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Back-analysis of the Parameters for a Creep Model using the Response Surface Methodology

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ABSTRACT

Realistic predication of the post-construction settlement is important for high-fill embankment design. In this paper, a creep model is proposed to capture the time-dependent behavior of coarse-grained soils and is implemented in the finite difference code FLAC 2D. Response Surface Methodology (RSM) is adopted to back-analyze the parameters of the creep model of materials of a high-fill embankment, based on the field monitoring data. The numerical results are found in good agreement with the field monitoring data for most observation points, which implies that the proposed creep model and the RSM are effective to back-analyze the post-construction settlement in high-fill embankment.

1. INTRODUCTION

In recent years, a large number of high-fill embankments and dams located in mountainous areas of China are under construction or being planned (Liu & Li et al., 2004; Chen & Song et al., 2011; Wu & Yuan et al., 2014). These projects have become increasingly high and complex. Thus, prediction about their post-construction settlement becomes extremely important. Usually a large number of observation instruments have been installed at important cross-sections.

In general, coarse-grained soils have been extensively used for the construction of these high-fill embankments and dams. Its mechanic properties play a dominate role in predicting their post-construction settlement (Bishop 1955; Xu et al., 2009; Xu & Song et al. 2012; Wu & Yuan et al. 2014). Finite element and finite difference analysis are usually adopted for deformation prediction (Duncan, 1996; Costa & Alonso, 2009), which requires establishing reasonable constitutive models to capture the creep behavior of coarse-grained soils observed in the lab and the field, including the

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nonlinear stress-strain relationship, time-dependent behavior, and stress-dependent behavior. Therefore, there is a general need to establish a reasonable creep model for coarse-grained soils.

In this paper, a creep model which can reflect the influence of confining pressure is proposed firstly. Response Surface Methodology, or RSM, is employed to back-analyze the model parameters. Based on the results of back-analysis, the long-term settlement distribution is plotted and analyzed.

2. PROJECT DESCRIPTION

The high-fill embankment is located on the northwest mountainous area of Sichuan province in China. There are several engineering challenges in this project: 1) the large volume of construction (excavation $3.093\times10^7\,\mathrm{m}^3$ and filling $2.763\times10^7\,\mathrm{m}^3$); 2) the large filled height (the airport has a maximum height of 104 m); 3) the fast loading process (only 14 months); 4) the thick soft original soil layers. In this paper, the cross-section (with the largest filled height and thickest original soft soil) is chosen for numerical simulation.

The filled process of this cross-section is extremely complex and fast, which is divided into 6 steps (seen in Fig. 1). The construction is started at the middle of April 2001 and completed at the end of November 2002, which lasts only 14 months except for the shutdown period in winter. To ensure safety and learn the settlement characters, observation instruments are installed on chosen cross-sections (Fig. 1). The monitored settlement curves of the runway at different positions (the inside line, the center line and the outside line) are plotted in Fig. 2.

