Reliability analysis and numerical simulation on grillage flexible supporting structure with prestressed anchors

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ABSTRACT

Based on the numerical simulation software FLAC3D, a project example was analyzed. For the grillage flexible supporting structure with prestressed anchors, based on the influence scope of single anchor destruction at different positions, the single anchor bar tensile failure function and the anti-slip stability function of the supporting structure as a whole after the invalidation of a single anchor were established. And the checking point method was used to solve the reliability index. The reliability index analysis show that the greater the safety factor, the more reliable the structure and the single anchor destruction at different positions will not lead to the destruction of the whole supporting structure. The effects of the variability in soil cohesion, internal friction angle and soil mass on reliability index were studied and the results show that internal friction angles have great influence on the reliability index. Finally, the influences of different values of soil cohesion, internal friction angle and anchor prestress values on anchor axial force and slope displacement were analyzed in the numerical simulation.

Key words: slope engineering; prestressed anchor; grillage supporting structure; reliability; numerical simulation

1. INTRODUCTION

In recent years, the grillage supporting structure with pre-stressed anchor bar is widely used in slope supporting engineering because it has many advantages, such as serviceability safety, convenient installation, economical, the small slope disturbance
and its effectiveness to control the deformation of slope body (Zhu Yan-peng 2008). In the design of slope supported with frame prestressed anchor structure, the fixed value method is generally used in traditional designs. Namely, to determine the degree of safety according to the quantitative distinguishing to cause damage and resist damage effects by the set failure criteria of soil. However, some practices show that the actual response of the anchor support structure is very different from the calculation results. The main reason for this kind of difference is that the corresponding geotechnical Parameters was treated as deterministic values in the design, and then take the rock reaction as certain. In fact, the corresponding geotechnical parameter is obtained by finite sample test, the test method and the corresponding data sorting method test have uncertain factors, the soil nonuniformity, the change of load and the outside conditions also make geotechnical reaction uncertain.

Therefore, a concept of reliability was introduced into the analysis of retaining structure. It takes into account the randomness of the corresponding variable (Pan Jun 2011). In the present study of slope reliability, Fan Zhao-ping etc. (2010) analyzed contrastive reliability of the slope stability by Mont-Carlo simulation when strength parameters of cohesive force $c$, internal friction angle $\phi$ follow different probability distribution function. The relationship between the slope stability and the variability of strength parameters follow different probability distribution was analyzed, and then discuss the sensitivity of the instability probability to the intensity of parameter variability changes. Luo Xiao-hui etc. (2006) established the stability analysis limit state equation for soil nailing structure that considered the effects of conditional load in terms of theoretical analysis model. In the aspect of supporting structure design, they analyzed the random variables and their combinations on the influence of safety factor and reliability sensitivity by using the theory of reliability, as well as the relationship between safety factor and reliability. Wu Zhen-jun etc. (2010) introduced a new method for reliability optimization solving proposed by Low & Tang, and it was used for slope reliability analysis. This method is applicable to relevant variables of any probability distribution, and it searches the minimum reliability index and probability critical sliding surface of slope directly in the variables of the original space that don’t need to calculate the equivalent normal mean and variance, independent variables transformation. This method can use any suitable constrained optimization method to solve and it is clear and concise. Tan Xiao-hui etc. (2009) studied the fuzzy stochastic finite element reliability analysis method of slope stability based on finite element method of slip surface stress in slope stability analysis. This method can consider the fuzziness of both the basic variables and the limit states at the same time. Tang Renhua etc. (2012) presented a series-parallel model of rib beam-anchor system in reliability analyses of anchored retaining wall. In this model, the rib beam is regarded as a continuous beam and the anchors as an elastic support. A composite stiffness coefficient of anchor and the soil around anchorage section was introduced into this model, and the application loads acting on each anchor are obtained by displacement method. Considering the correlation of performance functions, the corresponding failure probability was calculated based on the system reliability theory.

This paper mainly focused on the reliability calculations of single anchor tensile failure and the overall anti-sliding stability, and the influence of corresponding parameter variability on reliability index was analyzed as well. Finally, the effect of
cohesive force, internal friction angle and prestress value on the axial force of anchor and displacement of slope mass was studied.

