A simplified model of vortex-induced vertical force on bridge decks for predicting stable amplitudes of vortex-induced vibrations

*Le-Dong Zhu\(^1\), Xiao-Liang Meng\(^2\), Lin-Qing Du\(^3\) and Ming-Chang Ding\(^4\)

\(^1\), \(^4\) State Key Laboratory of Disaster Reduction in Civil Engineering / Key Laboratory for Wind Resistance Technology of Bridges, Ministry of Transport / Department of Bridge Engineering, Tongji University, Shanghai 200092, China

\(^2\) Shanghai Urban Construction Municipal Engineering (Group) Co., LTD, Shanghai 200065, China

\(^3\) Tongji Architectural Design (Group) Co. Ltd, Shanghai 200092, China

ABSTRACT

Wind tunnel tests of large scale sectional model with synchronous measurements of force and vibration responses were carried out to investigate the nonlinear behaviors of vertical vortex-induced forces (VIF) on three typical box decks (full closed box, centrally-slotted box, semi-closed box). The mechanisms of the onset, the development and the self-limiting phenomenon of the vertical vortex-induced vibration (VIV) were also explored via analyzing the energy evolution of different VIVF components and their contributions to the vertical VIV responses. The results show that the nonlinear components of the vertical VIF are often somewhat different from deck to deck; the most important components of the vertical VIF, governing the stable amplitudes of the vertical VIV responses, are the linear and cubic components of velocity contained in the self-excited aerodynamic damping forces. The former provides a constant negative damping ratio to the vibration system and is thus the essential power driving the development of the VIV amplitude, whilst the latter provides a positive damping ratio proportional to the square of the vibration velocity and is actually the inherent factor making the VIV amplitude self-limiting. On these bases, a universal simplified mathematical model of the vertical VIV on bridge decks is then presented in this paper for predicting the stable amplitudes of the vertical VIV of long-span bridges with satisfied accuracy.

1. INTRODUCTION

Steel box decks are very prevalent in the constructions of long-span bridges,
especially those built in strong wind-prone regions because of their good performance against flutter. However, they often suffer from various degrees of vortex-induced vibrations (VIV) (Bureden, 1991; Larsen et al., 2000; Larsen et al., 2008; Ge, 2011), which may cause discomfort of driving and fatigue problem of bridge structural members. Therefore, accurate predictions on VIV responses are very important to the wind-resistant designs of long-span steel bridges, and correct or reliable mathematical models of vortex-induced forces (VIFs) are very necessary to fulfill this end.

VIV is very easy to occur at low wind speeds on long-span bridges, and is always self-limiting in amplitude because of the nonlinearities of VIF. Although the surface mechanism of the VIV nonlinearity may be intuitively attributed to the continuous change of the transient wind attack angle and thus the aerodynamic shape of the bridge deck relative to the incident wind direction during the vibration, the inherent one is actually rather complicated and has not been ascertained thoroughly. The Scanlan’s empirical nonlinear model for vortex-induced force (VIF) is the most famous one frequently used in the research of bridge VIV (Ehsan and Scanlan, 1990; Simiu and Scanlan, 1996). It uses a nonlinear item of aerodynamic damping force, expressed as a product function of the velocity and displacement square of the vibration, to attempt to reproduce the self-limiting behavior of VIV. Larsen (1995) revised the Scalnan’s empirical nonlinear model into a generalized empirical nonlinear model by introducing a shape parameter to adjust the nonlinear order of aerodynamic damping. However, it was found that the Scanlan’s and Larson’s empirical nonlinear models of VIF are not adequate to depict the nonlinear vertical VIF acting on closed box deck and centrally-slotted box deck (Zhu et al., 2013; Meng, 2013).