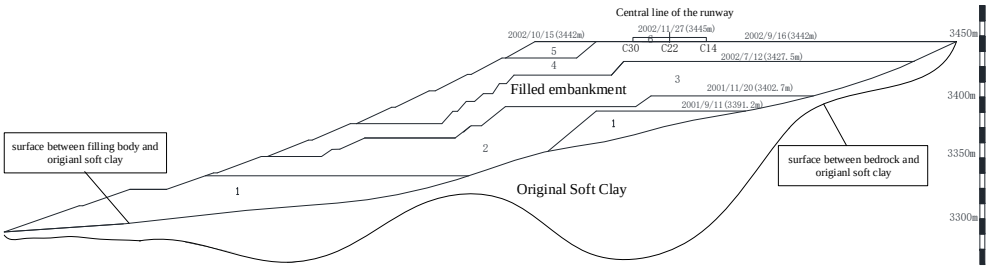


Fig. 1 the filled process of Yuanshanzi Valley

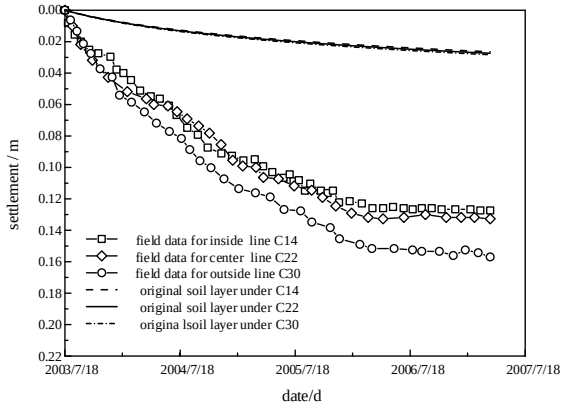


Fig. 2 the settlement curve of monitoring data and original soil layers

3. MATERIAL MODELS AND DISPLACEMENT BACK-ANALYSIS METHOD

3.1 Material models

The filled embankment and original soil layers are the two main components contributing to the settlement. Thus, the numerical calculation employs two models: 1) a creep model, which is employed to capture the time-dependent behavior of the filled embankment; 2) the secondary consolidation theory of clay, which is used to calculate the vertical deformation of original soil layers.

3.1.1 The creep model for coarse-grained soils

Coarse-grained soil is a frictional material consisted with soil particles. Therefore, the confining pressure has a determinant impact on its time-dependent behavior. In this paper, a creep model is proposed to capture the creep character of the coarse-grained soils. The proposed model is schematically characterized by modified components in Fig. 3, where a Kelvin unit, a Newton dashpot unit and a nonlinear elastic unit is connected in series. The Kelvin unit is characterized by the shear modulus G^K and viscosity η^K; the Newton dashpot unit is characterized by the viscosity η^M; the nonlinear elastic unit is characterized by the Young's Modulus E and Poisson's ratio υ. The total strain is formulated as Eq. (1)

$$\varepsilon_i = \varepsilon_i^K + \varepsilon_i^N + \varepsilon_i^S \tag{1}$$

where, ε_i^K is the strain of Kelvin unit; ε_i^N is the strain of Newton dashpot unit; ε_i^S is the strain of nonlinear elastic unit; the subscript ($i=1,2,3$) represents the three orthogonal direction in principle stress space. Eq. (2) to Eq. (4) are employed to reflect the influence of confining pressure:

$$G^K = 10^{K_1} P_a \left(\frac{\sigma_3}{P_a}\right)^{n_1} \tag{2}$$

$$\eta^K = 10^{K_2} \left(P_a \cdot h\right) \left(\frac{\sigma_3}{P_a}\right)^{n_2} \tag{3}$$

$$\eta^M = 10^{K_3}\left(P_a \cdot h\right)\left(\frac{\sigma_3}{P_a}\right)^{n_3} \tag{4}$$

where, K_1 , η_1 , K_2 , n_2 , K_3 , n_3 are the parameters of the proposed model; P_a is the atmospheric pressure; h is standard hour.

To capture the nonlinear elastic behavior of coarse-grained soils, the Duncan-Chang Model (1970), shown in Eq. (5), is used to calculate the time-independent deformation.

$$E = \left(1 - R_f \frac{1-\sin\varphi}{2c\cos\varphi + 2\sigma_3 \sin\varphi}\left(\sigma_1 - \sigma_3\right)\right)^2 K_4 P_a \left(\frac{\sigma_3}{P_a}\right)^{n_4} \tag{5}$$

where, K_4 , n_4 , R_f are the parameters. c and φ are the two parameters of Mohr-coulomb criterion. The nonlinear strength behavior can be represents as

$$\varphi = \varphi_0 - \Delta\varphi \log\left(\frac{\sigma_3}{P_a}\right) \tag{6}$$

where, φ_0 is the value of φ for $\sigma_3 = P_a$ and $\Delta\varphi$ is the reduction in φ for a 10-fold increase in σ_3. It can be seen that there are six parameters (c, φ_0, $\Delta\varphi$, K_4, n_4, R_f) in Duncan-Chang Model, which can be determined using a set of conventional triaxial tests.