2. THE ESTABLISHMENT OF PERFORMANCE FUNCTIONS FOR SUPPORTING STRUCTURE

2.1 The establishment of the performance function after single anchor invalidation

If the effect of the corresponding anchor failure on reliability index of retaining structure was considered only, the anchor destruction form in the grillage supporting structure with prestressed anchor bars are as follows: the cementation damage between the anchoring mortar and the surrounding rock, the bond damage between the anchoring mortar and the anchor bar, the tensile failure of the anchor bar. Taking the tensile failure of the anchor bar as an example, the performance function after the failure of single anchor was established, as shown in Eq. (1).

\[ Z = \frac{\pi d^2}{4} f_y - T_i \]  

(1)

Where: \( d \) is the diameter of anchor bar; \( f_y \) is the tensile strength of anchor bar; \( T_i \) is the axial force of anchor in the \( i \) row.

2.2 The establishment of the internal stability performance function after single anchor invalidation

The invalidation of any single anchor would lead to the appearance of local weak phenomenon in the grillage supporting structure that could lead a drop of internal stability in local parts of the supporting structure. The influence range of single anchor failure in different positions could be obtained by the numerical simulation, and the added value of anchor axial force is positive within the impact scope. Therefore, the weakest position is one of the anchor bars in the failure position of the anchor bars. Fig.1 shows the circular slip surface diagram of anchor supporting structure.

Fig.1 Circular slip surface diagram of anchor support structure
The internal stability performance function after any single anchor invalidation in the \( k \) row as shown in Eq. (2):

\[
M_k = R \cdot S_h \cdot \sum_{i=1}^{n} c_i l_i + R \cdot S_h \cdot \sum_{i=1}^{m} (W_i + q_i b_i ) \cos \theta_i \tan \varphi_i + \\
R \cdot \sum_{j=1, j \neq k}^{n} T_{nj} \times \left[ \cos \left( \beta_j + \theta_j \right) + \sin \left( \beta_j + \theta_j \right) \times \tan \varphi_j \right] \\
+ F (Y + H) - R \cdot S_h \cdot \sum_{i=1}^{n} (W_i + q_i b_i ) \sin \theta_i
\]

Where: \( H \) is the height of the slope; \( Y \) is the distance from circle center to ground surface; \( n \) is soil strips number of slide mass; \( m \) is the layer number of anchor; \( w_i \) is the weight of the \( i \)th soil strips; \( b_i \) is the width of \( i \)th soil strips; \( c_i \) is the standard value of cohesion at the sliding surface of \( i \)th soil strips; \( \varphi_i \) is the standard value of internal friction angle at the sliding surface of \( i \)th soil strips; \( \theta_i \) is the angle between tangent line and horizontal plane at the sliding surface of \( i \)th soil strips; \( \alpha_j \) is the angle between the anchor and horizontal plane; \( L_i \) is the arc length of the sliding surface for \( i \)th soil strips; \( S \) is the thickness of the sliding body unit; \( R \) is the arc radius of slip plane; \( T_{nj} \) is the axial force of the anchor in \( i \)th row.

Fig.2 The scope of internal force

Fig.3 The schematic sectional view of

Take anchor invalidation in the neutral position as example, Fig.2 shows the influence range after the single anchor failure, and Fig.3 shows the compatible deformation of diagrammatic cross-section. The added value of anchor axial force within the influence scope after the single anchor failure can be obtained by the corresponding deformation coordination and force balance equations.
3. ENGINEERING EXAMPLE ANALYSIS

This project is a nature slope at the northern end of Baitai Home that is located in Chongxin county of Gansu province. The slope is east-west direction and lean to north, the slope angle is about 85°~90° and the slope is a soil slope. The segment for reinforcement is at the northern side of the slope and the height of the slope is 13 meter. The important coefficient is 1.0, and the safety factor is 1.3. The name of the slope soil are as follows: ①loess like silt; ②loess like silty clay. The physical properties of the soil are showed as follows:

