Opto-Thermal Properties of Fibres using Digital Holographic Interferometry

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Abstract

A hot stage attached to the phase shifting digital holographic interferometry (DHI) is used to study the effects of temperature on the optical properties of basalt fibres. The phase shifting Mach-Zehnder interferometer is used to obtain five phase-shifted holograms, in which the phase difference between two successive holograms is π/2. These holograms are numerically reconstructed using the convolution approach into amplitude and phase distributions. The obtained phase is used to calculate the refractive indices for light polarizing parallel and perpendicular to the fibre axis. The influence of temperature on the dispersion parameters, polarizability per unit volume and dielectric susceptibility are obtained. The values of the dispersion and oscillation energies and Cauchy’s constants are provided at different temperatures. The obtained results show that DHI is seen to be useful new tool for studying opto-thermal properties of fibres.
1. Introduction:

The interaction between light and matter leads to refraction effects, which are essentially a forced vibration of electrons with natural wavelength $\lambda_n$ by an oscillatory electric field of wavelength $\lambda$. The refractive index varies with the variation of the wavelength of the incident light beam because of these interactions. The variation in refractive index with wavelength of incident light gives rise to the dispersion properties of the fibre [1]. The refractive index is a function not only of wavelength, but also of temperature, chemical composition and the physical state of matter. The changes of the refractive index with the temperature are called temperature dispersion [2]. The study of the optical properties of fibres at different temperatures throws light on the opto-thermal behavior of the investigated fibre.

Numbers of interferometric techniques have been largely applied to measure the spectral dispersion curves of birefringence and refractive indices $n^{11}$ and $n^\perp$ of textile and highly oriented fibres [3-5]. These techniques are also applied to investigate opto-thermal properties of fibres. Hamza et al. [6] designed a heating device attached with the multiple-beam Fizeau fringes technique to study the influence of temperature on the optical properties of undrawn polypropylene fibres. Also, Belal et al. [7] designed a hot-stage connected with the double-beam interference (Pluta) microscope to investigate the effect of temperature on the optical properties of fibres in the range (room temp. to 60°C). Recently, El-Dessouky et al. studied the effects of heating-cooling cycle on the opto-structural properties of low-density polyethylene fibres using two-beam microinterferometry [8].

Digital holography has been experiencing rapid development in recent years because of many technical advantages in comparing with number of interference techniques [9]. Holographic interferometry is an optical technique for obtaining interference of two wave fields that are scattered from an object in different states simultaneously [10]. In this technique, the photographic films were used to record the holograms. These films need chemical wet developing and their handling is time consuming. Also, the reconstruction process to obtain the phase requires additional experimental efforts. Digital holographic interferometry (DHI) was suggested to overcome the mentioned difficulties [11,12]. A CCD camera is used to record a hologram onto a computer and numerical methods are subsequently applied to reconstruct the hologram to enable direct access to both phase and amplitude information. The main advantage of digital holography is that not only the intensity but also the phase of a holographically stored wave front can be calculated directly in the numerical reconstruction.

DHI was used to calculate shape and deformation of an object [9] or the refractive index variations of an object [13, 14]. DHI has been combined with a phase shifting approach and the resulting technique was used to measure the non-homogeneous refractive index distribution in graded index optical as well as polymer fibers [15, 16]. Recently Yassien and Agour [17] studied the opto mechanical properties of polymer fibres using DHI.

The aim of the present study is to investigate the influence of temperature on the optical properties and dispersion parameters of basalt fibre using a hot-stage attached to the phase shifting digital holographic interferometric set-up. The dispersion parameters, polarizabilily per unit
volume, dielectric susceptibility, dispersion and oscillation energies are provided at different temperatures.

