Modal identification of Canton Tower using the fast Bayesian FFT method

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ABSTRACT

The Bayesian theory provides a promising perspective in modal identification. The fast Bayesian FFT (FBFFT) method is capable of identifying the most probable value of modal parameters and providing the reasonable estimation of the posterior uncertainty of the variables. The FBFFT method, which is originally proposed for a stationary and wide-band process, is extended to deal with the non-stationary response data. With the full-scale measurement data under Heyuan earthquake and Haima typhoon, the modal parameters of the Canton Tower are identified by the random decrement technique and the fast Bayesian FFT method. Then the aerodynamic damping ratio of the tower under the typhoon condition is estimated by subtracting the total damping from the structural damping.

1. INTRODUCTION

Nowadays with the development of the measurement technique, structural healthy monitoring (SHM) has attracted growing attention in civil engineering. With the development of the measurement technique during the past decades, a large number of SHM systems have been conducted (Sohn 2002, Kijewski-Correa 2006, Guo 2012, Zhang 2016). Li et.al conducted a systematic research on modal identification, wind field observation and wind-induce vibration measurement for a series of high-rise buildings including Jin Mao Building, Taipei 101 tower (Li 1998, 2006, 2011). Ni at.al conducted a sophisticated long-term SHM system with more than 700 sensors of Canton Tower and proposed an SHM benchmark for high-rise structures that has involved tremendous research efforts throughout the word (Ni 2009, 2012).
Modal parameter identification is a major part of the SHM problems which can be employed for finite element model updating, damage detection and structural vibration control. Due to the limitation of full-scale measurement, the input excitations are usually unavailable. Therefore extensive attention are devoted to output-only modal identification and quite a number of algorithms are proposed which can be divided into two main parts, in frequency domain and in time domain. In frequency domain, the peak-pick technique (PP), frequency domain decomposition (Brincker 2001), PolyMAX (Peeters 2004) are widely accepted. In time domain, there are more modal identified methods, for instance, time series analysis, random decrement technique (RDT), natural excitation technique (James 1993), stochastic subspace identification (Peeters 1999), etc.

Nevertheless, the majority of the proposed modal identification method are based on empirical statistical theory which can’t account for the uncertainty of the modal parameters and the system errors. The Bayesian theory provides a promising perspective in modal identification (Katafygiotis 2001, Yuen 2003, Au 2011). The approach under Bayesian framework is capable of identifying the most probable value (MPV) of modal parameters and providing the reasonable estimation of the posterior uncertainty of the variables. With the coefficient of variation, the MPVs of modal parameters are more reliable when evaluating the dynamic performance (Au 2012).

The FBFFT method is a frequency domain method in the Bayesian framework and originally proposed for a stationary and wide-band process. In this paper, it is extended with the empirical mode decomposition (EMD) to deal with the non-stationary earthquake response. Based on the full-scaled vibration date under earthquake and typhoon, the FBFFT and EMD-based FBFFT are employed to obtain the modal parameters of Canton Tower. For comparison, HHT combined the RDT are used to identify the modal parameters as well. Considering the posterior uncertainty, the result of two method are basically consistent with each other. Then the aerodynamic damping ratio of the tower under the typhoon condition is estimated by subtracting the total damping from the structural damping.

2. BRIEF REVIEW OF THE FBFFT METHOD

Based on the Bayesian inference, the measured acceleration time history of n DOFs of a structure \( \{ \tilde{x} \in \mathbb{R}^n \} \) can be modeled as:

\[
\tilde{x} = \dot{x}(\theta) + \epsilon
\] (1)

Where \( \dot{x}(\theta) \) is the modal acceleration response of the real modal parameters \( \theta \), \( \epsilon \) is the prediction error resulting from the measuring error and the modeling error.

The FFT of \( \tilde{x} \) is expressed as:

\[
F_k = F_k + iG_k \pm \sqrt{2\Delta t/N} \sum_{j=1}^{N_q} \hat{x}_j \exp \{-2\pi i[(k-1)(j-1)/N]\}
\] (2)

Where \( F_k \) and \( G_k \) is the real and imaginary part of the FFT. \( \Delta t \) = the sampling interval \( k = 2,3, \ldots, N_q, N_q = \text{int}[N/2] + 1. \)