<table>
<thead>
<tr>
<th>Numerical Order</th>
<th>Thickness of Soil $h$ (m)</th>
<th>Unit Weight $\gamma$ (kN/m$^3$)</th>
<th>Cohesion $c$ (kPa)</th>
<th>Internal Friction Angle $\varphi$ (°)</th>
<th>Bond Strength $\tau$ (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>①</td>
<td>7.4</td>
<td>14.4</td>
<td>9.1</td>
<td>21.5</td>
<td>50</td>
</tr>
<tr>
<td>②</td>
<td>&gt;10</td>
<td>15.6</td>
<td>16</td>
<td>22</td>
<td>50</td>
</tr>
</tbody>
</table>

The field soil is simplified to an ideal homogeneous soil layer for the convenience of calculation. The cohesive force, internal friction angle, the soil heavy, the diameter and tensile strength of anchor bar are random variables in calculations, and the corresponding mean value are: 12.07kPa. 21.7°. 14.9kN·m. 360MPa. 36mm, the corresponding variable coefficient are: 0.1. 0.15. 0.05. 0.1 and 0.1. There are five rows of anchor, and the free length of the anchor are 7.5m. 6m. 4m. 2.5m. 1m from top to bottom, the anchoring segment lengths are 8.5m. 10m. 10m. 10m. 10m. The vertical spacing and horizontal spacing of the anchor are 2.6 m and 2.7 m, the first row of anchor is 1.3m from the top. The anchor angle is 15°, and the diameter of anchoring body is 150mm. The section sizes of column and beam are $b \times h = 300$ mm$\times$300mm, the strength grade of concrete is C25, and HRB400 reinforced bar are used as anchor bar.

Based on these data, the single anchor tensile failure reliability is calculated in this paper as well as the reliability calculation of overall stability against sliding, specifically showing in Tab.2 and table Tab.3:

<table>
<thead>
<tr>
<th>Location of anchor damage</th>
<th>The first row</th>
<th>The second row</th>
<th>The third row</th>
<th>The fourth row</th>
<th>The fifth row</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reliability index</td>
<td>3.3534</td>
<td>1.966</td>
<td>2.2316</td>
<td>2.0935</td>
<td>2.4388</td>
</tr>
</tbody>
</table>

Tab.1 Physical-mechanical properties of soils

Tab.2 Reliability calculation after the destroy of a single anchor in different positions
Tab.3 The internal stable reliability index of slope supporting structure under different working conditions

<table>
<thead>
<tr>
<th>Failure mode</th>
<th>Safety factor</th>
<th>Reliability index</th>
<th>Failure probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>no damaging</td>
<td>1.3241</td>
<td>3.6477</td>
<td>0.0132</td>
</tr>
<tr>
<td>The first row</td>
<td>1.2744</td>
<td>3.1809</td>
<td>0.0734</td>
</tr>
<tr>
<td>The second row</td>
<td>1.2561</td>
<td>3.0146</td>
<td>0.13</td>
</tr>
<tr>
<td>The third row</td>
<td>1.2615</td>
<td>3.0646</td>
<td>0.11</td>
</tr>
<tr>
<td>The fourth row</td>
<td>1.2640</td>
<td>3.0687</td>
<td>0.11</td>
</tr>
<tr>
<td>The fifth row</td>
<td>1.2462</td>
<td>2.9680</td>
<td>0.15</td>
</tr>
</tbody>
</table>

The tab.2 shows that the reliability index of a single anchor tensile failure relates to the corresponding axial force. The axial force is greater, the reliability index is smaller. The tab.3 shows that the internal stability reliability index of slope supporting structure under different conditions has a certain relationship with the slope safety factor, and the safety factor is larger, the corresponding reliability index is greater.

3.1 The influence of the variation coefficient on reliability index

If the different variation coefficient is taken to internal friction angle, cohesion and the soil heavy, the internal stability reliability index of supporting structure is different, which is showed in Fig.4, Fig.5 and Fig.6. The general trend of these three figures shows that with the increase of the variable coefficient of corresponding parameters, the corresponding reliability index decreases.

3.2 The change of axial force

The axial forces of anchor 13, 23, 33, 43, and 53 are shown in Fig.7. According to the anchor length, if a measuring point for per meter is arranged, from the picture you can see the general trend of anchor axial force. The axial force of anchor declines from the anchorage segment, and the value goes greatly at first, but become smooth finally.