As mentioned above, the nonlinearity of VIF can be intuitively regarded to be caused by the continuous change of the transient wind attack angle during the deck vibration. Therefore, for the vertical VIV the vertical velocity of deck motion should be included into the auto variables of the nonlinear aerodynamic damping coefficient in the mathematical model of vertical VIF, but it was unreasonably discarded in the most empirical model of VIF, including in the Scanlan’s and Larson’s model. In this connection, new nonlinear models were presented by the authors for the vertical VIFs on different types of box deck, based on a series of wind tunnel tests of large-scale spring-suspended sectional model with simultaneous measurements of dynamic force and displacement response (Zhu et al., 2013; Meng, 2013; Zhu et al., 2015). These models were then proved to be able to depict well the measured nonlinear vertical VIFs on the bridge box-decks and predict the vertical VIV responses with enough accuracy. However, the presented new VIF models contain different nonlinear components for different types of bridge deck. The corresponding identification of the model parameters needs the synchronous measurement of the time history signals of VIF and the dynamic displacement on oscillating sectional models, which is relatively more complicated and difficult compared with the conventional wind tunnel tests with only convenient measurements of dynamic displacement.

In view of this, a simplified nonlinear mathematical model of the vertical VIF was then proposed to reduce the requirements for the techniques and instruments of wind tunnel test for the parameter identification, also to make them easy to be applied to practice. This simplified model, to be introduced in this paper, needs only the convenient measurements of dynamic displacements of sectional model for its
parameter identification and seems to be applicable to different types of bridge decks, including at least the flat box decks.

2. WIND TUNNEL TESTS OF SYNCHRONOUS MEASUREMENTS OF VERTICAL VIFs AND DYNAMIC DISPLACEMENTS

2.1 Three tested typical box decks

Three typical box decks, the full-closed box deck of Xiangshan Harbor Bridge in Ningbo of Zhejiang Province (a cable-stayed bridge with a main span of 688m), the centrally-slotted box deck of Xihoumen Bridge in Zhoushan of Zhejiang Province (a suspension bridge with a main span of 1650m) and the semi-closed box deck of old Haihai Bridge in Tanggu of Tianjin (a single-tower cable-stayed bridge with a main span of 310m) were taken as examples in this study. Their cross sections are shown in Fig.1.

![Deck cross sections of three typical bridges](image)

Fig.1 Deck cross sections of three typical bridges

2.2 Wind tunnel and test facilities and sectional models

Spring-suspended large-scale sectional model tests for synchronous measurements of the VIFs and dynamic displacements on the three typical bridge
decks were carried out in TJ-3 WIND Tunnel at Tongji University. This wind tunnel is a boundary layer wind tunnel of vertical closed-circuit with a closed testing section of 15 m in width, 2 m in height and 14 m in length. The range of wind speed is 1.0 to 17.6 m/s. As an example, Fig.2 shows the schematic diagrams of the large-scale sectional model test of the semi-closed box deck. The sectional model was suspended between two supporting frames in the wind tunnel with 8 helical springs through two suspending arms fixed at the two end of the model. To reduce the disturbance of the frames on the wind flow, the two supporting frames were wrapped with fairings, constituting an internal supporting and fairing wall system. The two supporting and fairing walls were 3.5 m long in the downwind direction and separated at a net distance of 3.63 m in crosswind direction. The windward ends of the walls were of arc shape to improve the flow quality in the testing area between the two walls. The measurement results show that in the case with out any model, the non-uniformity of mean wind speeds is lower than 2%, both the longitudinal turbulent intensity and vertical turbulent intensity are lower than 2%, and both the wind inclination and yaw angles approach to 0°.