Let \( Z_k = [F_k \ G_k]^T \) be the augmented vectors. For a high sampling rate and long duration, \( \{ Z_k \} \) is prove to be asymptotically and zero mean Gaussian vector (Yuen 2002). According to the Bayes' theorem, assuming no prior information, the posterior PDF of \( \theta \) given \( \{ Z_k \} \) is proportion to the likelihood function:

\[
P(\theta|\{ Z_k \}) \propto P(\{ Z_k \} | \theta)
\]
\[ L(\theta) = -\ln(\mathcal{P}(\{Z_k\}|\theta)) = \frac{1}{2} \left[ \sum_k \ln(\det C_k(\theta)) + \sum_k Z_k^T C_k(\theta)^{-1} Z_k \right] \]  

(4)

The most probable value of \( \theta \) can be obtained by minimizing the NLLF, and the posterior covariance matrix can be estimated by the inverse of the Hessian of the NLLF. As a matter of fact, the numerical optimization of the NLLF is extremely difficult owing to the large number of variables involved. For well-separated modes, more convenient and efficient algorithm is put forward by selecting a single mode to identify the corresponding modal parameters (Au 2001). After simplifications, Eq.4 can be written as

\[ L(f, \xi, S, \sigma^2, \Phi) = -n N \ln 2 + (n-1) N \ln \sigma^2 + \sum_k \ln \left( SD_k + \sigma^2 \right) + \sigma^{-2} (d - \Phi^T A \Phi) \]  

(5)

\[ D_k = \left[ \left( f^2 / f_k^2 - 1 \right)^2 + \left( 2 \xi f / f_k \right)^2 \right]^{-1} \]  

(6)

\[ A = \sum_k (1 + S_k / SD_k)^{-1} E_k \]  

(7)

\[ E_k = F_k F_k^T + G_k G_k^T \]  

(8)

\[ d = \sum_k (F_k^T E_k + G_k^T G_k) \]  

(9)

The natural frequency and damping ratio can be identified in the optimality of Eq.5. The uncertainty of parameters can be characterized by the posterior covariance matrix. The level of uncertainty is evaluated by the posterior coefficient of variation (COV).

Note that the FBFFT method is proposed for a stationary and white noise excitation and is approximately credible to deal with the wide-band process, such as the response under wind load (Au 2012). For earthquake excitation is generally regarded as strong non-stationary and narrow-band process, the FBFFT method is unavailable to identify the modal parameters from the earthquake-excited response. To meet the condition, the FBFFT is extended with the empirical mode decomposition (EMD). EMD is a signal processing technique proposed by Huang (1998). Based on the envelop curve of successive extreme values, any data can be decomposed into several intrinsic mode functions (IMF). Considering the signals at the level of their local oscillation, EMD is a self-adaptive decomposition method and reliable to dealing with non-stationary signals.

The EMD-based FBFFT can be conducted as following steps. Firstly the non-stationary structural response under earthquake is decomposed into the intrinsic mode
functions (IMF) by using the EMD. Then pick up the IMFs of the specific frequencies and employ the FBFFT to the decomposed signals to identify the modal parameters.

3. FULL-SCALE MEASUREMENT OF CANTON TOWER

3.1 SHM system of the Canton Tower

Canton Tower, located at Guangzhou, is a 610m tube-in-tube structure, with 454m of major structure and 156m of antenna mast. Currently it is the highest tower in China and the third in the world. Since its construction period, a long-term and sophisticated structural healthy monitoring system has been installed in the Canton Tower with more than 700 sensors by the Hong Kong Polytechnic University (Ni 2009).

Twenty uni-axial accelerometers are installed at 8 different sections to acquire the vibration date of the buildings, with a 50Hz sampling frequency, as shown in Fig.1. Each section is at least equipped with two orthogonal accelerometers. The 4th and 8th sections are equipped with four accelerometers, two for the long axis and the other two for the short axis. Fig.2 shows the location and measurement direction of the accelerometers in a plan. The SHM system has acquired a large plenty of the response data under ambient and extreme excitations, including several earthquakes and typhoons.

Fig.1 Monitoring system of the Canton Tower
3.2 Full-scaled vibration measurement

In this paper the acceleration time history during the Heyuan earthquake and Haima typhoon of the Canton Tower are adapted for modal identification during its service period, which are the two typical and significant load in the design of the high-rise buildings.

Heyuan earthquake was 4.2 magnitude and occurred at 13:50 PM China Standard Time on 31th August, 2012. The epicenter was in Heyuan, about 160km away from Guangzhou. The SHM system monitored the acceleration response from 13:00 to 14:00. Fig.3 shows the acceleration time histories of the top and bottom section.