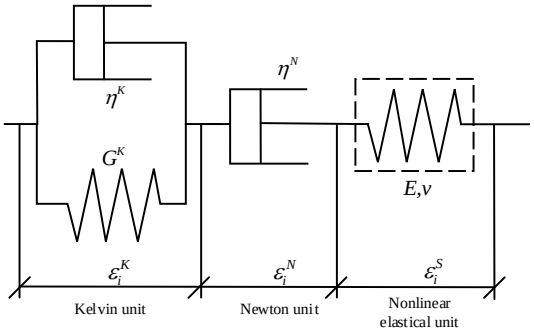


Fig. 3 the shear creep model for coarse-grained soils

3.1.2 Secondary consolidation for original red clay layer

The original soils contain clay, silt, etc. All these soil layers are below groundwater level. As mentioned before, the construction process of the filled embankment lasts almost 550 days (2011/4-2002/11). During this time, the Primary consolidation of the original soil layers has been largely completed. Therefore, it is the secondary consolidation of the original soil layers that has an important effect on the creep deformation of the runway in the monitoring period (2003/7/18-2007/4/18). In this paper, a widely used logarithmic empirical model for the secondary consolidation of soft clay, formulated as Eq. (7), is adopted to calculate the time-dependent settlement of original soil layers.

$$S_t = \frac{H}{1+e_0} C_\alpha \log\frac{t}{t_c} \tag{7}$$

where, *H* is the total thickness of original soft clay; e_0 is the initial void ratio C_α is the coefficient of secondary consolidation; t_c is the initial time of secondary consolidation;

The natural density of largest undisturbed soil (Q_1) is approximately equal to 2.1g/cm^3. Its unconfined deformation modulus is closely to 26.5MPa, which is corresponding to the effective stress range from 100kPa to 200kPa (Liu 2004). By assuming that the undisturbed soil is saturated, the initial void ratio can be roughly estimated,

$$e_0 = \frac{\rho_s - \rho}{\rho - \rho_w} = \frac{2.7 - 2.1}{2.1 - 1} = 0.55 \tag{8}$$

where, ρ_s is the density of soil particle; ρ is the density of undisturbed soil; ρ_w is the density of water. Thus, the compression factor C_c is easily obtained, as formulated in Eq. (9)

$$C_c = \frac{(1+e_0)}{\Delta \lg \sigma'} \frac{\Delta \sigma'}{E_s} = 5.641 \times 10^{-3} \tag{9}$$

where, $\Delta\sigma' = 100\text{kPa}$. The relationship between the coefficient of secondary consolidation and compression factor is shown in Eq. (10). In this paper, the ratio is assumed as 0.07, and then the coefficient of secondary consolidation is 4×10^{-4}.

$$\frac{C_\alpha}{C_c} = 0.025 \sim 0.1 \tag{10}$$

From Fig. 1, the thickness of the original soil below the runway on the inside line, the center line and the outside line is 85m, 88m, and 91m, respectively. Submitting to Eq. (7), the secondary consolidation curves of the three monitored points are obtained, as sketched in Fig. 2.

3.2 Response Surface Methodology

Response Surface Methodology, or RSM, is a collection of mathematical and statistical techniques that is useful for modeling the problem where a response of interest is influenced by several variables (Douglas 1997). That is, the objective is to optimize this response. In most RSM problems, the form of the relationship between the response and the independent variables is unknown. Usually, a stepwise searching process to find the response surface is shown in the following three steps.

1) **First-order model**. In a reasonable initial operating domain $\mathbf{R}(\xi)$, series of independent variables are chosen to determine certain responses. Based on these, the fitting function of the first-order model is formulated as Eq. (11).

$$y_{(1)} = \beta_0 + \sum_1^k \beta_i \xi_i \tag{11}$$

where, $y_{(1)}$ is the first-order response value; $\hat{\beta}_i \left(i = 0,...,k\right)$ is the coefficient.

2) **Steepest searching**. Based on the first-order model, the gradient $\nabla y_{(1)} = \left(\partial y_{(1)} / \partial x_1,...,\partial y_{(1)} / \partial x_k\right)$ is chosen as the optimum direction for steepest searching. If the maximum of the response is desired, the steepest ascent is used. On the contrary, if minimization is desired, the steepest descent is chosen. It is pointed out that this method just provides the direction of searching. The actual step size is determined by the designer based on the practical limitations. The steepest searching is conducted

along the path of steepest ascent/descent until no further increase/decrease in response is observed.