Fig.2 Schematic diagram of synchronous measurements of dynamic force and displacement on spring-suspended sectional model

Each of the deck sectional models was comprised of an internal rigid metal framework and an external coat system. The latter was divided into one middle measurement segment and two side non-measurement segments for the full-closed and semi-closed box decks whilst it was a whole segment for measurement for the centrally-slotted box deck. The model coat was made of a frame system of rectangular thin-wall stainless tubules covered by light thin aviation boards with inner liner of high-density foam. The purpose to use such a somewhat complex configuration for the model coat was to ensure it stiff enough and thus to avoid any perceivable local vibration of the coat while reducing its mass and inertia force as possible. The measurement segment of coat was supported on the internal metal framework through four single-component force balances inside the deck model, as shown in Fig.3. Thus, only the dynamic force on the measured coat segment was transferred on the force balances and the inertia force on the force balances was significantly reduced. The details about the model configuration and the balance installation can be referred to
Zhu et al. (2013), Meng (2014) and Zhu et al. (2015). The major parameters of the sectional models are listed in Table 1. The approach for identifying the non-wind-induced additional mass and damping coefficient of the model systems can be found in Zhu et al. (2013), Meng (2014).

Fig. 3 Single-component force balance used in the wind tunnel tests

Table 1 Major parameters of sectional models

<table>
<thead>
<tr>
<th>Deck type</th>
<th>Full-closed box deck</th>
<th>Centrally-slotted box deck</th>
<th>Semi-closed box deck</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length scale $\lambda L$</td>
<td>1/20</td>
<td>1/20</td>
<td>1/15</td>
</tr>
<tr>
<td>Length of model $L$ (m)</td>
<td>3.600</td>
<td>3.600</td>
<td>3.600</td>
</tr>
<tr>
<td>Width of model $B$ (m)</td>
<td>1.600</td>
<td>1.700(1.800)(^1)</td>
<td>1.608</td>
</tr>
<tr>
<td>Depth of model $D$ (m)</td>
<td>0.175</td>
<td>0.175</td>
<td>0.189</td>
</tr>
<tr>
<td>Length of measured coat segment $L_C$ (m)</td>
<td>2.400</td>
<td>3.556</td>
<td>2.400</td>
</tr>
<tr>
<td>Vertical frequency of model $f$ (Hz)</td>
<td>2.808</td>
<td>4.358</td>
<td>2.577</td>
</tr>
<tr>
<td>Mass of whole model $M_S$ (kg/m)</td>
<td>182.178</td>
<td>165.500</td>
<td>215.000</td>
</tr>
<tr>
<td>Mass of model coat per unit length $m_c$ (kg/m)</td>
<td>7.943</td>
<td>10.575</td>
<td>7.925</td>
</tr>
<tr>
<td>Total mass of model system $M$ (kg)</td>
<td>202.450</td>
<td>181.614</td>
<td>234.224</td>
</tr>
<tr>
<td>Nominal total damping ratio of model system at zero wind speed $\xi$</td>
<td>0.55%, 0.73%</td>
<td>0.26%, 0.45%</td>
<td>0.19%, 0.43%</td>
</tr>
</tbody>
</table>

\(1\) 1.700m and 1.800m are respectively the widths without/with the two side cantilever split plates, including the central slot width of 0.3m.

\(2\) Including non-wind-induced additional mass

\(3\) Including non-wind-induced additional damping ratio. The actual damping ratios of the sectional model system are dependent on vibration amplitude.

2.2 Measured displacement responses of VIVs

The dynamic displacement responses of the sectional models were measured with laser displacement sensors. It was found that the most significant VIV occurred, respectively, at the wind attack angles ($\theta$) of +5° for the full-closed box deck, 0° for both the centrally-slotted box deck and the semi-closed box deck, therefore, only the
corresponding test results are to be discussed infra due to the space limitation. The stable amplitudes the vertical VIV in the lock-in ranges of wind speed are shown in Fig.4(a)-(c), respectively, for the full-closed box deck at $\theta=+5^\circ$, for both the centrally-slotted box deck and the semi-closed box deck at $\theta=0^\circ$.

![Graphs of VIV amplitudes for different deck types](image_url)

**Fig.4 Stable amplitudes of vertical VIV of sectional models**

For the full-closed box deck under the wind with an attack angle of $5^\circ$, it can be found that the lock-in ranges of wind speed are, respectively, about 6.44-10.06 m/s and about 6.69-9.90 m/s for the damping ratios of 0.55% and 0.73%. The maximal response of the vertical VIV is 0.0279 m for the test case with a damping ratio of 0.55% and occurs at the wind speed of about 9.10 m/s. It is 0.2612 m for the test case with a damping ratio of 0.73% corresponding to the wind speed of about 9.55 m/s.

