Vortex-induced traveling waves on marine risers

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ABSTRACT

To demonstrate the traveling waves associated with the vortex-induced vibration (VIV) of a marine riser, this paper employs a multi-signal complex exponential method. This method is an extension of the classical Prony’s method which decomposes one arbitrary signal into a number of complex exponential components. Because the proposed method processes multiple signals simultaneously, it can estimate the “global” dominating frequencies (poles) shared by those signals. The complex amplitudes (residues) corresponding to the estimated frequencies for those signals are also obtained in the process. When the signals are collected at different locations along the axial direction of a marine riser, the phenomena of the traveling wave would be analyzed through the obtained complex amplitudes of those signals. The proposed method completely avoids the frequency leakage and resolution problems associated with the traditional Fourier-based methods. Another advantage of the proposed method is that it requires only short duration signals to conduct the traveling wave analysis.

Keywords: traveling waves, marine risers, vortex shedding

1. INTRODUCTION

Understanding the vibration behavior of marine risers excited by vortex shedding is important to the offshore industry. Analyzing the data of field experiment on a long flexible cylinder densely instrumented with fiber optic strain gauges, (Vandiver 2009) revealed a previously unknown phenomenon: the dominance of traveling wave — rather than standing wave — on riser’s strain/stress response.

To demonstrate the traveling wave response associated with the fundamental vortex shedding frequency, (Vandiver 2009) utilized a colored image plot. The data used in this image plot required a 2-step signal processing on the raw sensor signals: (i)
carrying out the fast Fourier transform (FFT) of the time series at each sensor location one by one, and (ii) filtering the signals at the fundamental vortex shedding frequency which involves a relatively subjective peak-picking process. From these processed data, the image plot was generated with its horizontal axis being the passage of time, the vertical axis being the non-dimensional axial location on the riser, and the color being the instantaneous amplitude of the filtered strain signals. As one scanned the image plot vertically, it was quite striking to notice the diagonal direction of constant color — an indication of traveling wave. From the slope of the diagonal rows, the wave propagation speed associated with the fundamental vortex shedding frequency might be estimated.

This paper proposes a multi-signal complex exponential method to investigate the traveling waves. By processing all signals simultaneously, the proposed method — an extension of the classical Prony’s method — estimates the fundamental vortex shedding frequency through an eigen analysis of the raw signals. The complex amplitude associated with the fundamental vortex shedding frequency for each signal is also obtained in the process. Sequentially, the phenomena of the traveling wave can be studied by analyzing the obtained complex amplitudes. To investigate the phenomena of traveling waves by the proposed method, experimental vortex-induced vibration data from Norwegian Deepwater Program (NDP) high mode test are to be utilized.

2. PRELIMINARIES

2.1 pth order constant-coefficient linear differential equation

A pth-order homogenous linear ordinary differential equation (ODE) with constant real-valued coefficients \( a_n \) could be written as:

\[
\sum_{n=0}^{p} a_n y^{(n)}(t) = 0, \quad \text{for } 0 \leq t < T
\]  

(1)

where \( y^{(n)}(t) = \frac{d^n y}{dt^n} \). Substituting \( y = e^{\lambda t} \) into Eq. (1) yields the characteristic equation:

\[
\sum_{n=0}^{p} a_n \lambda^n = 0
\]  

(2)

If \( \lambda \) is a root of Eq. (2), then \( y = e^{\lambda t} \) is a solution of Eq. (1). And if all the \( p \) roots \( \lambda_1, \ldots, \lambda_p \) of Eq. (2) are different, then the \( p \) solutions \( y_1 = e^{\lambda_1 t}, \ldots, y_p = e^{\lambda_p t} \) constitute a basis for Eq. (1). Thus the corresponding general solution of Eq. (1) is a linear combination of those exponentials, i.e.,

\[
y(t) = \sum_{n=1}^{p} r_n e^{\lambda_n t} \quad 0 \leq t < T
\]  

(3)
In Eq. (3), when \( y(t) \) is a real-valued signal, \( \lambda_n \) must either be real numbers or occur in complex conjugate pairs. Let \( \lambda_n = -\alpha_n + i\omega_n \), and \( \alpha_n \) is defined as the damping factor in seconds\(^{-1} \) and \( \omega_n \) is the frequency in radians. The coefficients \( r_n \) corresponding to complex exponents \( \lambda_n \) must also appear in complex conjugate pairs. Let \( r_n = A_n \exp(i\theta_n) \), then \( A_n \) is the amplitude and \( \theta_n \) is the sinusoidal initial phase in radians associated with \( e^{\lambda_n t} \).

