Investigation on surface distributed vortex-induced force on flat-closed-box girder using POD

*Xingyu Chen¹, Yongle Li², Bin Wang³ and Ledong Zhu⁴

¹),²),³) Department of Bridge Engineering, Southwest Jiaotong University, 610031 Chengdu, Sichuan, P. R. China
³) State Key Laboratory of Disaster Reduction in Civil Engineering, Tongji University, 200092 Shanghai, P. R. China
¹⁾ysyfcxy@126.com

ABSTRACT

In order to guarantee the performance of bridges and the comfort of vehicles, the vortex-induced forces need to be investigated. The total vortex-induced forces of bridge decks are analyzed previously. This paper is focused on the characteristics of surface distributed vortex-induced force. Proper orthogonal decomposition (POD) method is applied on the surface distributed vortex-induced forces of a flat-closed-box girder. According to the contributions of each mode shape, only the first 5 modes and their time histories of principal coordinates are analyzed. The results show that there are some relationship between modes and physical phenomena. It can be found that the modes with base frequency or double-frequency can be associated to the distribution of the vortexes. Understanding of this relationship would allow to carry out certain measures to shatter the vortex, thereby to inhibit the occurrence of vortex-induced vibration.

1. INTRODUCTION

Flat-closed-box girders are widely adopted in long-span bridges, for the advantages of light weight, high strength and good aerodynamic performance against flutter instability. Nevertheless, this kind of girder is often subjected to vortex-induced vibration (VIV) at low wind speeds, such as Strobelt Bridge (Larsen et al. 2000), Trans-Tokyo Bay Crossing Bridge (Fujino and Yoshida 2002) and Xihoumen Bridge (Li et al. 2011, Laima et al. 2013). Although VIV is a kind of self-limited vibration without disastrous consequences, continual VIVs may cause structural fatigue and discomfort to pedestrians and vehicles. Thus, VIV of flat-steel-box girders should be taken seriously.
and an accurate and reasonable prediction is necessary.

Actually, limited research can be found for the vortex-induced force (VIF) of flat-closed-box bridge decks. The experimental results of Diana et al. (2006) indicated significant nonlinear behaviors of the VIF on a multi-box deck. Sun et al. (2009) testified that aerodynamic coefficients can be obtained via CFD with \( k-\omega \) RANS turbulence model. The mechanism and vibration reduction of VIV of a box cross section in the presence of aerodynamic countermeasures were discussed by Sarwar and Ishihara (2010), which showed the influences of fairings and flaps. Hallak et al. (2012) discussed the aerodynamic behavior of a girder in the presence of tall vehicles via 2D CFD model.

POD method is an effective statistics tool to describe the surface pressure of structures, its principle is decomposing the pressure field into the time related principal coordinates and the space related covariance mode shapes (Solari and Carassale 2000). Through this approach, it is always found that the first several modes contribute most energy of the pressure fluctuations, thus the hidden features of data could be found through several essential modes. The analysis of the covariance mode shapes could provide information about the mechanisms of the subject investigated, and the efforts to find the connection between modes and physical phenomena were made by some researchers (Kikuchi et al. 1997, Holmes et al. 1997, Baker 2000). They argued whether each mode is associated with vortex shedding, flow separation and reattachment, and so on. Kikuchi et al. (1997) made a conclusion that each dominant mode is associated with an aerodynamic phenomenon, however, Holmes et al. (1997) stated that it is impossible to associate each essential mode with a physical action. Aiming at this difference, Ricciardelli et al. (2002) claimed that the association between covariance modes and mechanisms of excitation is not always one-to-one, in some cases, a given mode also contributes to other phenomena.

In this connection, CFD was utilized to explore the vortex-induced force of a flat-closed-box girder. Then the simulated results were decomposed into principal coordinates and covariance mode shapes using POD method. Aimed at VIV of this bridge deck, an attempt was made to associate the mode shapes with a certain physical phenomenon.