For the centrally-slotted box deck under the wind with an attack angle of $0^\circ$, it can be seen that the lock-in ranges of wind speed are, respectively, about 5.15-7.07 m/s and about 5.32-7.06 m/s for the damping ratios of 0.26% and 0.45%. The maximal response of the vertical VIV is 0.0114 m for the test case with a damping ratio of 0.26% and occurs at the wind speed of 5.62 m/s. It is 0.0086 m for the test case with a damping ratio of 0.45% corresponding to the wind speed of 5.98 m/s.

For the semi-closed box deck under the wind with an attack angle of $0^\circ$, the lock-in ranges of wind speed are, respectively, about 4.54-7.12 m/s and about 4.53-6.66 m/s for the damping ratios of 0.19% and 0.43%. The maximal response of the vertical VIV is
0.0141 m for the test case with a damping ratio of 0.26% and occurs at the wind speed of 5.75 m/s. It is 0.0107 m for the test case with a damping ratio of 0.45% corresponding to the wind speed of 5.92 m/s.

Fig. 5 Time histories of measured vertical VIV on model coats per unit length

Fig. 6 Amplitude spectra of measured vertical VIFs on model coats per unit length

2.3 Measured vertical VIFs during VIVs

The vertical VIFs on the measured coat segments were extracted from the total dynamic forces measured by four small single component force balances installed inside the box decks. The details about the extraction approach of the vertical VIFs can be referred to Zhu et al. (2013), Meng (2014). The extracted time histories of the
vertical VIFs on the model coats per unit length during the GTR process of VIV at or near the wind speeds corresponding to the maximal VIV responses were plotted by using blue lines with small hollow circles in Fig.5(a)-(c), respectively, for the three typical box decks. The corresponding spectra of VIFs were plotted by same type of lines in Fig.6(a)-(c), respectively.

It can then be seen that, for all the three decks, the curve shapes of VIF time histories deviate far away from sinusoidal curves, and notable multiple-frequency components are contained in the measured vertical VIFs. This indicates that all the vertical VIFs acting on the three typical box decks are strongly nonlinear. However, the relation among the contributions of different multiple-frequency components is different for the three typical box decks. For example, for the full-closed box deck, the double-frequency component takes almost the same portion as the fundamental-frequency component, and the triple-frequency component also take a non-negligible portion; the rest multiple-frequency components are very small and can be ignored. For the centrally-slotted deck, the nonlinearity of the vertical VIF is relatively weaker than those for the other two types of box decks, but both the triple-frequency and quadruple-frequency components are considerable whilst the other multiple-frequency components including the double-frequency component provide very small contributions. For the semi-closed box deck, the double-frequency component is significant although it is clearly smaller than the fundamental one; the triple-frequency is also notable and cannot be neglected; the rest multiple-frequency components are insignificant.

3. NONLINEAR MATHEMATICAL MODELS OF VERTICAL VIFS

As proved in Zhu et al. (2013) and Meng (2014), Scanlan’s empirical nonlinear mathematical model is not suitable for expressing the vertical VIF acting on bridge decks. This is because the nonlinearity of vertical VIF can be understand, from viewpoint of quasi-steady theory, as caused by the continuous change of the deck aerodynamic shape, which is resulted in by the continuous change of the equivalent attack angle of the transient resultant wind due to the existence of the vertical velocity of the deck motion. Therefore, the nonlinear aerodynamic damping ratio should mainly depend on the vertical velocity, instead of the vertical displacement, of the deck motion. On this account, the following different refined nonlinear mathematical models were proposed by the authors, respectively, for describing the nonlinear vertical VIFs acting on the per unit length decks of the foregoing-mentioned three types of box deck.