Fig.8 shows the axial force trend at different row of the anchor from top to bottom. In this example, the slope height is 13 meter and arranged with 5 rows of prestressed anchors. From this figure you can see that the axial force of anchor increases gradually from top to bottom, the axial force of the anchor in the fourth row is the largest, and then it decreased.
Fig. 4 The reliability index of coefficient variation of the internal friction angle compatibility

Fig. 5 The reliability index of coefficient variation of cohesion
3.3 The changes of displacement

The corresponding displacement of slope in the anchor position is shown in Fig.9. The maximum displacement of the slope occurred in the corresponding position of anchor 33 that can be seen in the figure. That to say, the largest displacement is in the middle of the slope, and the general trend of slope displacement shift is parabolic, the displacement waves in the position of last row anchor.

3.4 Parametric analysis

(1) The change of cohesion

For this example, there are two layers of soil, the cohesive force are 9.1kPa and 16kPa respectively. On the basis of it, by changing the value of cohesive force while keeping other parameters constant, the maximum slope displacement corresponding to
different cohesive force and the axial force of anchor 33 is shown in Tab.4 and Tab.5 (The numbers in brackets show the change of cohesive force for the soil in the second layer). Fig.10 and Fig.11 show the maximal displacement and the axial force of anchor 33 under the condition of different cohesion.

![Fig.10](image1.png) ![Fig.11](image2.png)

**Fig.10** The maximum horizontal displacement of slope corresponding to different cohesion

**Fig.11** The axial force of anchor corresponding to different cohesion

<table>
<thead>
<tr>
<th>Cohesion (Cohesion(kPa))</th>
<th>11.1(16)</th>
<th>13.1(18)</th>
<th>15.1(20)</th>
<th>17.1(22)</th>
<th>19.1(24)</th>
<th>21.1(26)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement (mm)</td>
<td>52.334</td>
<td>38.545</td>
<td>28.697</td>
<td>21.877</td>
<td>16.895</td>
<td>13.46</td>
</tr>
</tbody>
</table>

It can be concluded from Tab.4 that if the cohesive force increases 2kPa, the corresponding displacement increment is 6.25%, 25.55%, 23.77%, 22.77% and 20.33% respectively. We can see that with the increase of the cohesive force, the maximum horizontal displacement of slope gradually decreases, and basically in a linear gradient.

<table>
<thead>
<tr>
<th>Cohesion (Cohesion(kPa))</th>
<th>11.1(16)</th>
<th>13.1(18)</th>
<th>15.1(20)</th>
<th>17.1(22)</th>
<th>19.1(24)</th>
<th>21.1(26)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axial force (KN)</td>
<td>203.7</td>
<td>181.5</td>
<td>160</td>
<td>140.7</td>
<td>122.9</td>
<td>105.9</td>
</tr>
</tbody>
</table>
It can be concluded from Tab.5 that if the cohesive force increases 2kPa, the corresponding axial force reduction is 22.2 kN, 21.5 kN, 19.3 kN, 17.8 kN and 17kN respectively. According to this changing trend, the axial force of anchor tends to a certain value when the cohesive force increases to a certain value.

Fig.10 and Fig.11 show the maximal displacement and the axial force of anchor 33 under the condition of different cohesion. The following conclusions can be obtained from the two figures: the axial force of anchor in the frame structure with prestressed anchor decreases with the increase of the cohesive force and basically decreases linearly; the overall trend for maximum horizontal displacement of slope is that it reduces with the increase of cohesive force. In terms of the maximum horizontal displacement, the change of displacement is more sharp when the cohesive force is within a certain range, then it goes gently when the cohesive force reaches a certain value, and basically changes linearly.

(2) The influence of internal friction angle
There are two layers of soil in this slope, the internal friction angles are 16.5° and 17° respectively. On the basis of it, by changing the value of internal friction angle while keeping other parameters constant, the maximum slope displacement corresponding to different cohesive force and the axial force of anchor 33 is shown in Tab.6 and Tab.7 (The numbers in brackets show the change of internal friction angle for the soil in the second layer). Fig.12 and Fig.13 show the maximal displacement and the axial force of anchor 33 under the condition of different internal friction angle.

