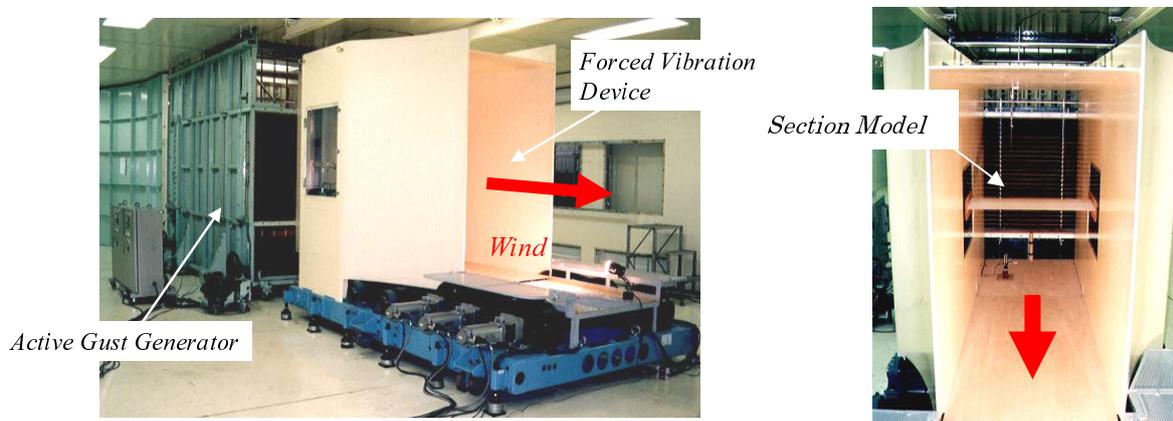


Fig.4 Experimental facilities

Fig.5 shows the flutter derivatives  $A_2^*$  and  $A_3^*$  measured in smooth and turbulent flows. The turbulent characteristics generated by our active gust generator are given as follows.

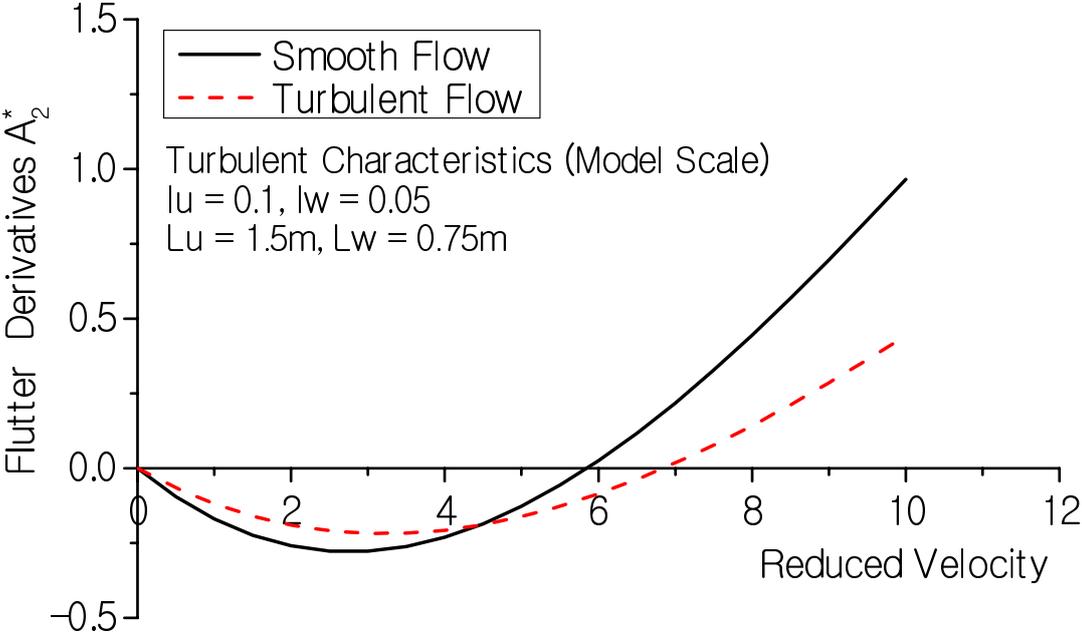
- Intensity of turbulence:  $I_u = 0.1$ ,  $I_w = 0.05$
- Scale of turbulence :  $L_u^x = 1.5\text{m}$ ,  $L_w^x = 0.75\text{m}$

The flutter derivatives  $A_2^*$  relates to aerodynamic damping term. The flutter derivatives  $A_2^*$  in the turbulent flow is smaller than the  $A_2^*$  in the smooth flow in Fig.5. Therefore this coincides with the conventional concept that the turbulent flow will raise flutter critical velocity and this is stabilizing effect to flutter.