For full-closed box deck (Zhu et al., 2013; Meng 2014):

\[
f_{v_i} = \rho U^2 D \left[ Y_i(K) \left( 1 + \varepsilon_{10}(K) \frac{y^2}{U^2} + \varepsilon_{11}(K) \frac{y}{U} \right) \frac{y}{U} \right.
\]

\[
+ Y_i(K) \frac{y}{D} + \frac{1}{2} \tilde{C}_i(K) \sin \left( K \frac{U}{D} t + \psi(K) \right) \]

(1)

For centrally-slotted box deck (Meng, 2014; Zhu et al., 2105):
\[ f_{vl} = \rho U^2 D \left[ Y_1(K) \left( 1 + \varepsilon_{i0}(K) \frac{y^2}{U^2} + \varepsilon_{i1}(K) \frac{y}{U} + \varepsilon_{i01}(K) \frac{y^2}{U^2} + \varepsilon_{i11}(K) \frac{y}{U} \right) \frac{\dot{y}}{U} \right. \\
\left. + Y_2(K) \left( 1 + \zeta_2(K) \frac{y}{D} \right) \frac{\dot{y}}{D} + \frac{1}{2} \tilde{C}_L(K) \sin \left( K \frac{U}{D} t + \psi(K) \right) \right] \]

For the semi-closed box deck (Ding, 2016):

\[ f_{vl} = \rho U^2 D \left[ Y_1(K) \left( 1 + \varepsilon_{i0}(K) \frac{y^2}{U^2} + \varepsilon_{i1}(K) \frac{y}{U} \right) \frac{\dot{y}}{U} \right. \\
\left. + Y_2(K) \left( 1 + \zeta_2(K) \frac{y}{D} \right) \frac{\dot{y}}{D} + \tilde{C}_L(K) \sin \left( K \frac{U}{D} t + \psi(K) \right) \right] \]

where, \( \rho \) is the air density, \( U \) is wind speed, \( D \) is the deck depth, \( y \) and \( \dot{y} \) are the vibration displacement and velocity, respectively. \( Y_i(K), Y_2(K), \varepsilon_i(K) \) \((i = 0, 1, 2; j = 1, 2, 3, 4)\), \( \zeta_2(K) \) are the \( K \)-dependent model parameters of the vertical self-excited force (SEF) to be identified through tests, and \( \tilde{C}_L(K), K_{vs}(K) = \omega(K)D/U \) and \( \psi(K) \) are the \( K \)-dependent amplitude coefficient, reduced vortex-shedding frequency and phase difference of the vertical pure vortex-shedding force (VSF) to be identified through tests, \( \omega_i(K) \) is circular frequency of vortex shedding. \( K = \omega D/U \) is the reduced frequency, \( \omega \) is the circular frequency of VIV.

The verifications of the above mathematical models will not be included in this paper because it is not the concerned topics of this paper. One can refer to Zhu et al. (2103) and Zhu et al. (2105) for the first two mathematical models and Ding (2016) for the last mathematical model.