**2. NUMERICAL SIMULATION**

**2.1 Governing equations and numerical simulation method**

The unsteady RANS method was adopted in this numerical simulation. The fundamental theory behind this method is to average the instantaneous governing equations in time domain. After being averaged, the governing equations become

\[
\frac{\partial u_i}{\partial t} + \rho \frac{\partial u_i u_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j^2} - \rho \frac{\partial u_i u_j}{\partial x_j},
\]

where \( \rho \) and \( \nu \) are the density and the dynamic viscosity coefficient of air, respectively;
subscript $i, j$ represent the coordinate axis in space with $i=1, 2$ and $j=1, 2$ for two-dimensional simulation; $x_i$ is the coordinate value along the $i$ axis; $u_j$ and $u'_j$ are the average velocity and the fluctuating velocity along $j$ axis, respectively; $t$ is the time; $p$ is the pressure; and $-\rho u'_i u'_j$ is the so-called Reynolds stress and this was represented by SST $k-\omega$ turbulence model in this simulation. The governing equations were discretized using the Second-order scheme based on the finite volume method. SIMPLEC algorithm was employed for the coupling of pressure and velocity. The pressure was discretized in a standard way. The momentum, turbulent kinetic energy and specific dissipation rate were performed utilizing the second-order form. Gradient terms were handled by the use of the least squares cell-based method. The commercial software FLUENT was employed to solve the governing equations.

![Computational domain sketch](image)

**Fig. 1** Computational domain sketch

### 2.2 Computational domain and boundary condition

Zhu et al. (2013) discussed the VIV of a flat closed-box bridge deck, and wind tunnel test was adopted in that research. To facilitate the validation of the simulation results, the bridge model and other parameters used in the wind tunnel test were also
utilized in this simulation. The length scale is 1:20; the mass and the stiffness of the girder are 50.61 kg/m and 15742.76 N/m, respectively. Moreover, as introduced in Zhu et al. (2013), the actual damping coefficient ratio varies linearly with the change of vibrating amplitude, but for improving the calculation efficiency, the damping coefficient ratio is defined as a constant of 0.5% when selecting the meshing and time step.

As can be seen from Fig. 1, the total size of the computational domain is 18B × 28D (B is the width of the scaled girder, D is the depth of the scaled girder). To improve the computational efficiency, the domain is divided into several parts, including rigid domain, dynamic-mesh domain, wake domain and exterior domain. The rigid domain moves synchronously with the girder, thus the dynamic-mesh domain is filled with moving mesh which is triangular gird. Because of the vortex street, the girds in wake domain are densified. The densities of structured girds in exterior domain are larger than others.

Fig. 1 shows the boundary conditions. As the wind attack angle is +5°, the left and the bottom are the sources of the upcoming wind, then a uniform wind speed of 9.1 m/s, turbulent intensity of 0.02 and turbulent viscosity ratio of 2 are assigned to the two boundaries. After the flow passes the bridge deck, wind blows out of the domain through the right and the top. Thus, the right and the top are assigned as pressure outlets with zero pressure. The deck surfaces are defined as no-slip wall boundaries.

2.3 Time step and meshing

In this study, three time steps 0.000223 s, 0.000445 s and 0.00089 s, are used to check the influence of time step on simulation results. These three time steps are corresponding to T/1600, T/800 and T/400, respectively. For all cases, the first 10 s is treated as a converging process and the corresponding results are ignored. The simulation results in the next 10 s are used to be compared. The amplitude of vertical displacement and the vibration frequency obtained using the three time steps are shown in Table 1. The results show that the displacement and frequency of the first two time steps (0.000223 s and 0.000445 s) are somewhat similar. Therefore, the time step of 0.000445 s could be accepted in the following numerical simulations.

Then three meshing schemes M1, M2 and M3 are generated to verify the influence of meshing scheme on simulation results. The overall mesh generation method keeps same for the three meshing schemes but the height of the first layer girds varied. In the numerical simulation, different heights of the first layer correspond to different y+, and the y+ value has great influence on the simulation results. The simulation results, using different meshing schemes, are shown in Table 2. It can be seen that, because the y+ value is below 1, the results of M1 and M2 are very similar. In consideration of both the accuracy of the simulation results and the computational efficiency, the meshing scheme M2, as shown in Fig. 2., is finally applied to the following numerical simulations.
2.4 Simulation results and validation

The simulated vertical displacement of the girder and its comparison with wind tunnel test are listed in Table 3. As introduced in Section 2.2, the linearly damping coefficient ratio is used in simulation results except the calculation for selecting the meshing and time step. Thus, the simulated vertical displacement listed in Table 3 is smaller than the result of M2 shown in Table 2. As shown is Table 3, it can be seen that the maximum vertical displacements and dominant frequencies of the two methods agree well with each other with the difference ratio below 5%.

Table 1 Computed results of three different time steps

<table>
<thead>
<tr>
<th>Time step (s)</th>
<th>Amplitude of Vertical displacement (m)</th>
<th>Vibration Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000223</td>
<td>0.0234</td>
<td>2.73</td>
</tr>
<tr>
<td>0.000445</td>
<td>0.0250</td>
<td>2.73</td>
</tr>
<tr>
<td>0.00089</td>
<td>0.0265</td>
<td>2.71</td>
</tr>
</tbody>
</table>

Table 2 Computed results of three different meshing schemes

<table>
<thead>
<tr>
<th>Meshing scheme</th>
<th>First layer girds height (m)</th>
<th>Max $y^+$ value</th>
<th>Amplitude of Vertical displacement (m)</th>
<th>Vibration Frequency (Hz)</th>
<th>Gird number (million)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>$3 \times 10^{-5}$</td>
<td>0.2</td>
<td>0.0335</td>
<td>2.75</td>
<td>0.35</td>
</tr>
<tr>
<td>M2</td>
<td>$1.5 \times 10^{-4}$</td>
<td>0.7</td>
<td>0.0335</td>
<td>2.76</td>
<td>0.24</td>
</tr>
<tr>
<td>M3</td>
<td>$3 \times 10^{-4}$</td>
<td>1.5</td>
<td>0.0250</td>
<td>2.73</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Fig. 2 Meshing scheme 2
Besides the vertical displacement, the VIF also is a key result for VIV. According to Zhu et al. (2013), VIF could be identified from the lift through the following formulas

\[
F_v^0(t) = -m_0 \ddot{y}^0(t) - c_0 \dot{y}^0(t), \tag{3}
\]

\[
F_{VI}(t) = F_v(t) + m_0 \ddot{y}^0(t) + c_0 \dot{y}^0(t), \tag{4}
\]

where the index 0 represents the zero wind speed condition; \( \ddot{y}^0 \) is the fluctuating part of the lift in the zero wind speed condition; \( \ddot{y}^0 \) and \( \dot{y}^0 \) are the acceleration and the velocity of the girder; \( m_0 \) and \( c_0 \) are the non-wind-induced additional mass and damping coefficient of the girder, respectively; \( F_v \) is the fluctuating part of the lift force; and \( F_{VI} \) is the VIF. \( m_0 \) and \( c_0 \) can be obtained from Eq. (3). Through a simulation with zero wind speed, they are 6.051 kg and 3.098 N·s/m, respectively.

Fig. 3 shows the VIF, the amplitude spectrum and their comparison with tested results. It can be seen that the simulated VIF agrees well with the tested one. Furthermore, the tested VIF is not smooth because of the measurement error, while the simulated VIF is smooth and stable.

![VIF and Amplitude Spectrum](a) VIF (b) Amplitude spectrum of VIF

**Fig. 3** VIF at resonance stability period and comparison with test results

As a summary, it can be concluded that the simulated results agree well with tested results, and that this numerical simulation is acceptable and accurate. Moreover, it could be found that the VIF on the flat-closed-box girder presents a fairly strong...
multiple-frequency characteristic.

3. ANALYSIS OF DISTRIBUTED VIF USING POD

3.1 POD method

POD method is an effective statistics tool to describe the surface pressure of structures, its principle is decomposing the pressure field into the time related principal coordinates and the space related covariance mode shapes. Through this approach, a series of stochastic processes could be described by several key modes, and these essential modes always relate to the hidden features of data.

Assuming the quantity of pressure points is \( n \), and then the fluctuating pressure time history with zero mean of any point could be decomposed as

\[
p_i(t) = \sum_{j=1}^{n} a_{ij}(t) \varphi_j,
\]

where \( i \), ranged from 1 to \( n \), represents the number of the pressure point; \( j \), also ranged from 1 to \( n \), is the number of the mode; \( a_{ij}(t) \) is the \( j \)th principal coordinate; \( \varphi_j \) is the \( j \)th covariance mode corresponding to \( i \)th pressure point. After decomposing all the pressure time histories, the results could be integrated as

\[
P(t) = \sum_{j=1}^{n} a_{ij}(t) \Phi_j,
\]

\[
\Phi_j = [\varphi_j \cdots \varphi_n],
\]

where \( a_{ij}(t) \) and \( \Phi_j \) are the \( j \)th principal coordinate and the \( j \)th covariance mode of the surface distributed pressure for the structure. Moreover, the eigenvalue \( \lambda_j \) and the eigenvector \( \Phi_j \) of the \( n \times n \) covariance matrix \( C_P \) of the pressure field are calculated simultaneously. The eigenvalue \( \lambda_j \) is the variance of the principal coordinate \( a_{ij}(t) \) associated with the mode \( \Phi_j \), and the contribution of \( j \)th mode could be expressed via \( \lambda_j \) as follows:

\[
\Lambda_j = \frac{\lambda_j}{\sum_{j=1}^{n} \lambda_j},
\]

3.2 POD modes and their corresponding physical phenomena

In this study, 301 pressure points are distributed on the surface of the girder, after numerical simulation, the pressure time histories of these points are obtained. Then through Eqs. (3) and (4), the VIF of every point could be acquired. Aimed at the VIFs,
301 modes and their time histories of principal coordinates are obtained through POD techniques introduced in Section 3.1. As shown in Fig. 4, the first 5 modes give a contribution of nearly 100% to the total aerodynamic excitation. Then it can be concluded that the first 5 modes could describe the main features of the vertical vortex-induced vibration. Thus, only the first 5 modes and their time histories of principal coordinates are analyzed later.

In Figs. 5 to 7 the spectrums of principal coordinates, the contour of modes and the corresponding vortex distributions and are shown, for the first 3 covariance modes shapes.

As shown in Fig. (4), the contribution of the first mode is about 75%. The spectrum of the 1-st principal coordinate is shown in Fig. 5(a), it can be seen that the predominant frequency is 2.715 Hz, corresponding to the Strouhal frequency. As shown in Fig. 5(b), the VIFs distributed at the upstream roof are positive or negative, actually, most of the VIFs distributed around the bridge deck are negative, and the negative VIFs with large value are distributed at the downstream roof. Aimed at this distribution law, a corresponding physical phenomenon has been found, as shown in Fig. 5(c). It can be seen that there are several small vortexes locate at the upstream roof, and that a huge vortex locates at the downstream roof. Moreover, the airflow near the floor of this deck is relatively stable, except the location of the downstream track maintenance. This suggests that the distribution of vortexes agrees well with the distribution of VIFs of the first covariance mode shape. It can then be concluded that, for this flat-closed-box girder, the huge vortex located at the downstream roof may have a significant influence on the vortex-induced vibration and the total vortex-induced force.

Fig. 4 also shows that the second mode gives a contribution of more than 20% to the total aerodynamic excitation. In Fig. 6(a), the spectrum of the 2-nd principal coordinate is plotted, and it can be found that the predominant frequency is 2.710 Hz, also corresponding to the vertical natural frequency. As shown in Fig. 6(b), only the VIFs distributed at the middle portion of the upper surface are negative with relative large value, and the remaining VIFs distributed around the girder are positive, especially, the values of VIFs distributed at the upstream roof and the underside of downstream web are relative large. Fig. 6(c) shows a corresponding physical phenomenon for this distribution law. A huge vortex locates at the middle portion of the upper surface, meanwhile, since the obstruction of the safety barrier, this vortex will be divided into two parts. Also there is a vortex of medium size located at the upstream roof, and a relative small vortex locates at the underside of downstream web. Furthermore, the airflow near the downstream roof is relatively stable with no vortex. It illustrates that this physical phenomenon agrees well with the distribution of VIFs of the second covariance mode shape. It can be found that, for the second mode, the roof of this flat-closed-box girder influences the vortex-induced vibration and the total vortex-induced force obviously.

As can be seen from Fig. 4, the contribution of the third mode is less than 2%. Fig. 7(a) shows the spectrum of the 3-rd principal coordinate, it can be seen that the
predominant frequency is 5.426 Hz, equal to double of the vertical natural frequency. As shown in Fig. 7(b), the positive VIFs are distributed at the initiation of the upper surface and part of the downstream roof, and the VIFs distributed at the roof are negative with relative large value except the above-mentioned positions. Meanwhile, the VIFs distributed at the latter part of the downstream floor are negative with relative large value. Corresponding to this distribution law, a physical phenomenon is shown in Fig. 7(c). A huge vortex locates at the upstream roof adjacent to the middle portion, and another relative huge vortex locates at the downstream roof. There is obvious interaction between the underside of downstream web and the airflow, the same to the ends of the upper surface. Furthermore, there is no vortex located at the ends of downstream floor, but since the affect of the downstream track maintenance, the airflow flows around the track maintenance with intensive streamline. It can be seen that this physical phenomenon does not match well with the distribution of VIFs of the third covariance mode shape. It can also be found that, for the third mode, the roof of this flat-closed-box girder also influences the vortex-induced vibration and the total vortex-induced force obviously.
(a) Amplitude spectrum of principal coordinate

(b) The covariance mode shape

(c) Corresponding physical phenomenon

Fig. 5 The POD results of the first mode
Fig. 6 The POD results of the second mode

(a) Amplitude spectrum of principal coordinate

(b) The covariance mode shape

(c) Corresponding physical phenomenon
Fig. 7 The POD results of the third mode

(a) Amplitude spectrum of principal coordinate

(b) The covariance mode shape

(c) Corresponding physical phenomenon
In Fig. 4, it can be found that both the forth and the fifth modes give a contribution of less than 1% to the total aerodynamic excitation. From Fig. 8(a) and Fig. 9(a), it can be seen that the outstanding frequencies of the forth and the fifth modes are double or triple of the vertical natural frequency. As shown in Fig. 8(b) and Fig. 9(b), for the forth and the fifth modes, the distribution of VIFs is messy and irregular, thus, there is no corresponding physical phenomena which agree well with the distribution of VIFs. It means that the correspondence is not always one-to-one, especially for these modes with high-order frequencies, these modes may combine with other key modes, and then correspond to a certain physical phenomenon. Actually, it is just for these modes that there are high-order terms of the total VIF. However, although the distribution of VIFs is messy and irregular, the value of VIFs distributed at the roof is relative large. It also illustrates that the VIFs of the roof are most important for the vortex-induced vibration and the total vortex-induced force.

4. CONCLUSIONS

Studies on the vortex-induced force of a flat-closed-box girder are carried out through CFD simulation with SST k-ω turbulence model in this study. Then POD method has been adopted to analyze the simulated results, and the following conclusions are drawn.

(1) The numerical simulation method is acceptable and accurate, the simulated results agree well with tested results.

(2) The VIF on the flat-steel-box girder presents a fairly strong multiple-frequency characteristic.

(3) The first 5 modes could describe the main features of the vertical vortex-induced vibration for this flat-closed-box girder.
There is a certain correspondence between the covariance mode shapes and the physical phenomenon. The mode shapes with base frequency or double-frequency can be associated to the distribution of the vortexes.

(5) The correspondence is not always one-to-one, especially for these modes with high-order frequencies, these modes may combine with other key modes, and then correspond to a certain physical phenomenon.

(6) The roof of this flat-closed-box girder influences the vortex-induced vibration and the total vortex-induced force obviously.

ACKNOWLEDGEMENTS

The authors are grateful for the financial supports from the Open subject of Key Laboratory of Disaster Prevention in Civil Engineering under Grant SLDRC14-01, and the Sichuan Province Youth Science and Technology Innovation Team (2015TD0004).

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