Haima typhoon was the fourth tropical storm in 2011 and landed in Guangdong province in 10:10, 23rd June. The anemograph and the accelerometers measured the wind speed and acceleration response from 00:07 to 19:07, 19 hours in total. Fig.4 shows the 10 minutes average wind speed and wind direction. During the measuring period, the majority of the average wind speed ranges from the 9m/s to 12m/s and the maximum average wind speed is 14.44m/s in 11hour. The average wind direction changes a little in the first 14 hours and the main direction is shown in Fig.1. Fig.5 shows the wind speed and the acceleration time histories in 11hour. As the X-direction
is more close to main wind direction, the maximum acceleration response is 3.22milli-g, much larger than that of Y-direction, 1.59milli-g.

![Fig.4 10 minutes average wind speed and direction of typhoon Haima](image)

![Fig.5 time histories of the fluctuating wind velocity and acceleration response in 11 hour](image)

### 3.3 Analysis of the acceleration time histories

The power spectrum density (PSD) of the acceleration response under earthquake and typhoon is shown in Fig.7. Due to the dynamic characteristics of two excitations, the PSDs of the acceleration response differ clearly in frequency domain. Under earthquake there lie several obvious peaks in higher modes and the peak values decline insignificantly with the increasing of frequency. On the contrary, under typhoon the first two modes are the dominant in the response and the peak values decrease rapidly with the increase of frequency. The difference of the dominant frequencies under earthquake and typhoon shows the discrepancy of the dynamic characteristics of the excitations (Chen 2011). Heyuan earthquake, as a short-distance earthquake, can be more likely to cause the high vibration modes, while the typhoon is generally considered to be a long-periodic process hence the frequencies of the highest PSD peaks is less than 1Hz.

By the peak-pick method the natural frequencies of the Canton Tower can be easily identified. The frequencies of the first five vibration modes are listed in Table 1. The first four modes is translational and the fifth mode is torsional. The frequency under two excitations differs a little and the difference are quite small.
Fig. 6 PSD of the acceleration under earthquake and typhoon

Table 1. The identified frequency by the Peak-Pick method

<table>
<thead>
<tr>
<th>Mode</th>
<th>1X</th>
<th>2Y</th>
<th>3X</th>
<th>4Y</th>
<th>5T</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earthquake</td>
<td>0.0906</td>
<td>0.1331</td>
<td>0.3706</td>
<td>0.4611</td>
<td>0.4969</td>
</tr>
<tr>
<td>Typhoon</td>
<td>0.0916</td>
<td>0.1342</td>
<td>0.3681</td>
<td>0.4617</td>
<td>0.4983</td>
</tr>
</tbody>
</table>

Difference* = (f_{Typhoon} - f_{Earthquake}) / f_{Earthquake} * 100%

4. MODAL PARAMETER IDENTIFICATION

4.1 Bayesian parameters

Using the fast Bayesian FFT method, each mode is identified individually. Table 2 shows the initial frequency and the relative frequency band of the first five vibration modes. The initial frequency is selected by the peak-pick method directly from the PSD of acceleration and the frequency band of a single mode is selected manually.

Table 2. The initial frequency and frequency band

<table>
<thead>
<tr>
<th>Num</th>
<th>Mode</th>
<th>Frequency (PP) /Hz</th>
<th>Lower /Hz</th>
<th>Upper /Hz</th>
<th>Bandwidth</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>X</td>
<td>0.0916</td>
<td>0.087</td>
<td>0.096</td>
<td>9.8%</td>
</tr>
<tr>
<td>2</td>
<td>Y</td>
<td>0.1342</td>
<td>0.128</td>
<td>0.139</td>
<td>8%</td>
</tr>
<tr>
<td>3</td>
<td>X</td>
<td>0.3681</td>
<td>0.340</td>
<td>0.380</td>
<td>11%</td>
</tr>
<tr>
<td>4</td>
<td>Y</td>
<td>0.4617</td>
<td>0.430</td>
<td>0.480</td>
<td>11%</td>
</tr>
<tr>
<td>5</td>
<td>T</td>
<td>0.4983</td>
<td>0.480</td>
<td>0.510</td>
<td>6%</td>
</tr>
</tbody>
</table>

4.2 Modal shape

Based on the measured accelerations under typhoon Haima and Heyuan earthquake, the mode shapes of Canton tower are identified by the FBFFT method. Fig. 7 shows the first four sway mode shapes, in which 1(X) denotes the first sway mode of X direction. Apparently each mode has a reasonable consistence between earthquake and typhoon, indicating that the different excitations have a little impact on the mode shapes of the structure.
4.3 Damping ratio and frequency

For comparison the HHT combined with RDT is also applied in identifying the natural frequencies and damping ratios. Table 2 shows the identified frequencies and damping ratios of first five modes during typhoon. As a reference, the result identified by Guo (2012) is also listed together. It is noted that the damping identified under typhoon excitation should be regarded as total damping, consisting of two parts, the structural and aerodynamic damping. Fig. 2 shows the identified damping ratios of the first five modes under typhoon. The green bars indicate the MPVs of damping by the FBFFT method and the dash line represents the corresponding ±σ confidence intervals.