3) **Second-order model**. Usually near the region of the optimum, the linear function is not proper because the response system is curvature. Thus, a polynomial of higher degree is needed. In most second-order design, the central composite design (CCD) is the most popular design, as shown in Eq. (12).

$$y = \beta_0 + \sum_{1}^{k} \beta_i x_i + \sum_{1}^{k} \beta_{ii} x_{ii}^2 + \sum_i \sum_j \beta_{ij} x_i x_j \tag{12}$$

To find the maximum/minimum response, the Eq. (12) is rewritten in matrix notation,

$$y = \beta_0 + \mathbf{x}'\mathbf{b} + \mathbf{x}'\mathbf{B}\mathbf{x} \tag{13}$$

Solving the equation $\frac{\partial y}{\partial \mathbf{x}} = 0$, the optimum solution is formulated as Eq. (14).

$$\mathbf{x_0} = -\frac{1}{2}\mathbf{B}^{-1}\mathbf{b} \tag{14}$$

Almost all RSM problems are carried out through these procedures. Of course, it is not true that the polynomial model can be a reasonable prediction of the true response over the entire space of the independent variables. But in a relatively small region, it usually works quite well.

4. BACK-ANAYLSIS OF MODEL PARAMETERS

4.1 FDM Model

A FDM model is used to simulate the settlement response of the embankment. Fig. 4 shows the FDM mesh, which contains 1085 elements. Furthermore, the actual construction process is simulated. The stress-strain relationship of original soil layers is simplified as a rigid body and the settlement of original soil layers has been removed from the surface settlement (see 3.1.2). The stress-strain relationship and time-dependent behavior of the filled embankment is modeled using the proposed creep model.

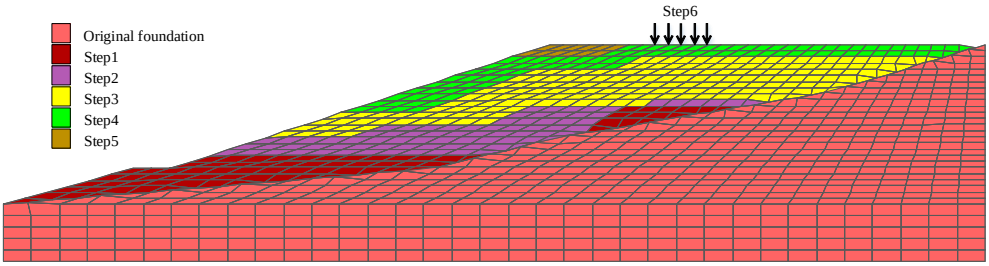


Fig. 4 FDM mesh of The high-fill embankment

To establish an effective response, an error function, formulated in Eq. (15), is defined. It represents the difference between monitoring data and corresponding results obtain from numerical calculation.

$$\vartheta(\mathbf{P}) = \sqrt{\frac{1}{n}\sum_{i}^{n}\left(\frac{u_i^h(\mathbf{P}) - u_i}{u_i}\right)} \tag{15}$$

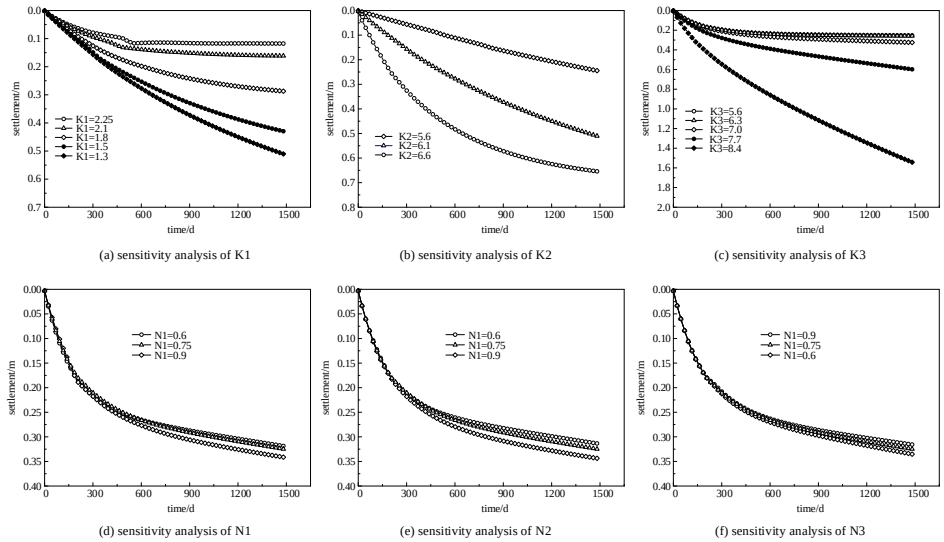
where, u_i and u_i^h (i=1,2,…,n), are the monitoring data and the numerical results, respectively. Obviously, the aim of the back-analysis is to find the absolute minimum of ϑ for the reason that the minimum provides the best agreement between in-situ monitoring data and the numerical results.