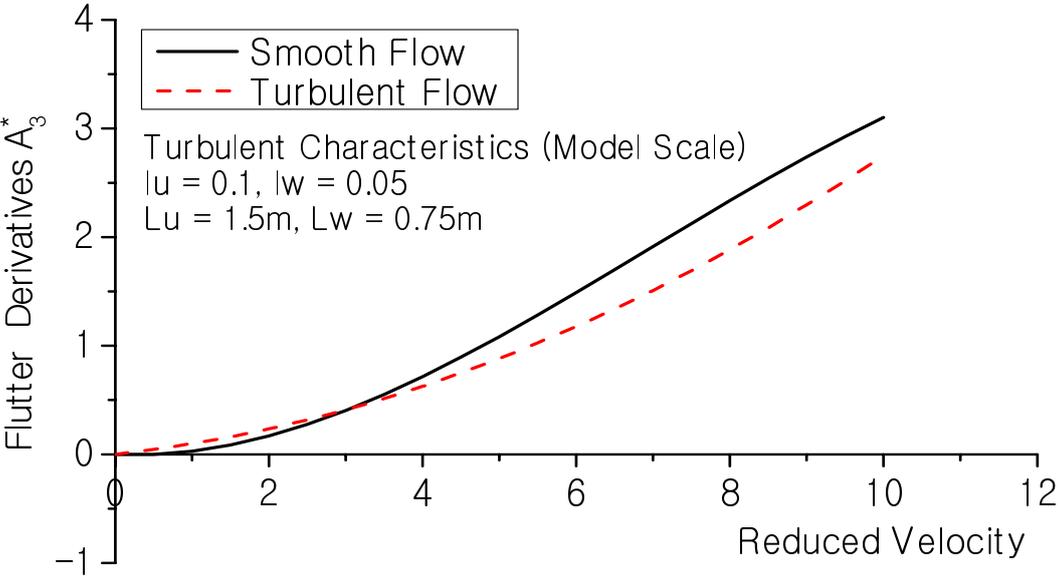
## 3.2 Flutter Stability Analysis

Flutter stability analysis is carried out using the above  $A_2^*$  and  $A_3^*$  in smooth and turbulent flows. First, turbulent characteristics used in this stability analysis are  $I_u = 0.1$ ,  $I_w = 0.05$ ,  $L_u^x = 1.5\text{m}$ ,  $L_w^x = 0.75\text{m}$ . Table 1 gives the numerical results of stability

analysis for 1-st and 2-nd moments. Also, the flutter critical velocity by complex eigen-value analysis is included in this table. Paying attention to flutter stability for 2-nd moment, the critical wind velocity for the flutter derivatives measured in turbulent flow is 18% higher than in smooth flow.



(a)  $A_2^*$



(b)  $A_3^*$

Fig. 5 Flutter derivatives measured in smooth and turbulent Flows

Table 1 Results of stability analysis for 1-st and 2-nd

Flutter derivatives	① $V_{cr}$ by complex Eigen-value analysis (m/s)	②1-st moment (m/s)	③2-nd moment (m/s)	③/①
Smooth	37.8	37.8	33.6	0.89
Turbulent	45.5	45.5	39.8	0.87
Turbulent/Smooth	1.20	1.20	1.18	-

N.B) • Turbulent intensity :  $I_u = 0.1$ ,  $I_w = 0.05$   
 • Turbulent scale :  $L_u^x = 1.5m$ ,  $L_w^x = 0.75m$

Furthermore, parametric studies of the flutter stability analysis changing turbulent conditions were carried out. Figs.6 and 7 respectively show the results of the flutter stability analysis using the F.D. measured in smooth and turbulent flows. With increasing turbulent intensity and scale, it is confirmed that flutter critical velocity will decrease.

The zones above the dash line (i.e. green and blue zones) in Fig.8 indicate that the ratio is less than 1. This means that the critical wind velocity solved by the F.D. in a turbulent flow is lower than that in a smooth. These zones are where Intensity of turbulence and its scale are large. To fulfill the ratio less than 1, if the scale is small, the intensity must be large and on the contrary when the scale is large, small intensity may be enough. Physical interpretation to our results can be as follows:

According to Scanlan (1988), bridge responses grow up according to the duration time of the high wind velocity beyond critical wind velocity. Fig.9 shows the definition of the duration time of the high wind velocity beyond critical wind velocity. Mean wind velocity used in this analysis is  $\bar{U} = 43.2m/s$ . The average values of the duration time beyond  $1.05 \times \bar{U} = 45.5m/s$  ( $=V_{cr}$ ) are calculated.