Taking the Laplace transform of Eq. (3) yields

\[
Y(s) = \sum_{n=1}^{p} \frac{r_n}{s - \lambda_n} \tag{4}
\]

Usually \( r_n \) and \( \lambda_n \) are also referred to as the residues and poles associated with the function \( Y(s) \).

### 2.2 pth order constant-coefficient linear difference equation

In digital signal processing problems, one is almost always interested in a time signal \( y(t) \) only for the discrete values. Denote \( y_k = y(t_k) \), \( t_k = k\Delta t \), for \( k = 0, 1, \ldots, N-1 \), in which \( \Delta t \) = time step increment, and \( N \) = the number of time steps. In parallel to the ordinary differential equation shown at Eq. (1), a p th-order homogenous linear, constant-coefficient difference equation can be written as

\[
\sum_{n=0}^{p} b_n y_{n+k} = 0 \tag{5}
\]

In the above equation, if both \( b_0 \) and \( b_p \) are different from zero, the positive integer \( p \) is called the order of the equation.

To find the solutions of Eq. (5), one might try, as with the analogous differential equation, the substitution

\[
y_k = e^{\lambda_k t} = (e^{\lambda t})^k \tag{6}
\]

However, it is more convenient to write

\[
y_k = z^k \tag{7}
\]

where \( z = e^{\lambda \Delta t} \). Substituting Eq. (7) into Eq. (5), and dividing out \( z^k \), one obtains the characteristic equation of the difference equation Eq. (5) to be:

\[
\sum_{n=0}^{p} b_n z^n = 0 \tag{8}
\]
If the roots of Eq. (8) are distinct, then

\[ y_k = \sum_{n=1}^{p} r_n z_n^k \]  

(9)

As Eq. (9) is the discrete counterpart of Eq. (3), the two equations must have the same residues \( r_n \). Since \( z_n = \exp(\lambda_n \Delta t) \), one has \( \lambda_n = \ln(z_n)/\Delta t \).

3. EXTENDED PRONY’S METHOD FOR THE DECOMPOSITION OF MULTIPLE SIGNALS

Prony’s method is a technique for fitting the exponential model Eq. (3) (or Eq. (9)) using equally spaced data \( y_k \). In other words, Prony’s method is to determine \( r_n \) and \( \lambda_n \) (or \( z_n \)) based on the set \( y_k \), \( k = 0, 1, \ldots, N-1 \). While the original Prony’s method presented a method of exactly fitting as many purely damped exponentials as needed to fit the available data points, the modern version of Prony’s method generalizes to damped sinusoidal models and also makes use of the least squares analysis to approximately fit an exponential model for cases where there are more data points than needed to fit to the assumed number of exponential terms.

The exponential model Eq. (9) for Prony’s method readily suggests that the corresponding difference equation model is Eq. (5). In fact, the first step of Prony’s method is to use the equally spaced data \( y_k \) to estimate the constant coefficients of the linear difference equation. Once the difference equation Eq. (5) is determined, one can proceed the calculation for poles and residues associated with the difference equation. In summary, three sequential steps, using Eqs. (5), (8) and (9), respectively, are implemented in Prony’s method:

Step 1: Without losing the generality, let \( b_0 = 1 \). By repeatedly using Eq. (5), one determines the linear prediction parameters \( b_0, b_1, \ldots, b_{p-1} \) that fit the given discrete signal. In matrix form, it is written as:

\[ Yb = -y' \]  

(10)

where

\[ Y = \begin{bmatrix} y_0 & y_1 & \cdots & y_{p-1} \\ y_1 & y_2 & \cdots & y_p \\ \vdots & \vdots & \ddots & \vdots \\ y_{N-p+1} & y_{N-p} & \cdots & y_{N-2} \end{bmatrix}, b = \begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ b_{p-1} \end{bmatrix}, \text{and } y' = \begin{bmatrix} y_p \\ y_{p+1} \\ \vdots \\ y_{N-1} \end{bmatrix} \]  

(11)

Step 2: With the linear prediction coefficients \( b \) obtained from Step 1, the roots (denoted as \( z_n, n = 1, \ldots, p \)) of the characteristic equation Eq. (8) are computed.
Step 3: Once $z_n, n=1,\ldots,p$, have been computed, the third step involves the solution of a second set of linear equations to yield the estimates of the exponential amplitude and sinusoidal initial phase. Based on Eq. (9), a matrix form is written as:

$$
\begin{bmatrix}
0 & 0 & \ldots & 0 \\
1 & z_2 & \ldots & z_p \\
\vdots & \vdots & \ddots & \vdots \\
1 & z_{N-1} & \ldots & z_{N-1}^p
\end{bmatrix}
\begin{bmatrix}
r_1 \\
r_2 \\
\vdots \\
r_p
\end{bmatrix}
= 
\begin{bmatrix}
y_0 \\
y_1 \\
\vdots \\
y_{N-1}
\end{bmatrix}
$$

(12)

Recall that the roots of Eq. (2), namely $\lambda_n$, are related to those of Eq. (8) through $\lambda_n = (\ln z_n) / \Delta t$. Appropriate linear least squares procedures for the first and third steps of the three-step Prony's method has sometimes been called the extended Prony’s method.

### 3.1 Multiple signals decomposition

Prony’s method can be further extended to perform the decomposition for multiple signals (Trudnowski 1999). Denote the column vector $y_k \in \mathbb{R}^{N_s \times 1}$ for the signal data at $t = t_k$ where $N_s$ is the number of signals. An intended feature is that all $N_s$ signals share the same model — a $p$ th-order difference equation as Eq. (5). Thus, one writes

$$
\sum_{n=0}^{p} b_n y_{n+k} = 0
$$

(13)

With this feature, a consistent set of "global" poles (natural frequencies and damping factors) is among all signals.

The extension of Prony’s method from the single signal to multiple signals is quite straightforward for the three steps:

**Step 1:** By repeatedly using Eq. (13), one determines the linear prediction parameters $b_0, b_1, \ldots, b_{p-1}$ (noting $b_p = 1$) from $y_k, k = 0,1,\ldots,N-1$:

$$
\begin{bmatrix}
y_0 & y_1 & \ldots & y_{p-1} \\
y_1 & y_2 & \ldots & y_p \\
\vdots & \vdots & \ddots & \vdots \\
y_{N-p-1} & y_{N-p} & \ldots & y_{N-2}
\end{bmatrix}
\begin{bmatrix}
b_0 \\
b_1 \\
\vdots \\
b_{p-1}
\end{bmatrix}
= 
\begin{bmatrix}
y_p \\
y_{p+1} \\
\vdots \\
y_{N-1}
\end{bmatrix}
$$

(14)

**Step 2:** With the linear prediction coefficients $b$ obtained from Step 1, the roots (denoted as $z_n, n=1,\ldots,p$) of the characteristic equation Eq. (8) are computed.
Step 3: Once $z_n, n = 1, \ldots, p$, have been computed, the third step involves the computation for the residues of each signal. Denote $r_n \in \mathbb{R}^{N_s, s}$ for the nth residue vector for all $N_s$ signals. A straightforward extension of Eq. (12) leads to:

$$
\begin{bmatrix}
z_1^0 & z_2^0 & \cdots & z_p^0 \\
z_1^1 & z_2^1 & \cdots & z_p^1 \\
\vdots & \vdots & \ddots & \vdots \\
 z_1^{N_s-1} & z_2^{N_s-1} & \cdots & z_p^{N_s-1}
\end{bmatrix}
\begin{bmatrix}
r_1^T \\
r_2^T \\
\vdots \\
r_p^T
\end{bmatrix}
= 
\begin{bmatrix}
y_0^T \\
y_1^T \\
\vdots \\
y_{N_s-1}^T
\end{bmatrix}
$$

(15)

Since the second step of Prony’s method is an ill-conditioned problem and round-off errors must exist for the linear prediction parameters $b$ computed in the first step, the estimation of $z_n$ in the second step of Prony’s method can have significant error.

3.2 Numerical implementation

To avoid dealing with an ill-conditioned problem on determining the roots of a high order polynomial encountered in Prony’s method, an alternative approach which begins with a first-order matrix difference equation — a state-space model — to replace a high order difference equation used in Prony’s method is advocated (Hu 2013). The state-space model approach also involves three steps. While its third step is identical to that of Prony’s method, the first two steps of the proposed method are: (1) obtaining a realization for the state matrix of first-order matrix difference equation from the Hankel matrices composed by the signal data; and (2) computing the eigenvalues of the realization matrix which are the global poles of the signals. One obvious advantage of the state-space model approach is completely avoiding the ill-conditioned problem of solving the zeros of a high order polynomial.

4. EXPERIMENTAL DATA STUDIES

The VIV signals investigated are from the Norwegian Deepwater Program (NDP) high mode test, which was a high length-to-diameter ratio (L/D) riser hydrodynamics testing program: a riser model of $L = 38$ m and $D = 0.027$ m. For each test run, the riser was setup in a way, and towed at a constant speed, to simulate the riser subjected to either uniform or sheared current. Measurements of micro bending strain and acceleration in the in-line (IL) and cross-flow (CF) directions along the riser were taken. Readers are referred to (Trim et al. 2005) for the details of the test.

In the present numerical study, eight acceleration data sets in the in-line direction from Test2350, where the sheared current profiles are corresponding to towing speed 0.7 m/s, will be analyzed. The eight acceleration signals were collected at $z/L = 0.1314, 0.2404, 0.3381, 0.4370, 0.5555, 0.6400, 0.7735$ and $0.8907$ ($z = 0$ at the bottom of the riser). The acceleration signal at $z/L = 0.1314$ is shown at Fig. 1. It seems that the signal between 20s and 30s is stationary. To implement the proposed method, all eight acceleration signals between 20s and 30s (see Fig. 2) are utilized. Taking the model
order $p = 40$, one obtains the most dominant pole to be $\lambda_1 = 0.0301 + 42.4008i$, and the second most dominant pole $\lambda_2 = 0.0675 + 62.5668i$.

Fig. 1 An acceleration signal in the IL direction from Test2350 of the NDP high mode test

Fig. 2 Chosen signals from 8 sensors for traveling wave analysis
4.1 Traveling wave of the most dominant component

The most dominant component has the global pole \( \lambda_i = 0.0301 + 42.4008i \). From the relationship of \( \lambda_n = -\alpha_n + i\omega_n \) and \( \omega_n = 2\pi f_n \), one obtains the corresponding frequency \( f_i = 6.7483 \) Hz and damping factor \( \alpha_i = 0.0007 \). Because the damping factor \( \alpha_i \) is very small, the dominant component of each signal is close to sinusoidal. Referring to Eq. (15) for \( N_s = 8 \), the first residue vector \( r_i \) for all 8 signals is listed at Table 1. From \( r_n = A_n \exp(i\theta_n) \), the corresponding amplitudes and phase angles are also listed in Table 1. Since the values of the 8 phase angles are different, it readily suggests that a traveling wave (propagating along the riser) exists for the most dominant component. In other words, the vibration mode of the riser with \( f_i = 6.7483 \) Hz is a complex-valued mode, not a real-valued mode.

Shown in Fig. 3 are the extracted 8 acceleration components with frequency 6.7483 Hz, where the displayed components from top to bottom are corresponding to the physical locations from top to bottom as well. By looking at the crests of each component, one can easily conclude that waves are traveling along the marine riser. Shown in Fig. 4 is an image display of the components—similar to that of (Vandiver 2009), where the horizontal axis is the passage of time, the vertical axis is the non-dimensional axial location on the riser, and the color is the amplitude of signals. As one scans Fig. 4 vertically, one notices that constant colors are shown in the diagonal direction: an indication of traveling wave. From the slope of the diagonal line, the wave propagation speed associated with this dominant frequency could be estimated, which is roughly 7.19 m/s.

<table>
<thead>
<tr>
<th>( z/L )</th>
<th>( r_n )</th>
<th>( A_n )</th>
<th>( \theta_n )</th>
</tr>
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<tr>
<td>0.8907</td>
<td>1.3680+1.0021i</td>
<td>3.3916</td>
<td>-0.6322</td>
</tr>
<tr>
<td>0.7735</td>
<td>-1.5183-1.1307i</td>
<td>3.7861</td>
<td>2.5015</td>
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<tr>
<td>0.6400</td>
<td>2.3001+0.9996i</td>
<td>5.0158</td>
<td>-0.4100</td>
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<tr>
<td>0.5555</td>
<td>-1.9490-1.0609i</td>
<td>4.4380</td>
<td>2.6431</td>
</tr>
<tr>
<td>0.4370</td>
<td>3.6834+0.5057i</td>
<td>7.4359</td>
<td>-0.1364</td>
</tr>
<tr>
<td>0.3381</td>
<td>-2.6623-0.8685i</td>
<td>5.6007</td>
<td>2.8263</td>
</tr>
<tr>
<td>0.2404</td>
<td>3.7254-0.0119i</td>
<td>7.4508</td>
<td>0.0032</td>
</tr>
<tr>
<td>0.1314</td>
<td>-3.3014-0.0798i</td>
<td>6.6047</td>
<td>3.1174</td>
</tr>
</tbody>
</table>
Fig. 3 The 8 acceleration components with frequency 6.7483 Hz

Fig. 4 Time series at all sensor locations showing traveling wave behavior with speed approximately 7.19 m/s
4.2 Traveling wave of the second most dominant component

(Vandiver 2009) utilized a Fourier-based method to reveal the dominance of traveling wave from an image plot associated with the most dominant frequency component. Their Fourier-based method, however, required a 2-step signal processing on the raw sensor signals: (i) carrying out the fast Fourier transform (FFT) of the time series at each sensor location one by one, and (ii) filtering the signals at the dominant frequency which involves a relatively subjective peak-picking process. In contrast, the proposed method objectively estimates not only the residues associated with the most dominant pole, but also the residues associated with other dominant poles at the same time. For the second most dominant pole \( \lambda_2 = 0.0675 + 62.5668i \), the corresponding frequency \( f_2 = 9.9578 \) Hz and damping factor \( \alpha_2 = 0.0011 \). The residue vector \( r_2 \) for all 8 signals is listed at Table 2. From Table 2, one can conclude that the vibration mode of the riser with \( f_2 = 9.9578 \) Hz is also a complex-valued mode, which suggests that non-standing waves along the marine riser exist for this component as well. Repeating the same task as getting Fig. 3 and Fig. 4 for \( f_1 = 6.7483 \) Hz, one get Fig. 5 and Fig. 6 for \( f_2 = 9.9578 \) Hz. Both figures clearly show the traveling wave phenomena for the second most dominant component also.

<table>
<thead>
<tr>
<th>( z/L )</th>
<th>( r_n )</th>
<th>( A_n )</th>
<th>( \theta_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8907</td>
<td>0.1129+0.0353i</td>
<td>0.2366</td>
<td>-0.3026</td>
</tr>
<tr>
<td>0.7735</td>
<td>0.0681+0.0541i</td>
<td>0.1740</td>
<td>-0.6717</td>
</tr>
<tr>
<td>0.6400</td>
<td>-0.1285-0.0812i</td>
<td>0.3041</td>
<td>2.5781</td>
</tr>
<tr>
<td>0.5555</td>
<td>0.0508+0.0489i</td>
<td>0.1411</td>
<td>-0.7663</td>
</tr>
<tr>
<td>0.4370</td>
<td>0.1079+0.0320i</td>
<td>0.2251</td>
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</tr>
<tr>
<td>0.3381</td>
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<td>0.2404</td>
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<td>0.3051</td>
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</table>
Fig. 5 The 8 acceleration components with frequency 9.9578 Hz

Fig. 6 Time series at all sensor locations showing traveling wave behavior at frequency 9.9578 Hz
5. CONCLUSIONS

This paper proposed a multi-signal complex exponential method which simultaneously processed multiple signals and could estimate the "global" dominating frequencies (poles) shared by those signals. In comparison to the traditional Fourier-based methods, the proposed method completely avoids the problems associated with frequency leakage and resolution. Another advantage of the proposed method is that it requires only short duration signals to conduct the analysis. In the numerical studies, eight acceleration data sets in the in-line direction from the Norwegian Deepwater Program (NDP) high mode test was utilized, and the traveling wave evidence was effectively revealed by using the proposed multi-signal method.

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