4.2 Parameter sensitivity analysis

In the proposed model, there are 13 parameters in total. c, φ_0, $\Delta\varphi$, K_4, n_4, R_f and υ can be easily determined by the conventional triaxial tests, as listed in Tab. 1. To get a better understanding of the effects of parameters K_1, η_1, K_2, n_2, K_3, n_3 and avoid back-analyzing too many parameters, sensitivity analysis have been carried out. The numerical settlement curve of the highest-fill point is sketched (Fig. 5). n_1, n_2 and n_3 have litter influence on the deformation. Thus, K_1, K_2, K_3 will be back analyzed later.

Tab. 1 the parameters of the nonlinear elastic unit

c / kPa	φ_0 / degree	$\Delta\varphi$ / degree	K_4	n_4	R_f	υ
0	48.3	8.26	925.8	0.73	0.9	0.35



(a) sensitivity analysis of K1 (b) sensitivity analysis of K2 (c) sensitivity analysis of K3

(d) sensitivity analysis of N1 (e) sensitivity analysis of N2 (f) sensitivity analysis of N3

Fig. 5 the parameter sensitivity of the shear creep model

4.3 Displacement Back-Analysis Based on Response Surface Methodology

4.3.1 First-order Design

The region of exploration for fitting the first-order model is K_1 (1.55, 1.65), K_2 (5.55, 5.65) and K_3 (7.35, 7.45). To simplify the fitting, the independent variables will be coded to the usual interval (-1, 1). The coded variables are

$$x_1 = \frac{K_1 - 1.6}{0.1}, \qquad x_2 = \frac{K_2 - 5.6}{0.1} \qquad \text{and} \qquad x_3 = \frac{K_3 - 7.4}{0.1}$$

The detailed design is shown in Tab. 2.

Then the following model in the coded variables is obtained:

$$\vartheta = 1.023 - 0.236x_1 - 0.092x_2 - 0.189x_3 \tag{16}$$

4.3.1 Steepest descent searching analysis

Based on the first-order model, the path of steepest descent, $\Delta X=(1, 0.4, 0.8)$ is calculated. Then, we start at the point ($x_1 = 0$, $x_2 = 0$, $x_3 = 0$) and search along the path until an increase in response is observed, as shown in Tab. 3.

Decreases in response are observed through the eight steps. Now, we are near the optimum region, and we need a higher order to capture the curvature in this response system.

4.3.1 Steepest descent searching analysis

Based on the results of steepest descent searching, the central composite design (CCD) design is employed, seen in Tab. 4. Then the corresponding vector and matrix is obtained.

$$\mathbf{b} = \begin{bmatrix} -3.1272 \\ 0.9725 \\ -0.0186 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 0.8272 & -0.5987 & 0.1562 \\ -0.5987 & 0.0792 & -0.0703 \\ 0.1562 & -0.0703 & -0.0198 \end{bmatrix}$$

Finally, the parameters are obtained: K_1=1.748, K_2=5.720 and K_3=7.532.

Tab. 2 Process data for fitting the first-order model

Natural variables			Coded variables			response
K_1	K_2	K_3	x_1	x_2	x_3	ϑ / %
1.550	5.550	7.350	-1	-1	-1	136.609
1.550	5.550	7.450	-1	-1	1	133.363
1.550	5.650	7.350	-1	1	-1	115.346
1.550	5.650	7.450	-1	1	1	112.132
1.650	5.550	7.350	1	-1	-1	86.381
1.650	5.550	7.450	1	-1	1	83.362
1.650	5.650	7.350	1	1	-1	70.968
1.650	5.650	7.450	1	1	1	67.971
1.600	5.600	7.400	0	0	0	97.141

Tab. 3 The steepest descent searching

step	Natural variables			Coded variables			response
	K_1	K_2	K_3	x_1	x_2	x_3	ϑ / %
Origin	1.600	5.600	7.400				97.141
Δ	0.020	0.008	0.016	1	0.4	0.8	
Origin+ Δ	1.620	5.608	7.416	2	0.8	1.6	84.466
Origin+2 Δ	1.640	5.616	7.432	3	1.2	2.4	75.098
Origin+3 Δ	1.660	5.624	7.448	4	1.6	3.2	66.691
Origin+4 Δ	1.680	5.632	7.464	5	2	4	57.483
Origin+5 Δ	1.700	5.640	7.480	6	2.4	4.8	48.772
Origin+6 Δ	1.720	5.648	7.496	7	2.8	5.6	40.346
Origin+7 Δ	1.740	5.656	7.512	8	3.2	6.4	32.152
Origin+8 Δ	1.760	5.664	7.528	9	3.6	7.2	26.043
Origin+9 Δ	1.780	5.672	7.544	10	4	8	31.233

Tab. 4 Process data for fitting the second-order model

Natural variables			Coded variables			response
K_1	K_2	K_3	x_1	x_2	x_3	ϑ / %
1.740	5.656	7.512	-1	-1	-1	20.952
1.740	5.656	7.544	-1	-1	1	19.568
1.740	5.672	7.512	-1	1	-1	16.651
1.740	5.672	7.544	-1	1	1	17.856
1.780	5.656	7.512	1	-1	-1	16.876
1.780	5.656	7.544	1	-1	1	17.682
1.780	5.672	7.512	1	1	-1	19.356
1.780	5.672	7.544	1	1	1	16.833
1.789	5.664	7.528	1.682	0	0	22.569
1.731	5.664	7.528	-1.682	0	0	15.987
1.760	5.678	7.528	0	1.682	0	16.983
1.760	5.650	7.528	0	-1.682	0	17.214
1.760	5.664	7.552	0	0	1.682	15.586
1.760	5.664	7.504	0	0	-1.682	15.027

4.4 Analysis Based on Back-Calculated Parameters

To illustrate the efficiency of the back-analysis method and the creep model, Fig 6 compares the numerical results with the observed data. It can be seen that three settlement curves of calculated results are in good agreement with the monitoring data. Furthermore, using the back-determined model parameters, the settlement distribution after twenty year are predicted (Fig 7), with a maximum settlement of 0.46 m.

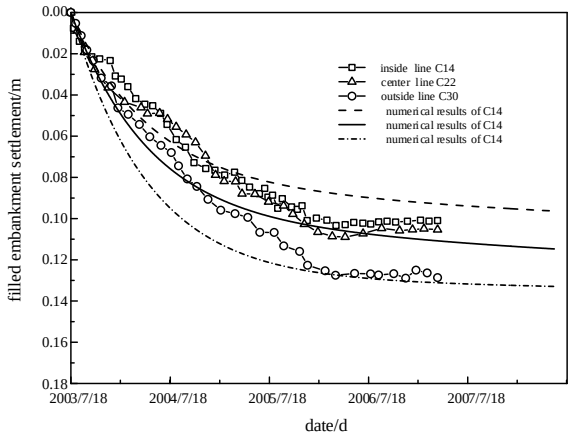


Fig 6 the comparison of nummerical results and the field data

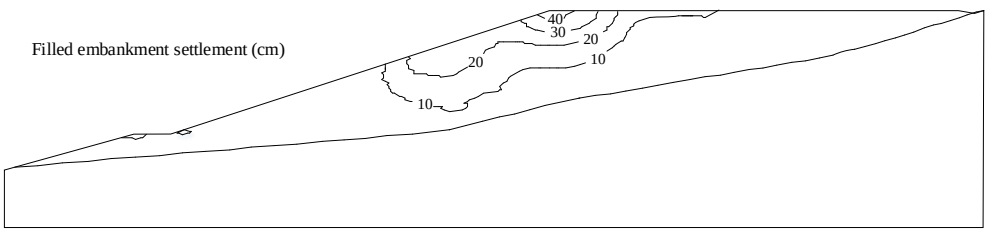


Fig 7 the displacement distribution of the airport after 20 years

5. CONCLUSION

A creep model is adopted to simulate the time-dependent behavior of the coarse-grained soils. The parameters for the proposed model of a high-fill embankment are back-analyzed employing the response surface methodology, based on the in-situ monitoring data. The displacement calculation is consistent with the monitoring data, which implies the back-analysis method is efficient. With the back-determined parameters, the deformation distribution at twenty years later are predicted and analyzed.

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