It can be seen that the MPV of frequency differs a little with the HHT under different excitations and it agrees with the results of reference, revealing that the results identified by two methods are reliable. Meantime the coefficients of variation (COV) of frequency is quite small, less than 0.5%, which indicates little uncertainty and high confidence level of frequency. It must be pointed that the most probable values (MPV) of frequency identified by FBFFT method are slightly less than those by the HHT of all modes.

On the contrary, note that the COVs of damping ratio are very high in contrast with those of frequency, indicating much greater posterior uncertainty of damping. This might be attributed to the complicacy of damping ratio. The COVs of x-direction are
much larger than those of y-direction and the maximum COV lies in the first sway mode, more than 20%. Meantime the damping has much discrepancy by different methods, especially in contrast with that in Guo (2012). However, considering the posterior uncertainty, the MPVs of damping are still consistent with those determined by the HHT to some extent.

Table 2. Identified modal parameters under typhoon

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency</th>
<th>Damping</th>
<th>Guo (2012)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MPV*/Hz</td>
<td>COV*/%</td>
<td>HHT/Hz</td>
</tr>
<tr>
<td>1(X)</td>
<td>0.0912</td>
<td>0.24%</td>
<td>0.0980</td>
</tr>
<tr>
<td>2(Y)</td>
<td>0.1340</td>
<td>0.32%</td>
<td>0.1377</td>
</tr>
<tr>
<td>3(X)</td>
<td>0.3693</td>
<td>0.07%</td>
<td>0.3735</td>
</tr>
<tr>
<td>4(Y)</td>
<td>0.4601</td>
<td>0.07%</td>
<td>0.4730</td>
</tr>
<tr>
<td>5(T)</td>
<td>0.4962</td>
<td>0.08%</td>
<td>0.5008</td>
</tr>
</tbody>
</table>

*MPV = most probable value, COV = coefficient of variation, identified in FBFFT method.

As shown in Fig. 1, the Heyuan earthquake occurred at 52’, so the first 51.5 minutes before the seismic wave affected the building, contains little information about Heyuan earthquake. Therefore it can be regarded as the ambient excitations. To make full use the record during the earthquake, two different conditions, the ambient response consisted of first 51.5 minutes and the entire earthquake acceleration response record, are defined to assess the influence of this earthquake. For the acceleration response under earthquake is apparently a non-stationary process, the aforementioned EMD based FBFFT is employed for dealing with the entire earthquake record data.

Identified modal parameters under earthquake and the ambient excitation are listed in table 3 and table 4. As mentioned before, the epicenter of Heyuan earthquake
is only about 160km away from Guangzhou, so this earthquake should be classed as short-distance earthquake. As a reference another result under short-distance earthquake, Shenzhen earthquake (Li 2015), is listed in Table3 as well. It is identified by the frequency domain decomposition algorithm (FDD). It is seen that the damping ratios by the FDD are basically larger than others, which is similar as Taipei 101 tower (Li 2011). This may be attributable to the spectral leakage of Fourier transform, which will over-estimate the damping when utilizing the FDD (Brincker 2001).

Familiar with the result under typhoon the frequencies identified by different means under two conditions differ a little and damping ratios remain high discrete levels. As Fig.3 shown, the COVs of earthquake are much larger than those of ambient excitation. Considering ±σ confidence intervals, the damping ratios identified by different methods are consistent well under two excitations. The difference of damping ratios between the earthquake and ambient excitations is relatively small, showing that Heyuan earthquake has little impact on the structure due to the small magnitude and low-intensity.

Table3. Identified modal parameters under earthquake

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency</th>
<th>Damping ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MPV/Hz</td>
<td>COV</td>
</tr>
<tr>
<td>1(X)</td>
<td>0.0909</td>
<td>0.60%</td>
</tr>
<tr>
<td>2(Y)</td>
<td>0.1332</td>
<td>0.49%</td>
</tr>
<tr>
<td>3(X)</td>
<td>0.3702</td>
<td>0.30%</td>
</tr>
<tr>
<td>4(Y)</td>
<td>0.4611</td>
<td>0.27%</td>
</tr>
<tr>
<td>5(T)</td>
<td>0.4939</td>
<td>0.28%</td>
</tr>
</tbody>
</table>