4. SIMPLIFIED MATHEMATICAL MODEL OF VERTICAL VIF AND VERIFICATION

4.1 Simplified mathematical model

The authors also carried out the evolution analyses of the work done by different components of nonlinear vertical VIF on the vibration system and the parametric analyses about the influences of the different vertical VIF components on the vertical VIV displacement responses (Zhu et al., 2103; Meng 2014; Zhu et al., 2105; Ding, 2016). The results demonstrate that the most important components to the stable amplitudes of VIV displacement responses in all the above three nonlinear VIF models are only the linear item of velocity (\( \dot{y} \)) and the nonlinear cubic item of velocity (\( y^3 \)). Ignoring the other components may cause the evident or even remarkable changes of the vertical VIFs and the notable changes of the cumulating phase of the long-term displacement responses, but leads to little changes of the stable amplitudes of VIV displacements. Furthermore, it is also found that during the vertical VIVs the linear item of \( \dot{y} \) provides a constant negative aerodynamic damping ratio to the vibration system while the nonlinear cubic item of \( y^3 \) provides a time-varying positive aerodynamic damping ratio increasing with the development of VIV. Obviously, the vertical VIV will get stable when the average value of the time-varying damping ratio of the vibration
system which consists of the positive structural damping ratio, the constant negative aerodynamic damping ratio and the time-varying positive nonlinear aerodynamic damping ratio. Therefore, it can be concluded that the linear negative aerodynamic damping force is the primary power driven the VIV development while the nonlinear positive aerodynamic damping force related to $y^3$ is the inherent factor of the self-limiting phenomenon of the vertical VIV. In view of this, the above nonlinear mathematical models as shown in Eqs.(1)–(3) can be simplified into a unified nonlinear model as Eq.(4), or even to a more simplified one as Eq.(5).

$$f_{v1} = \rho U^2 D \left[ Y_1(K) \left( 1 + \varepsilon_{03} \frac{y^2}{U^2} \right) \frac{\dot{y}}{U} + Y_2(K) \frac{y}{D} \right]$$  \hspace{1cm} (4)

$$f_{v1} = \frac{1}{2} \rho U^2 (2D) \left[ Y_1(K) \left( 1 + \varepsilon_{03} \frac{y^2}{U^2} \right) \frac{\dot{y}}{U} \right]$$  \hspace{1cm} (5)

4.2 Parametric identification of simplified mathematical model

The approximate formula for estimating the amplitude of the vertical VIR based on the above simplified nonlinear mathematical model of vertical VIF can be deduced under the assumption that both the amplitude and phase functions of VIR have slow variation behaviors (Zhu et al., 2103).

$$\eta(s) = \frac{\beta}{\sqrt{1 - (\beta^2 / A_0^2)}} e^{-\alpha \beta^2 s / 4} \cos \{ Ks - \psi_0 \} = A(s) \cos \{ Ks - \psi_0 \}$$  \hspace{1cm} (6)

Where, $A_0$ and $\psi_0$ are the initial amplitude and phase of the “Decay to Resonance (DTR)” or “Grow to Resonance (GTR)” of VIV, respectively. $A(s)$ is the time-varying amplitude of DTR or GTR. $\beta$ is the steady amplitude of the VIR displacement response. $\alpha$ is a parameter reflecting the varying rate of the vibration amplitude of displacement during the decay or growing stage of the VIR. The values of $\alpha$ and $\beta$ can be obtained with least square fitting method by using only the measured displacement response. $K_0$ is the reduced frequency of the vibration system under zero wind speed.

Then one can derive that the following relationships for identifying the model parameters.

$$Y_1 = \frac{\beta^2 \alpha + 8 \xi K_0}{4} \cdot \frac{m}{\rho D^2}$$  \hspace{1cm} (7)

$$\varepsilon_{03} = - \frac{4 \alpha}{3K \left( \beta^2 \alpha + 8 \xi K_0 \right)}$$  \hspace{1cm} (8)

$$Y_2(K) = \left( K_0^2 - K^2 \right) \cdot \frac{m}{\rho D^2}$$  \hspace{1cm} (9)

Where, $m$ is the distributed mass.

Model parameters of full-closed box deck and verification

The values of $Y_1$ and $\varepsilon_{03}$...
of the concerned full-closed box deck at various reduced wind speeds within the whole lock-in range of vertical VIV under the condition of wind attack angle being +5°, which are identified with the simplified vertical VIF model as Eq.(5) based on the measured displacement responses, are shown in Fig.9. The corresponding values of $Y_1$ and $\varepsilon_{03}$, identified with the refined vertical VIF model as Eq.(1) based on the measured time histories of both the displacement responses and the vertical VIF (see Zhu et al., 2013; Meng 2014), are also plotted in Fig.9 for a comparison. One can then see that the two sets of identified parameters are quite close to each other. Obviously, comparing with the parameter identification of the refined VIF model, the parameter identification of the simplified VIF model is much more simple from both aspects of the identification algorithm and the requirement for the wind tunnel testing techniques.