Fig.10 shows the relationship among the duration time and scale and intensity of turbulence. Duration time becomes large in the zones where turbulent scale and intensity are large. This corresponds to the tendency of Fig.8. In other words, when the duration time of the high wind velocity beyond critical wind velocity becomes longer, the bridge response amplitude will grow enough to become flutter. It can be obvious that the aerodynamic responses are apparently destabilized by a turbulent flow with above mentioned conditions.

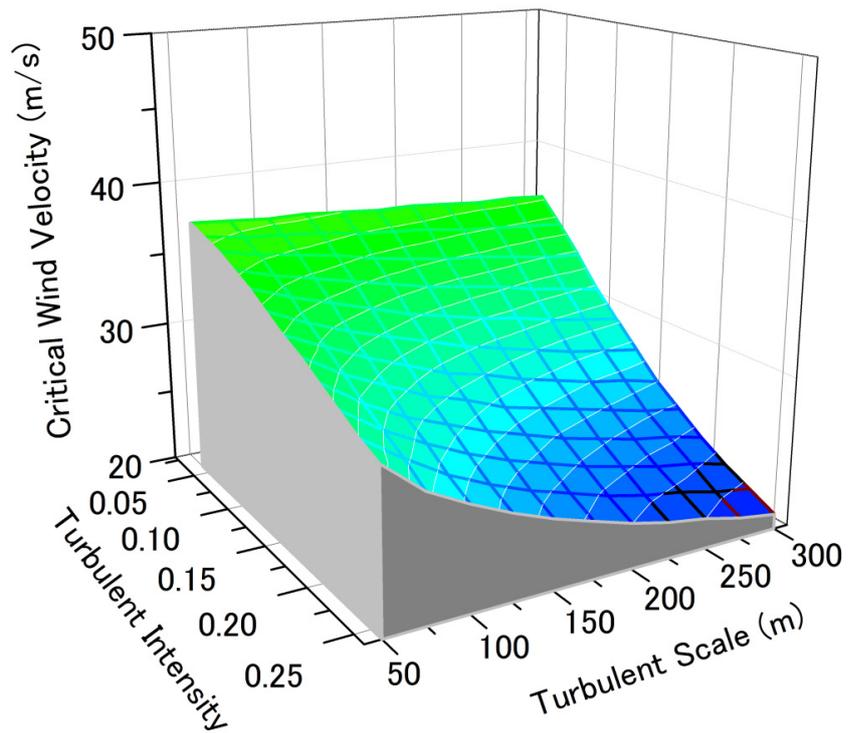


Fig. 6 Critical wind velocity by 2-nd moment stability vs. turbulent properties  
(Use of F.D. measured in a smooth flow)

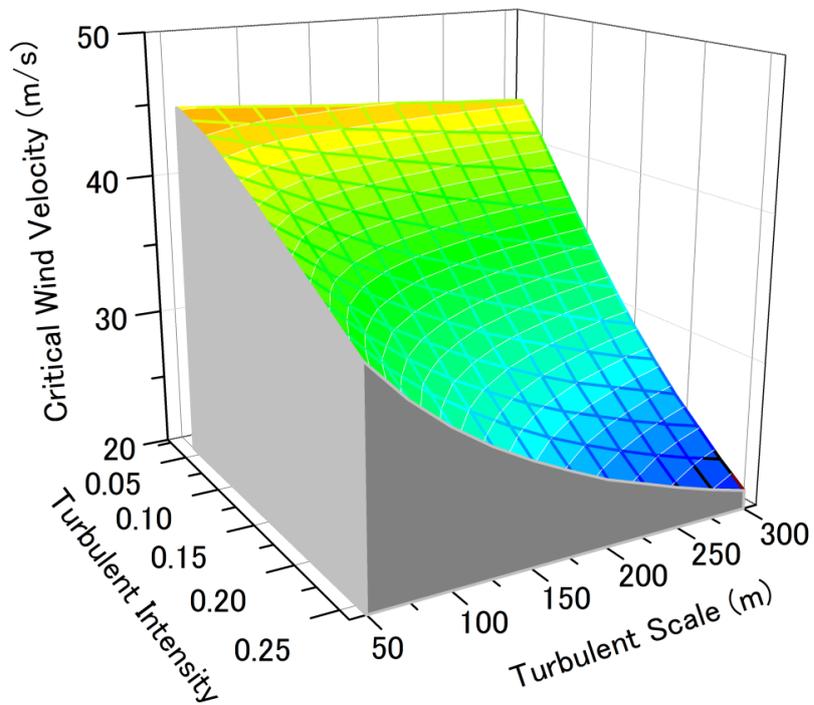


Fig. 7 Critical wind velocity by 2-nd moment stability vs. turbulent properties  
(Use of F.D. measured in a turbulent flow)