2. Experimental Techniques, Results and Discussions:

A hot-stage [7] attached with phase shifting Mach-Zehnder interferometer se-tup [15] is used to study the influence of temperature on the spectral dispersion curves of refractive indices (n∥ and n⊥) and dispersion properties of basalt fibre over the temperature range (28-60 °C). The fibre (basalt fibre) is fixed with a suitable adhesive material on a glass microscope slide. A drop of immersion liquid with refractive index nL is placed upon the fixed fibre and then the covering glass is placed on it. The fibre is placed on the hot-stage and then it is transferred to the interferometer. A Piezo-transducer phase shifter PZT in the reference arm, of the interferometer, acting as a phase shifting tool to obtain five phase shifted holograms with phase steps π/2 between two successive holograms. These digital holograms were reconstructed numerically using the convolution method [11]. The reconstruction process was performed by multiplication of the stored digital hologram with the numerical description of the reference wave and then by convolution of the result with the impulse response function [11]. Digital holograms were numerically reconstructed by means of changing the reconstruction distance, which is the distance between the hologram plane and the reconstruction plane. The phase distribution (φ) of the basalt fibre is calculated using the following equation [12]:

\[
\phi(\xi, \eta) = \arctan \left( \frac{\text{Im} \{U(\xi, \eta)\}}{\text{Re} \{U(\xi, \eta)\}} \right).
\]

Where:

\[
U(\xi, \eta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x, y) E^*(x, y) g(\xi, \eta, x, y) dx dy.
\]

The parameters (x, y) and (ξ, η) are the coordinates at the CCD and the reconstructed planes, respectively. h(x, y) is the wavefield obtained from the combination of the phase shifted holograms, \(E^*(x, y)\) is the conjugated reference beam and \(g(\xi, \eta, x, y)\) is the impulse response. Fig. 1(a,b), for example, shows the wrapped phase map of the basalt fibres for light polarizing parallel and perpendicular to the fibre axis. The standard phase unwrapping algorithm has been applied to convert the interference phase modulo 2π into a continuous phase distribution [12]. Fig. 1(c,d) shows the unwrapped phase, in the two directions of light polarization, representing the changes in the optical path length which is caused by the fibre as in comparison with the surrounding medium taking into consideration the thickness of the sample. The unwrapped phase of the basalt fibre is determined by the same mentioned method at different temperatures and different wavelengths.
Using the obtained unwrapped phase maps, the mean refractive index of the fiber’s sample at different draw ratios can be calculated using the following equation [5]:

\[ n^i = n_L \pm \frac{\phi^i \lambda}{2 \pi} \tag{3} \]

Where \( i \) denotes to the polarization state (parallel || or perpendicular \( \perp \)). Positive and negative signs are used when the mean refractive index of the fiber \( n \) is greater or less than the refractive index of the immersion liquid \( n_L \), respectively.

Fig. 2(a,b) gives the variation of the mean refractive indices (\( n^\parallel \) and \( n^\perp \)) with the elevated temperatures at different wavelengths. It is found that \( n^\parallel \) and \( n^\perp \) of the basalt fibre decrease linearly with increasing temperature.

Fig. 1. The wrapped phase (a,b) and unwrapped phase (c,d) of basalt fibres for light polarizing parallel and perpendicular to the fibre axis.

Fig. 2. The relationship between temperature and refractive indices \( n^\parallel \) (a) and \( n^\perp \) (b) at different wavelengths; 546, 550, 577 and 590 nm for basalt fibre.
Dispersion properties play an important role in the characterization and performance of natural and synthetic fibres. The influence of temperature on the spectral dispersion curves are studied, to obtain more useful information about the optical characteristics of basalt fibre. Fig. 3(a,b) shows these curves for light polarizing parallel and perpendicular to the fibre axis. It is obvious from these curves that these curves of the used sample decrease as temperature increase; this is due to that temperatures have a direct effect on the internal structure of the fibre. Also, from these dispersion curves it is clear that basalt fibre have normal dispersion curves in this region of the spectrum at different temperatures. By using these curves one can reads out the index of refraction n for any used wavelength at any elevated temperature. To calculate the oscillation energy $E_o$ and the dispersion energy $E_d$ [19] of basalt fibre at different temperatures, we plotted the relationship between $(n^2-1)^{-1}$ and the square of photon energy $E^2$ for light polarizing parallel and perpendicular to the fibre axis. We obtained a straight lines as shown in Fig. 4 (a,b) for light polarizing parallel and perpendicular to the fibr axis.