![Fig.7 Identified parameters of simplified and refined models for full-closed box deck](image)

![Fig.8 Stable amplitudes of vertical VIV displacement calculated with the simplified VIF model as Eq.(5) and the measured ones for full-closed box deck](image)

The vertical responses of the vertical VIV of the sectional model system within the whole lock-in range of VIV can then be calculated with the simplified VIF model as Eq.(5) and the corresponding identified parameters mentioned above. The calculated dimensionless stable amplitudes of vertical VIV displacement within the lock-in range
together with the measured ones are plotted in Fig. 10. The two sets of responses agree very well to each other, demonstrating the reliability of the simplified vertical VIF model and the feasibility of such simplification for predicting the stable amplitudes of vertical VIV of the full-closed box deck.

Model parameters of centrally-slotted box deck and verification

Fig. 10 shows a comparison between the curves of $Y_1$ and $\varepsilon_{03}$ of the concerned centrally-slotted box deck vs. the reduced wind speed within the whole lock-in range, which were identified, respectively, with the simplified vertical VIF model as Eq. (5) and with the refined vertical VIF model as Eq. (2) (see Meng 2014). Obviously, the two sets of identified parameters agree well with each other.

Fig. 10 Identified parameters of simplified and refined models for centrally-slotted box deck

Fig. 11 Stable amplitudes of vertical VIV displacement calculated with the simplified VIF model as Eq. (5) and the measured ones for centrally-slotted box deck

Fig. 11 shows the dimensionless stable displacement amplitudes of the vertical VIV of the sectional model system of the centrally-slotted box deck within the whole lock-in range, which are calculated according to the simplified VIF model as Eq. (5). The corresponding measured ones are also plotted in this figure. One can find again that
the two sets of amplitudes are very well consistent with each other, indicating out that the simplified vertical VIF model is also reliable for the centrally-slotted box deck.

Model parameters of semi-closed box deck and verification. Fig. 13 exhibits a comparison between the curves of $Y_1$ and $\varepsilon_{\theta_3}$ of the concerned semi-closed box deck vs. the reduced wind speed within the whole lock-in range, which were, respectively, identified with the simplified vertical VIF model as Eq. (5) and with the refined vertical VIF model as Eq. (3) (see Ding 2016). Obviously, the two sets of identified parameters agree well with each other.

Fig. 13 Identified parameters of simplified and refined models for semi-closed box deck

Fig. 14 Stable amplitudes of vertical VIV displacement calculated with the simplified VIF model as Eq. (5) and the measured ones for semi-closed box deck

The dimensionless stable displacement amplitudes of the vertical VIV of the sectional model system of the semi-closed box deck within the whole lock-in range, which are calculated according to the simplified VIF model as Eq. (5), are shown in Fig. 14. The corresponding measured ones are also plotted in this figure. It is evident that the two sets of amplitudes are well consistent with each other, indicating out that the simplified vertical VIF model is also reliable for the semi-closed deck.
5. CONCLUSIONS

Considering that the linear negative aerodynamic damping force is the primary power driven the VIV development while the nonlinear positive aerodynamic damping force being a cubic function of vibration velocity is the inherent factor of the self-limiting phenomenon of the vertical VIV, a simplified mathematical model of nonlinear vertical VIF acting on bridge decks was then presented and discussed in this paper for the purpose of predicting stable displacement amplitudes of vertical VIV of bridges with an enough accuracy. The simplified VIF model was verified to be applicable to the three typical box decks discussed in this paper and has a good prospect to be applied to other types of bridge decks.

5. ACKNOWLEDGEMENT

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REFERENCES

Ding M.C. (2016), Study on the Nonlinear Vortex-induced Forces on Semi-closed Box Decks, Master Dissertation, Tongji University, Shanghai, China.