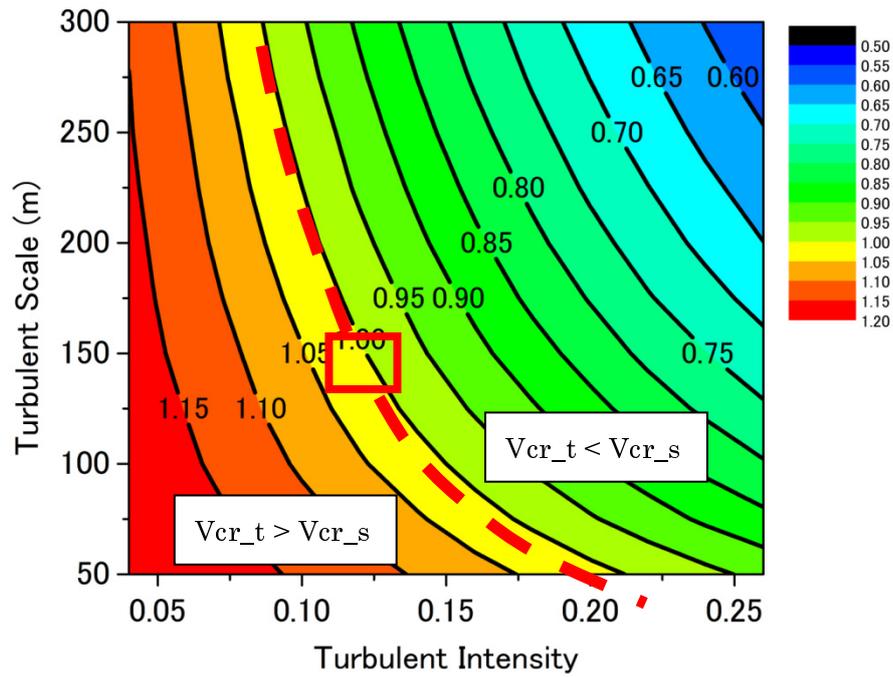


Fig. 8 Ratio of critical wind velocity ( $V_{cr\_t}$ ) vs. ( $V_{cr\_s}$ )

N.B. ( $V_{cr\_t}$ ) indicates  $V_{cr}$  with use of F.D. measured in turbulent flow.  
 ( $V_{cr\_s}$ ) indicates  $V_{cr}$  with use of F.D. measured in smooth flow