Fig.3 .The spectral dispersion curves of refractive indices $n_{||}$ (a) and $n_{\perp}$ (b) at different temperatures for basalt fibre.

Fig. 4 . The relationship between $(n^2-1)^{-1}$ and the square of photon energy $E^2$ for light polarizing parallel (a) and perpendicular (b) to the fibre axis.
From their slopes and intersects, the oscillation energy $E_o$ and the dispersion energy $E_d$ are calculated at different temperatures as shown in table (1). The dispersion and oscillation energies are unique parameters that display general structural trends [20].

Table (1). The dispersion and oscillation energies for light vibrating parallel and perpendicular to the fibre axis $E_d^{\|}$, $E_o^{\|}$, $E_d^\perp$ and $E_o^\perp$ for basalt fibre at different temperatures.

<table>
<thead>
<tr>
<th>Temperature T (°C)</th>
<th>$E_d^{|}$ (ev)</th>
<th>$E_o^{|}$ (ev)</th>
<th>$E_d^\perp$ (ev)</th>
<th>$E_o^\perp$ (ev)</th>
</tr>
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<tbody>
<tr>
<td>28</td>
<td>13.15</td>
<td>9.75</td>
<td>12.54</td>
<td>9.39</td>
</tr>
<tr>
<td>34</td>
<td>12.99</td>
<td>9.7</td>
<td>13.17</td>
<td>9.86</td>
</tr>
<tr>
<td>40</td>
<td>12.83</td>
<td>9.63</td>
<td>12.89</td>
<td>9.7</td>
</tr>
<tr>
<td>45</td>
<td>12.95</td>
<td>9.74</td>
<td>13.04</td>
<td>9.83</td>
</tr>
<tr>
<td>50</td>
<td>12.7</td>
<td>9.6</td>
<td>12.06</td>
<td>9.21</td>
</tr>
<tr>
<td>55</td>
<td>12.76</td>
<td>9.68</td>
<td>13.26</td>
<td>10.05</td>
</tr>
<tr>
<td>60</td>
<td>12.57</td>
<td>9.58</td>
<td>12.79</td>
<td>9.77</td>
</tr>
</tbody>
</table>

Fig. 5 (a,b) gives the relationship between refractive indices $n^{\|}$ and $n^\perp$ with the inverse square of the wavelength of the incident light beam, $1/\lambda^2$ at different temperatures for basalt fibre. From the slopes and the intersects parts of the straight lines, the Caushy’s dispersion constants A and B that characterizing the dispersion activity of the material are calculated at different temperatures and the results are given in table (2). From this table it is clear that the constant A for light polarizing parallel and perpendicular to the fibre axis have the same behavior of refractive index. The values of the constants A and B refer to the high degree of the molecular orientation parallel and perpendicular to the fibre axis.

Fig.5 . The relationship between $1/\lambda^2$ and refractive indices $n^{\|}$ (a) and $n^\perp$ (b) at different temperatures
Table (2): Constants A and B of Cauchy’s dispersion formula of refractive indices $n_{\|}$ and $n_{\perp}$ of basalt fibre at different temperatures.

<table>
<thead>
<tr>
<th>Temperature T (°C)</th>
<th>$A_{|}$</th>
<th>$B_{|} \times 10^3$ (nm$^2$)</th>
<th>$A_{\perp}$</th>
<th>$B_{\perp} \times 10^3$ (nm$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>28</td>
<td>1.531</td>
<td>7.9</td>
<td>1.528</td>
<td>8.5</td>
</tr>
<tr>
<td>34</td>
<td>1.528</td>
<td>7.9</td>
<td>1.527</td>
<td>7.6</td>
</tr>
<tr>
<td>40</td>
<td>1.526</td>
<td>8.01</td>
<td>1.525</td>
<td>7.9</td>
</tr>
<tr>
<td>45</td>
<td>1.525</td>
<td>7.82</td>
<td>1.524</td>
<td>7.7</td>
</tr>
<tr>
<td>50</td>
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<td>8.01</td>
<td>1.523</td>
<td>7.5</td>
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<td>1.521</td>
<td>7.9</td>
<td>1.522</td>
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<tr>
<td>60</td>
<td>1.519</td>
<td>8.01</td>
<td>1.518</td>
<td>7.4</td>
</tr>
</tbody>
</table>