<table>
<thead>
<tr>
<th>Internal friction angle</th>
<th>16.5(17)</th>
<th>18.5(19)</th>
<th>20.5(21)</th>
<th>22.5(23)</th>
<th>24.5(25)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement (mm)</td>
<td>133.91</td>
<td>88.045</td>
<td>62.329</td>
<td>44.228</td>
<td>31.335</td>
</tr>
</tbody>
</table>

It can be concluded from Tab.6 that if internal friction angle increases 2°, the corresponding reduction of displacement is 34.25%, 29.21%, 29.04% and 29.15% respectively. We can see that with the increase of internal friction angle, the maximum horizontal displacement of slope decreases gradually, and when the internal friction angle reaches a certain value, the maximum horizontal displacement of slope basically reduces with the increase of internal friction angle.

<table>
<thead>
<tr>
<th>Internal friction angle</th>
<th>16.5(17)</th>
<th>18.5(19)</th>
<th>20.5(21)</th>
<th>22.5(23)</th>
<th>24.5(25)</th>
</tr>
</thead>
</table>
Axial force (kN) \[ \begin{array}{cccccc}
281.1 & 242 & 215.2 & 191.6 & 169.2 \\
\end{array} \]

It can be concluded from Tab.7 that if internal friction angle increases 2°, the corresponding reduction of axial force is 39.1 kN, 26.8 kN, 23.6 kN and 22.4 kN respectively, and basically changes linearly.

![Fig.12](image1.png) The maximum horizontal displacement of slope corresponds to different internal friction angle

![Fig.13](image2.png) The axial force of anchor corresponds to different internal friction angle

Fig.12 and Fig.13 show the maximal displacement and the axial force of anchor 33 under the condition of different internal friction angle. And the following conclusions can be obtained from the two figures: the axial force of anchor in the frame structure with prestressed anchor decreases with the increase of the internal friction angle and basically decreases linearly; the overall trend for maximum horizontal displacement of slope is that it reduces with the increase of internal friction angle.

(3) The influence of prestress value

About the influence of prestress value to the axial force of anchor and slope, considering prestress value 80 kN, 100 kN, 120 kN, 140 kN, 160 kN and 180 kN to the anchor, the influence of different prestress values on the axial force of anchor and slope is shown in Fig.14 and Fig.15.
The Fig.14 shows that the prestress value on anchor basically has no influence on the axial force of anchor, that is to say the axial force of anchor only related to the parameters of soil. It can be concluded from Fig.15 that the prestress value on anchor has influence on the displacement of slope, and the maximum displacement decreases with the increase of prestress value. For this engineering example, when the prestress value increases from 80kN to 180kN, the maximum horizontal displacement of slope decreases from 53.731mm to 50.015mm. Therefore, we can say that the prestress value on anchors should be controlled in the range of 80 kN to 100 kN, the increase of prestress value over this range has no obvious influence on the slope displacement restriction.

**Fig.14** The axial force of anchor  
**Fig.15** The maximum horizontal displacement

### 4. CONCLUSIONS

1. The results of reliability index analysis of internal stability on the supporting structure shows that the safety factor is larger, the corresponding reliability index is larger, and the variability of the parameters is greater, the reliability index is smaller.

2. The general trends of the axial force of freedom segment and anchorage segment are as follows: the axial force in the freedom segment is almost invariant, and it gradually decreases in the anchorage segment, and it reaches a certain value finally; the change trend of axial force of anchor is parabolic from top to bottom, it gradually increase at first, become biggest in the middle, and then decreases.

3. The roughly change trend of slope displacement increases at first and then decreases after reaches a certain value. In this project example, the displacement in the position of anchor 53 has a tendency to increase.
(4) The change of the slope displacement and axial force in the condition of different cohesive force or internal friction angle decreases with the increase of the cohesive force or internal friction angle.

(5) Seen from the change of prestress value, with the increase of prestress value, the change of axial force of anchor is small. Although the corresponding displacement of slope reduces gradually, the decrease scope is small, so the prestress value of 100kN is relatively appropriate.

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