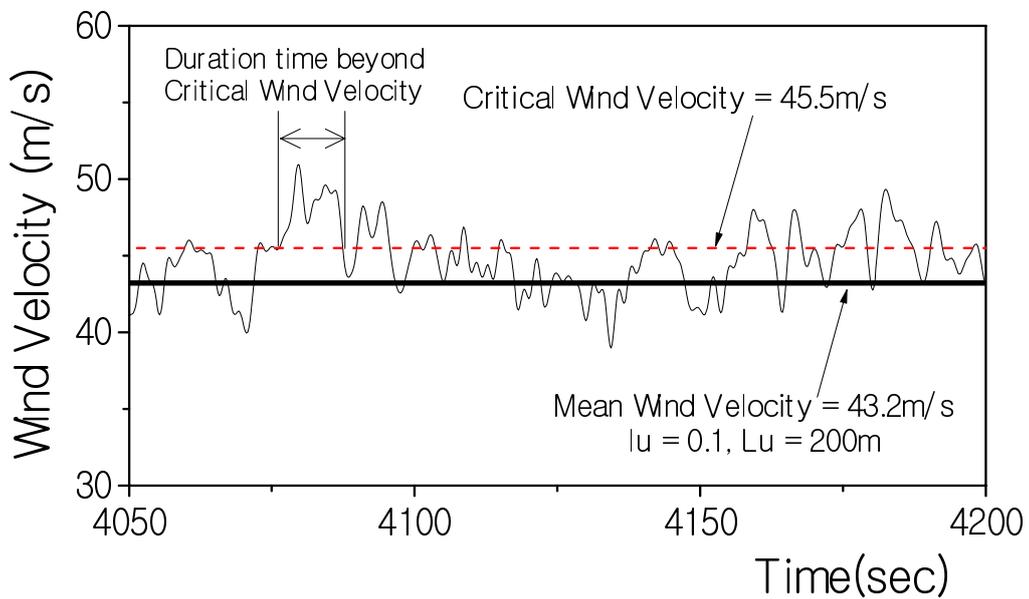


Fig. 9 Definition of duration time

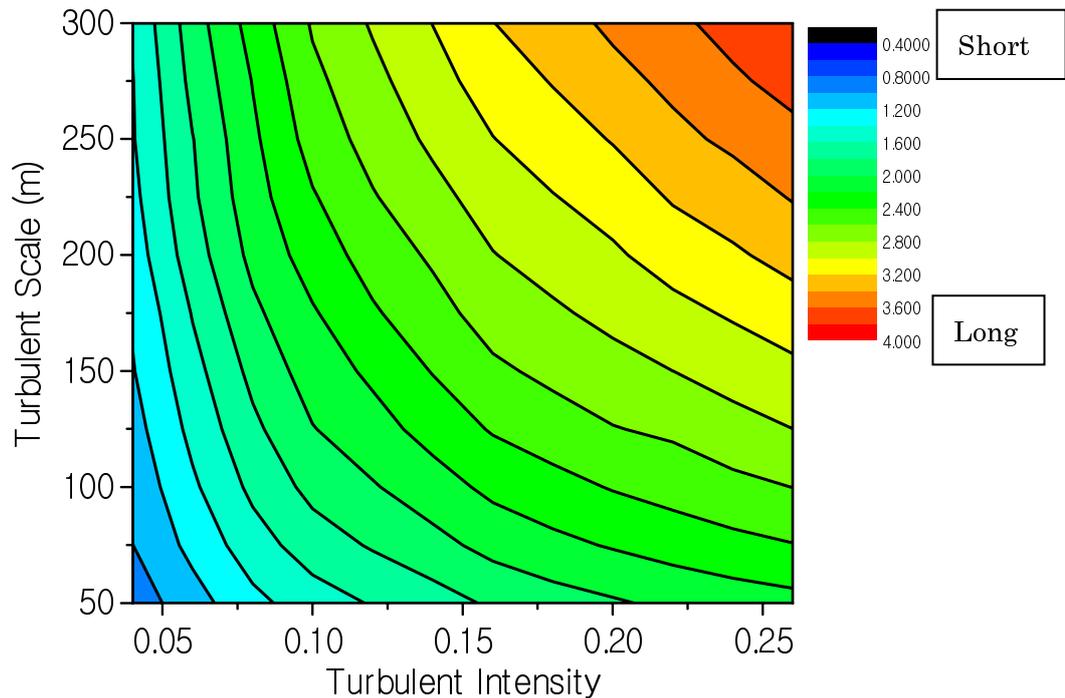


Fig. 10 Duration time of wind gust beyond flutter critical velocity ( $V_{cr} = 45.5\text{m/s}$ ) Vs. turbulent characteristics

#### 4 CONCLUSIONS

This paper presents a stochastic method of flutter stability analysis of long span bridge subjected to random wind load based on the theory of Markov processes. The main conclusion remarks are summarized as follows:

- 1) The spectral density function of strong wind gust was modeled by 1-st order linear filter. The power spectral density given by 1-st order linear filter roughly approximates the original power spectral density of von Karman's formula.
- 2) Flutter stability analysis was carried out using the flutter derivatives of the rectangular deck with  $B/D = 7.5$  measured in smooth and turbulent flows. When turbulent intensity and scale become large, it is confirmed that flutter critical velocity will decrease by judging the stability criterion of 2-nd moment.
- 3) The critical wind velocity given by the flutter derivatives in a turbulent flow can be lower than that in a smooth flow according to the wind turbulent parameters (e.g. scale and intensity).

It is thought that the flutter derivatives in a turbulent flow change according to the various turbulent characteristics. Therefore more flutter derivatives of the various turbulent characteristics must be examined.

The results of our research may give warning to the common consensus of engineers that wind tunnel tests (e.g., 2D section model ones ) in smooth flow is safety side to estimate flutter velocity of long span bridges. Our results show that if the intensity and scale of turbulence is small (i.e. the limit is smooth flow), the flutter critical wind velocity obtained by the wind tunnel tests in smooth flow can be dangerous side for the flutter prediction.

However this paper shows only one example of a box deck ( $B/D = 7.5$ ). Configurations of bridge decks or towers are many. Therefore deeper research is necessary. Time-domain approach may show more clear interpretation for our flutter stability analysis then it will be our future research reservation.

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