In polymeric fibres studies, the attempts are considered to relate the molecular structure of fibre to its thermal properties which have direct effect on the structural parameters such as refractive index and polarizability per unit volume. The obtained values of the refractive indices $n_{\|}$ and $n_{\perp}$ of the fibre material are used to calculate the polarizability per unit volume in case of light polarizing parallel $P_{\|}$ and perpendicular $P_{\perp}$ to the fibre axis by application of the following Lorantz-Lorenz equation [21]:

$$\frac{n^2 - 1}{n^2 + 2} = \frac{4\pi}{3} P$$  \hspace{1cm} (4)

Fig. 6(a,b) illustrates the relationship between polarizability and temperature at different wavelengths of basalt fibre. This figure show that the mean values of poalarizapilities $P_{\|}$ and $P_{\perp}$ are decreased with increasing of temperature and this behavior is similar to that occur for refractive indices $n_{\|}$ and $n_{\perp}$.

Fig. 6. The relationship between temperature and polarizabilities $P_{\|}$ (a) and $P_{\perp}$ (b) at different wavelengths.

The dielectric properties are considered one of the most conventional and sensitive methods of studying polymer structure [22]. The dielectric
constant \( \varepsilon \) is calculated with the aid of the values of refractive indices using the following relationship [23]:

\[
\varepsilon = n^2
\]

(5)

The dielectric susceptibility \( \eta \) is related to the dielectric constant by the following equation [23]:

\[
\eta = \frac{\varepsilon - 1}{4 \pi}
\]

(6)

Using Eqs. (5) and (6), the dielectric susceptibility \( \eta \) can be calculated for light polarizing parallel and perpendicular to the fibre axis. The relationship between dielectric susceptibility \( \eta \) and temperature at different wavelengths of basalt fibre is given in Fig. 7(a,b).

![Fig.7. The relationship between temperature and dielectric susceptibility \( \eta^\parallel \) (a) and \( \eta^\perp \) (b) at different wavelengths.](image)

From the above results it is clear that phase shifting DHI is an accurate method for determining the optical and dispersion properties of fibres. The DHI has many advantages such as [12,24,25]: The phase distribution can be calculated at all pixels, not only at the fringe centers. Therefore a best spatial resolution is possible with the available electronics. Due to the multiple recorded holograms the sign ambiguity is resolved automatically. Therefore, the evaluated value of samples phases increases and decreases in the same manner as the original one. Numerical focusing of the DHI emulates the manual focusing control of conventional microscopes and overcomes the limitations of the focusing (working) distance of their objectives. The phase accuracy achieved by phase shifting DHI method is \( \lambda/100 \) [24]. Therefore the accuracy in the measurement of refractive index is better than \( 4 \times 10^{-4} \) and this value is found to be in agreement with that obtained by literature [26].

**Conclusions:**

Phase-shifting digital holographic interferometry (DHI) was successfully used to study the opto-thermal properties of basalt fibres. DHI was applied to access phase information from the recorded holograms. The results show that refractive indices \( (n^\parallel \text{ and } n^\perp) \) and
polarizabilities ($P_{||}$ and $P_{⊥}$) of the basalt fibre decrease linearly with increasing temperature at different wavelengths. From the spectral dispersion curves $n(\lambda)$ it is clear that basalt fibre have normal dispersion curves in this region of the spectrum at different temperatures. By using these curves one can reads out the index of refraction $n$ for any used wavelength at any elevated temperature. The oscillation energy $E_o$, the dispersion energy $E_d$ and constants of Cauchy's dispersion formulae are provided at different temperatures. The molecular structure of fibre relates with its thermal properties which have direct effect on the structural parameters such as refractive index and polarizability per unit volume.

References: