The 2014 World Congress on
Advances in Civil, Environmental, and Materials Research (ACEM14)
Busan, Korea, August 24-28, 2014

Bayesian Approach in Structural Tests with Limited Resources

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ABSTRACT

Destructive Testing and Nondestructive Testing are often utilized by forensic structural engineers to understand the existing condition of the structure and estimate the value of random variables for reliability modeling. The classical approach for estimating these values is conducting N tests and adding up the values from the tests and dividing by N. With this classical approach, our prior knowledge through educational background and professional experience cannot provide any benefit to the forensic structural engineers to make decisions. This paper notes that the Bayesian approach in structural reliability theory can be used to update test results by incorporating professional knowledge and structural engineering literature.

INTRODUCTION

As structural engineers, we are educated to evaluate structural members and structural systems to quantify, in scientific terms, performance. We use structural analysis mathematics to quantify this performance. In the forensic engineering of an existing building, we must follow the same evaluation approach.

The area of structural engineering called structural reliability provides a scientific basis for a forensic evaluation. In structural reliability, we use limit states that define the performance of the structural system and its structural members as thresholds of system and member behavior under specified load hazard levels. We use two basic types of limit states and they are called structural system limit states and structural member section limit states.

An example of a structural system limit state is when the displacement of the top floor, or roof, reaches the level at which the building will collapse. This is called an

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ultimate limit state and is referred to as “Ultimate Limit State: collapse of structure”. Another example of a structural system limit state is when building occupants start feeling the building move in high winds. We call this a serviceability limit state and it is referred to as “Serviceability Limit State: human perception”.

The building also has what we call in our structural reliability language structural member section limit states. For example, a structural member section limit state is when the internal member forces and deformations reach cracking level of the concrete occurs and there is a corresponding significant loss of stiffness of the structural member. This is called “Structural Damage Limit State: cracking of concrete”. Another structural member section limit state occurs when the strain in a steel reinforcing bar is at the yield strain and when the load is removed the steel bar will not return to its undeformed condition. This is called the “Structural Damage Limit State: first yield of reinforcing steel”.

UNCERTAINTY IN STRUCTURAL ENGINEERING

In a forensic investigation of a structural system or structural member, we must realize that we have uncertainty in many parts of the evaluation. Table 1 provides examples of sources of uncertainty. (Bulleit 2008, Elnashai et al, 1993, Melchers 1999)

Table 1 Sources of Uncertainty

Ground Motion and Wind Loading Uncertainty. Record-to-record (random) variability exists in the structural element forces and deformations due to different design ground motion time histories and wind time histories.
Test Data Uncertainty. Test data uncertainty is related to the test plan, number of test specimens, type of testing and quality of the tests. For example, a test can be a visual and field analysis of a concrete cylinder removed from a concrete slab or a wood member saw cut from a wood building. This test data is used as information to define/determine/assess the performance of the building.
Structural Member and Section Modeling Uncertainty. Modeling uncertainty is related to how well the analysis models capture the actual structural behavior of a single structural member or of a section of the member. For example, basic structural engineering models can be improved upon by using calibrated and validated finite element models thus reducing structural model uncertainty.
Structural System Modeling Uncertainty. Modeling uncertainty is related to how well the structural analysis models capture the actual total building system behavior. For example, building floors can be modeled as rigid diaphragms or improved upon using finite element (flexible) models.
Constructed Building Uncertainty. The type and frequency of inspection by the structural engineer and others has an impact on the uncertainty of the characteristics of the as-built building. Also, testing during construction has an impact, for example, if concrete test cylinders are tested for only the minimum compressive strength of concrete or in addition for the concrete stress-strain curve.

QUANTIFICATION OF UNCERTAINTY

Uncertainty is typically quantified by structural engineers using the structural reliability term called *Coefficient of Variation*. Table 2 provides acceptable values for the coefficient of variation of several structural engineering variables (ACI 2002, ATC 2009, Ellingwood et al, 1999, Ghiocel et al, 1975). A larger value of the coefficient of variation corresponds to more uncertainty. It is very important to note that by using professional experience and training, we can use this table to guide us in selecting values for the coefficient of variation for our variables we consider at any phase of our forensic investigation.

Table 2 Coefficient of Variation (%)

Rolled Steel Yield Stress	8
Grade 50 Steel Tension Member	9
Reinforcing Bars (Grade 60) Yield Stress	9
Concrete Control Cylinder Compressive Strength (Excellent)	10
Concrete Control Cylinder Compressive Strength (Average)	15
Concrete Control Cylinder Compressive Strength (Poor)	20
Damping in Concrete Building	30
Concrete Modulus of Elasticity	20
Concrete Poisson Ratio	10
Steel Modulus of Elasticity	6
Damping in Steel Frame Building	20
Live Load	25
Maximum Annual Wind Speed	16
Maximum 50-Year Wind Speed	12
Demand and Capacity Prediction from Structural Element Analysis Models that have been verified with a large amount of test data (High Confidence)	20
Demand and Capacity Prediction from Structural Element Analysis Models that have been verified with limited test data (Limited Confidence)	30
Demand and Capacity Prediction from Structural Element Analysis Models that have been verified with very little test data (Little Confidence)	40
ATC 63 Quality Rating Superior Confidence	10
ATC 63 Quality Rating Good Confidence	20
ATC 63 Quality Rating Fair Confidence	35
ATC 63 Quality Rating Poor Confidence	50

COMMUNICATING WITH THE OTHER WORLD

Equations are our tools in our structural engineering analysis and structural reliability work. However, after all the work in our investigation is done, we must ask ourselves, even if the client does not ask us – How confident are we with our opinions? To bridge this potential communication gap, we can use the confidence scale in Table 3 to assign values based to a great extent on professional experience. This table is based on the American Society of Civil Engineers publication called *Degrees of Belief*. (Vick 2002)

Table 3 Confidence Scale

Confidence	Probability	Single Number Probability
Almost Certain	90 to 99.5% sure	90%
Very Likely or Very Probable	75 to 90% sure	80%
Likely or Probable	60 to 75% sure	70%
Medium Chance	40 to 60% sure	50%

DEMAND AND CAPACITY

When the demand exceeds the capacity for a limit state we call this, in structural reliability language, the *Failure of the Limit State*. In structural reliability theory, the Safety Factor (F) is defined as the Capacity (C) of the limit state divided by the Demand (D) on the limit state. That is F = (C/D) and a value of F less than one defines failure.

In a real world forensic study, we never exactly know the capacity or the demand because of limited information. Therefore, C, D and F are in structural reliability language called *Random Variables*. The end result is that failure is a random event. Our structural engineering building code and standard committees have recognized this for almost half a century in design and structural reliability theory has for decades been a part of the foundation of building codes and standards.

We also use a parameter called the Safety, or Reliability, Index to enable us to incorporate the different consequences of failure and the probability of failure accounting for the uncertainty in both the demand and the capacity.

The quantification of either the capacity or the demand corresponding to a limit state in a forensic study always starts with the structural engineer making a best estimate of the expected value of the capacity or demand. Typically, these original estimates are improved upon using results from structural analysis equations and results from experimental tests on structural members conducted at universities or similar laboratories.

ASCE 7-10 provides us with valuable input because many structural members we study in a forensic investigation require us to do structural member testing because test data does not exist on its performance. The following are quotations from ASCE 7-10, Section 1.3.1.3:

> *Assumptions of stiffness, strength, damping, and other properties of components and connections incorporated in the analysis shall be based on approved test data or referenced Standards.*
>
> *Testing used to substantiate the performance capability of structural and nonstructural components and their connections under load shall accurately represent the materials, configuration, construction, loading intensity, and boundary conditions anticipated in the structure…Evaluation of test results shall be made on the basis of the values obtained from not less than 3 tests, provided that the deviation of any value obtained from any single test does not vary from the average value for all tests by more than 15%. If such deviation from the average value for any test exceeds 15%, then additional tests shall be performed until the deviation of any test from the average value does not exceed 15% or a minimum of 6 tests have been performed.*

Also, ASCE 7-10 uses the resistance factor to quantify uncertainty. Even though this equation needs to be presented in a slightly different form in some forensic engineering work, it is important to quote here:

> *Similarly, resistance factors that are consistent with the above load factors are well approximated for most materials by*
>
> $$\phi = \left(\mu_R / R_n\right) \exp[-\alpha_R \beta V_R]$$

THE ART OF STRUCTURAL ENGINEERING AND THE BAYESIAN VIEW OF UNCERTAINTY

Due to the cost of testing and analysis, most forensic structural engineering studies cannot afford the unlimited testing and analysis. Most of the time, no testing is allowed to the forensic engineers due to the limit of budget. Even with this circumstance, forensic engineers can do calculations and analysis based on their educational background, working experience and technical advances made by others. Without the formal mathematics of Bayesian methods of analysis we all as experts offer our opinions and have done so for decades. However, with the mathematical equations of the Bayesian approach we can improve upon the accuracy and credibility of our forensic study opinion. It enables us to use this experience and learning in a well-developed scientific way with very limited test data.

In other perspective, if the forensic engineers just rely on the test data without using their educational background, working experience and technical advances made by others, it is loss of valuable source of information the engineers can use. The faulty

selection of limited test locations without using prior knowledge can also lead engineers to incorrect decisions.

In a forensic engineering study, we must perform the three basic parts shown in Figure 1. Figure 2 presents a more detailed view of the Expected Value Analysis and Uncertainty Analysis parts of a forensic structural engineering study. In performing the strengthening or report part of Figure 2, we used the Expected Capacity and Demand to develop a capacity reduction factor.

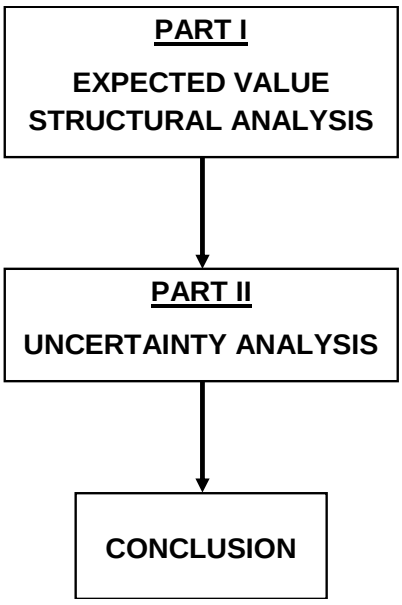

Figure 1 Parts of a Forensic Study

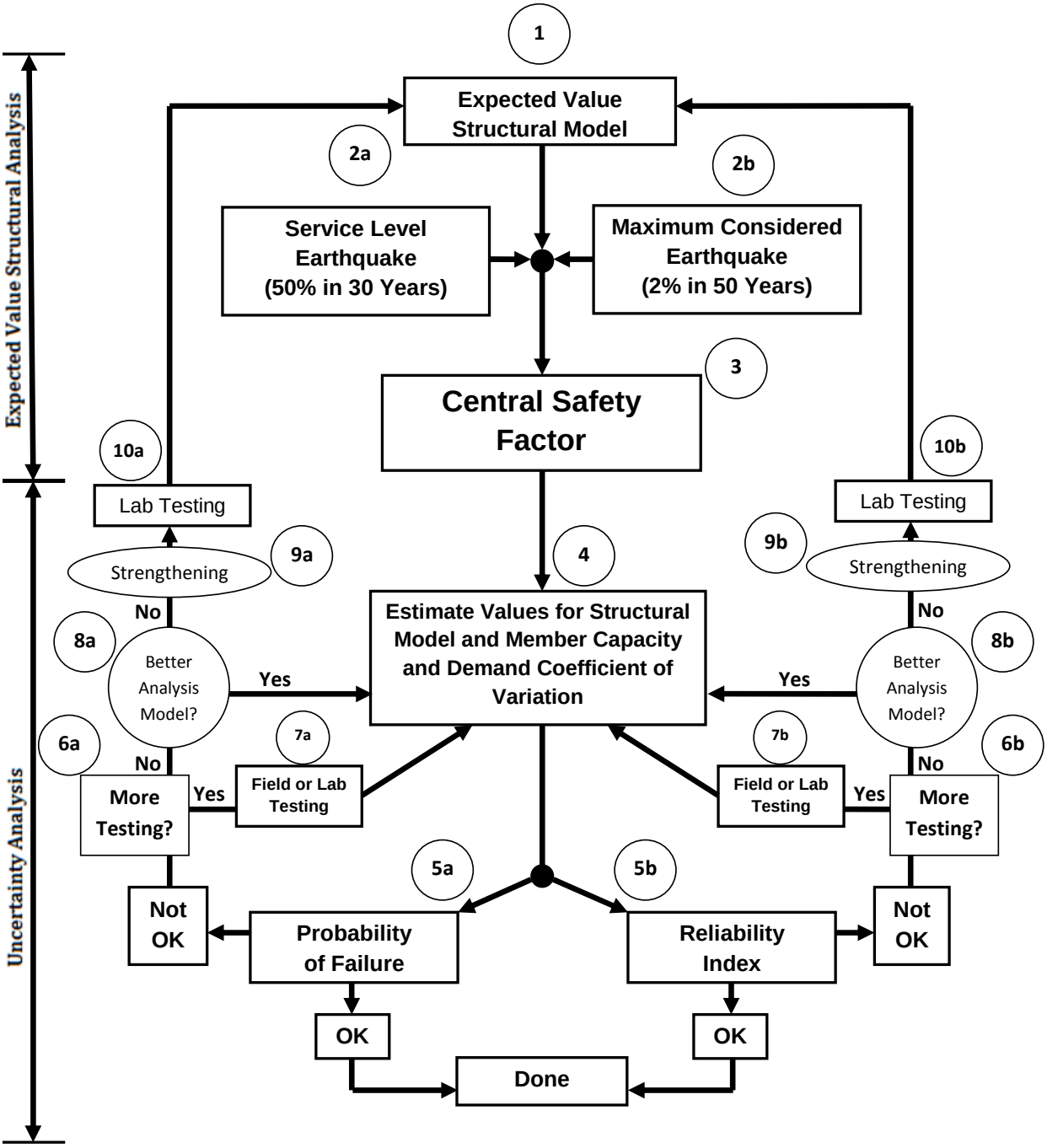

Figure 2 The Structural Analysis Procedure

TRANSPARENCY AND BAYESIAN ANALYSIS

The building code is like a box of Duncan Hines brownie mix in many ways. If one follows the directions on the back of the brownie box, then one gets OK brownies. Similarly, if one follows the code provisions, then one gets an OK but not great building design. To cook the brownie, and to design the structural members in a building using the directions / provisions, can be done without any or perhaps only superficial understanding of what is behind the directions / provisions. Stated differently, there is no transparency and the intent is satisfaction for the "typical" person / building.

We must not be "Duncan Hines cooks" in forensic engineering. But this requires us to make decisions that are based on professional experience or as the fore-fathers called it, the "Art of Structural Engineering." Consider the following quotation from the classic 1961 book by J. A. Blume titled Design of Multistory Reinforced Concrete Buildings for Earthquake Motions (Blume, 1961):

> *"Considerable knowledge has been gained in the last three decades about the phenomenon of ground motions, the characteristics of structures, and their behavior in earthquakes. Despite this progress the complexities are still so great that earthquake-resistant design is not yet capable of complete and rigorous execution solely by means of mathematical analysis, design codes or rules. It is an art as well as a science, and requires experience and judgment on the part of the engineer."*

The mathematics of Bayesian Updating is not simple to follow and understand without considerable effort. Following example will show how to use Bayesian Updating by take prior estimates of the Mean and Coefficient of Variation of Capacity and Demand based only on experience and then to update these estimates with the benefit of Destructive/Non-Destructive field testing, different levels of structural analysis, and laboratory testing of structural members.

EXAMPLE: Classic Case of Normal Base Variable with Unknown Mean and Known Standard Deviation

The Base Random Variable X is the Minimum Compressive Strength of Concrete, f_c', and the Standard Deviation of X is assumed to be known and equal to σ_x = 990 psi. Three (n = 3) test results are available, which are: f_c' = 5,471 psi, 7,823 psi, and 5,326 psi. Note that for this example, the true value of the mean of f_c' (used to simulate the test result data values of f_c', i.e. 5,471 psi, 7,823 psi, and 5,326 psi) is $\bar{X} = 5{,}500\,\text{psi}$, but it is assumed to be unknown.

<u>Step 1</u>: Y is the Nested Random Variable representing the unknown Expected Value of the Base Random Variable X. The Prior Expected Value of Y, i.e. $\bar{Y}_p$, is assumed by the structural engineer to be 5,000 psi.

Steps 2 and 3: The Standard Deviation of X, σ_x, is assumed to be known and equal to 990 psi.

Step 4: Prior Expected Value of X, i.e. $\bar{Y}_p$.

$\bar{Y}_p$ = Prior value of $\bar{Y}$ (i.e. the Expected Value of Mean of $f_c^{'}$).

= 5,000 psi

Step 5: Confidence in $\bar{Y}_p$.

In this example, we will assume a Coefficient of Variation of X. Prior estimated value of the Coefficient of Variation of the Nested Random Variable Y, ρ_{yp}, is 80%. Note this is a very large Coefficient of Variation and is only selected in this example to show convergence with essentially no confidence in the prior estimate of the Expected Value of X. Therefore,

σ_{yp} = Prior value of the Standard Deviation of the Nested Random Variable Y,

i.e. the Standard Deviation of the random variable Expected Value of $f_c^{'}$.

= $\rho_{yp}\bar{Y}_p$ = 4,000 psi

Step 6: Test Data

$\bar{Y}_t$ = Sample Mean = $\left[X^{(1)} + X^{(2)} + ... + X^{(n)}\right]/n$

$$= \frac{1}{3}\left[5471 + 7823 + 5326\right] = 6,207\, psi$$

Step 7: Updated Expected Value of X, i.e. $\bar{Y}_u$.

$\bar{Y}_u$ = Posterior, updated, Expected Value of the Mean of $f_c^{'}$

$$= \left[\frac{\bar{Y}_t\sigma_{yp}^2 + \bar{Y}_p\left(\sigma_x^2/n\right)}{\sigma_{yp}^2 + \left(\sigma_x^2/n\right)}\right] = \left[\frac{1}{1+\left[\left(\sigma_x^2/\sigma_{yp}^2\right)/n\right]}\right]\bar{Y}_t + \left[\frac{1}{1+\left[n\left(\sigma_{yp}^2/\sigma_x^2\right)\right]}\right]\bar{Y}_p$$

$$= C_1\bar{Y}_t + C_2\bar{Y}_p$$

where

$$C_1 = \left[\frac{1}{1+\left(R/n\right)}\right],\ C_2 = \left[\frac{1}{1+\left(n/R\right)}\right] \text{ and } R = \left(\sigma_x/\sigma_{yp}\right)^2$$

$$R = \left(990/4,000\right)^2 = 0.0613$$

The value of R is small because ρ_{yp} = 0.8 which has a very large uncertainty.

$$C_1 = \left[\frac{1}{1+\left(0.0613/3\right)}\right] = 0.98$$

$$C_2 = \left[\frac{1}{1+3/0.0613}\right] = 0.02$$

Therefore,

$$\begin{aligned}\overline{Y}_u &= 0.98\overline{Y}_t + 0.02\overline{Y}_p \\ &= 0.98(6,207) + 0.02(5,000) \\ &= 6,183\,psi\end{aligned}$$

Notice how the test data is very heavily weighted (i.e. 0.98) because the uncertainty of the Prior of the Mean of Y is very large.

<u>Step 8</u>: Updated Variance and Standard Deviation of Y

σ_{yu}^2 = Posterior, updated, Variance of the Expected Value of $f_c^{'}$

$$\sigma_{yu}^2 = \left[\frac{\sigma_{yp}^2\left(\sigma_x^2/n\right)}{\sigma_{yp}^2 + \left(\sigma_x^2/n\right)}\right] = \frac{\left(\sigma_x^2/n\right)}{\left[1+\left[\left(\sigma_x^2/\sigma_{yp}^2\right)/n\right]\right]} = \left[\frac{1}{\left[n+\left(\sigma_x^2/\sigma_{yp}^2\right)\right]}\right]\sigma_x^2 = C_3^2\sigma_x^2$$

where

$$C_3 = \sqrt{1/(n+R)} = \sqrt{1/(3+0.0613)} = \sqrt{0.327} = 0.57$$

Therefore,

$$\sigma_{yu} = 0.57\sigma_x = 0.57(990) = 566\,psi$$

<u>Steps 9 and 10</u>: Predictive Expected Value, Standard Deviation and Coefficient of Variation of X

Predictive Expected Value of X, $\overline{X}_u = \overline{Y}_u = 6,183\,psi$

Predictive Standard Deviation of X, $\sigma_{xu} = \sqrt{\sigma_x^2 + \sigma_{yu}^2} = \sqrt{990^2 + 566^2} = 1,140\,psi$

Predictive Coefficient of Variation of X, $\rho_{xu} = \sigma_{xu}/\overline{Y}_u = 1140/6183 = 0.184$

CONCLUSION

Bayesian updating method provides the users the chance of using their professional knowledge and the experience of others as documented in the published literature not just simply relying on Destructive/Non-Destructive field testing and laboratory testing of structural members. This method can prevent or minimize forensic engineers' incorrect decisions based on the faulty selection of limited test locations or members and allows the forensic engineer to share the benefits from information updating.

REFERENCES

ACI (2002). *Evaluation of Strength Test Results of Concrete* (ACI 214R-02), American Concrete Institute.

ASCE Standard (2010). *Minimum Design Loads for Buildings and Other Structures* (ASCE/SEI 7-10), American Society of Civil Engineers.

ATC (2009). *Quantification of Building Seismic Performance Factors* (ATC 63), Applied Technology Council

Blume, J. A. (1961). “Design of multistory reinforced concrete buildings for earthquake motions.” Portland Cement Association, Chicago.

Bulleit, W. M. (2008). “Uncertainty in Structural Engineering.” *Practice Periodical on Structural Design and Construction,* American Society of Civil Engineers.

Ellingwood B. R., and Tekie P. B. (1999). “Wind load statistics for probability-based structural design.” *Journal of Structural Engineering*, April

Elnashai, A. S., and Izzuddin B. A. (1993). “Sources of uncertainty and future research requirements in seismic analysis of structures.” *Nuclear Energy,* 32, No.4, Aug., 213-220.

Ghiocel D., and Lungu D. (1975). *Wind, snow and temperature effects on structures based on probability*, ABACUS Press

Melchers R. E. (1999). *Structural reliability analysis and prediction*, John Wiley & Sons, England

Vick, S. G. (2002). *Degrees of Belief: Subjective Probability and Engineering Judgment*, ASCE Press.