Table4. Identified modal parameters under ambient excitation

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency</th>
<th>Damping ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MPV/Hz</td>
<td>COV</td>
</tr>
<tr>
<td>1(X)</td>
<td>0.0912</td>
<td>0.36%</td>
</tr>
<tr>
<td>2(Y)</td>
<td>0.1334</td>
<td>0.20%</td>
</tr>
<tr>
<td>3(X)</td>
<td>0.3706</td>
<td>0.07%</td>
</tr>
<tr>
<td>4(Y)</td>
<td>0.4611</td>
<td>0.06%</td>
</tr>
<tr>
<td>5(T)</td>
<td>0.4951</td>
<td>0.08%</td>
</tr>
</tbody>
</table>
5 AERODYNAMIC DAMPING RATIO

Aerodynamic damping is a simplification of the mutual effect between the structure and wind. For a wind-excited structure, the identified damping is generally regarded as the total damping, consisting of two parts, structural and aerodynamic damping.

\[ \zeta = \zeta_s + \zeta_a \]  

(10)

where \( \zeta_s, \zeta_a \) and \( \zeta_a \) are the total, structural and aerodynamic damping ratio. Due to the complexity of damping mechanism and the uncertainty of modal identification algorithms, the structural damping is greatly influenced by the forms of excitation so \( \zeta_s \) is estimated with ambient data and assumed as a constant value (Aquino 2013). Based on the modal identified results under typhoon and ambient excitation, \( \zeta_a \) can be calculated.

The aerodynamic damping ratios of first five modes are individually identified by the HHT and the FBFFT method. In the same fashion as Fig.8 and Fig.9, Fig.10 shows the aerodynamic damping ratio by two algorithms. The Green bar represents the MPV of \( \zeta_a \) by the FBFFT method while the dash line indicates substantially the posterior uncertainty with the \( \pm \sigma \) confidence intervals. Fig.10 shows the identified \( \zeta_a \) has a reasonable agreement between two methods despite the difference of the second mode is relatively large. And it can be seen that the aerodynamic damping ratios of X-direction is larger than those of Y-direction and the torsion mode damping ratios is relatively small. In total the aerodynamic damping of the first five modes is positive except the third mode, which is still close to zero. It may ascribe to the structural form and shape of Canton Tower. It is a super-tall tube-in-tube structure while its outer tubes are inclined in the vertical direction with concrete-filled tube (CFT) columns. The hyperbolic shape might have a positive influence in its aerodynamic characteristic.

It is generally accepted that the aerodynamic damping of high-rise building is closely relative to the reduced wind velocity \( V_r \). In low reduced wind velocity, the across-wind direction aerodynamic damping is larger than along-wind direction’s,
though across-wind direction aerodynamic damping is more likely to turn to be negative in high reduced wind velocity (Marukawa 1996, Quan 2005).

\[ V_r = \frac{V_s}{f_s B} \]  

(11)

Where \( V_s \) is oncoming wind speed, \( f_s \) is natural vibration frequency and \( B \) is the reference width of the building.

The reduced wind velocity of this typhoon is about 3~4, in the region of low reduce wind velocity. This might explain why the aerodynamic of y-direction is larger than that of X-direction because Y-direction is close to the across-wind direction as show in Fig.1. So the Y-direction can be approximately regarded as across-wind direction.

![Fig. 10 Identified aerodynamic damping ratio by HHT and FBFFT methods](image)

6. CONCLUSIONS

This study presents the structural and aerodynamic parameter identification by the fast Bayesian FFT method. To deal with the non-stationary and narrow-band process, the FBFFT method is extended with the empirical mode decomposition (EMD). The non-stationary structural response is firstly decomposed into the several IMFs by EMD. Then pick up the IMFs of the specific frequencies and employ the FBFFT to identify the modal parameters.

With the full-scale measurement data under Haima typhoon and Heyuan earthquake, the modal parameters, such as mode shapes, natural frequencies and damping ratios are identified by the FBFFT and EMD-based FBFFT method. For comparison the HHT combined with RDT is also employed to identify the modal parameters. The identified frequencies agree well with each other while damping ratios have a relative high discrepancy under different excitations. Considering ±σ confidence intervals, the agreement of damping ratios between the FBFFT methods and the HHT are is acceptable.
Then regarding the damping under ambient excitation as the structural damping, the aerodynamic damping ratio of the tower under the typhoon condition is estimated by subtracting the total damping from the structural damping. It has a reasonable agreement between two methods despite the difference of the second mode is relatively large. Apart from the third mode, the aerodynamic damping ratios are positive indicating that the dynamic shape has a potential positive influence in its aerodynamic characteristics.

REFERENCES


