Axial Flow Patterns for Yawed Inclined Circular Cylinders for Cable-Stayed Bridge Cables

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ABSTRACT

The wind-induced vibrations of bridge stay cables has been a long studied and documented topic with various experimental and numerical investigations reported. The current research investigates in depth the characteristics and the patterns of the axial flow formed on the leeward surface of the stay cables inclined and yawed to clarify the potential causes of dry galloping vibrations of stay cables. CFD simulation was performed on cylinder models with high aspect ratios to model full-scale cables. The Large Eddy Simulation (LES) model was developed with a constant Smagorinsky model for simulating the turbulent flow for high Reynolds numbers (Re) in the range of $10^5$ for cable inclination angles of 0\degree to 60\degree, and yaw angles of 0\degree to 40\degree. Pressure around the circumference of the circular cylinder was monitored on 5 rings arranged along the cylinder, and velocities were monitored downstream the cylinder. Inclinations for which the stay cable is prone to developing aerodynamic instability and new flow characteristics for inclined cylinders are discussed.

1. INTRODUCTION
The construction of cable-stayed bridges has expanded beyond precedent, as these structures are preferred to the more expensive cable suspension bridges. During construction, or upon completion of these massive bridges, the stay cables have been known to vibrate. These vibrations might be caused by rain-wind effect, wind buffeting turbulent effect, parametric excitations, wake effects, or combination of these mechanisms. The stay cables vibrations which captured the particular attention of wind engineers and aerodynamicists worldwide areas caused by a phenomenon that is not well understood at this time and is known as dry cable galloping. Several researchers have carried out rigorous experimental programs in an attempt to better understand this phenomenon (Larose et al., 2003; Cheng et al., 2008, Matsumoto et al., 1990, Katsuchi et al., 2009, etc.). This new aerodynamic instability of the structural stay cable was found to occur around the Reynolds number of $Re = 2.2 \times 10^5$, for cable inclined at the angles of 57\degree and higher (Larose et al., 2003). A new formulation for the aerodynamic
forces for inclined cables has been developed for verifying the galloping criterion based on the variation of aerodynamic coefficients with the Re number, and the inclination of the cable MacDonald et al. (2003). A structural damping recommendation has been developed for cable-stayed bridges (FHWA, 2005) in an attempt to reduce the galloping of stay cables. However, the tests performed in the wind tunnel, and in the field on similar cable models tested within the same Reynolds number range resulted in different structural responses and different aerodynamic coefficients (Cheng et al., 2008; Katsuchi et al., 2009; Zuo, 2014). This phenomenon is of concern as it indicates that the increase in structural damping of inclined cables is not necessarily effective in reducing the dry inclined cable galloping vibration (Saito et al., 1994).

The presence of an axial flow on the leeward side of the cylinder inclined at 42.5°, and yaw varying between 0° and 45° has been shown through oil film visualization techniques in wind tunnel tests, for a Reynolds number of Re = 1.5 × 10^5 (Matsumoto et al., 1990), and noted in the wind tunnel studies performed by Matsumoto et al. (1995), for up to Re = 1.65 × 10^5. A connection has been established between the onset wind speed for dry inclined cable galloping, and the presence of the axial flow along the leeward side of the cylinder. This interpretation was similar to the presence of the water rivulet causing the on-set of rain-wind vibration, though the Re number, and the inclination of the cables for which the two phenomena, dry inclined galloping and rain-wind vibration, are different (Matsumoto et al. 2010).

The current research looks at numerically simulating the flow around an inclined circular cylinder and verify the formation of the axial flow along the leeward side of the cylinder, for clarifying the conditions for the occurrence of dry cable galloping of inclined cable. Reynolds numbers of 2.2 × 10^5 and 4.4 × 10^5 were used in the investigation to simulate flow around cylinder for inclination angles of 0° to 60° and yaw angles of 0° to 40°. The diameter of the cylinder was D = 0.089 m with a cylinder length of 30D. Pressure on the surface of the cylinder was monitored around 5 rings arranged along the cable at equal intervals.

2. COMPUTATIONAL MODEL

The computational domain employed for the present investigation is a closed space rectangular field extending 30D along the length of the cylinder, 22.5D in the direction of flow, 11.25D upstream and downstream from the center of the cylinder, and 22.5D in the direction perpendicular to the direction of flow centered around the cylinder, as illustrated in Figure 1 below. The height of the domain changed with the inclination of the cylinder to match the cylinder length of 30D. A tetrahedral mesh with over 3,000,000 cells were used for the entire domain with over 200,000 elements for the mesh of the cylinder surface (Fig.1). The cable models were exposed to various wind speeds ranging between 18.0 m/s to 90.3 m/s resulting in high Reynolds numbers of Re = 1.1 × 10^5 – 5.5 × 10^5. These values are characteristic to the precritical regime of the TrBL (Transition in Boundary Layers) flow regime as defined by Zdravkovich (1997). This regime is characterized by the abrupt decrease in drag force as a result of an increase in wind speed and is known as the drag crisis (Simiu and Scanlan, 1987; Zdravkovich, 1997).
The current study uses the turbulent LES model throughout the entire domain with Smagorinsky model for turbulent viscosity. Several cases of perpendicular and inclined/yawed circular cylinders of diameter $D = 0.089$ m and length of $30D = 2.67$ m were investigated (Fig. 1). Aerodynamic characteristics of inclined/yawed slender circular cylinders, and the three-dimensional characteristics of the axial flow which appeared for certain cases were analyzed. This size of mesh for the computational domain was chosen as a balance between the convergence rate of the model, computational time of the model, and accuracy of the model. Any coarser meshes in Fluent would significantly increase the computational time required for the model with minimal change on the results obtained. In deciding on the size of the computational domain for this study, two studies were used to in balancing the size of the domain with the computational requirements to model the flow. Qingkuan et al (2007) simulated the flow around a cable model with a water rivulet on the cable surface using large eddy simulation using a circular computational domain with dimensions of $20D \times 20D \times 0.2D$. Using a slightly larger domain, Yeo and Jones (2008) performed a three-dimensional detached eddy simulation to investigate the three-dimensional characteristics flow past a yawed and inclined circular cylinder using varying length of the cylinder. The resulting domain was $40D \times 40D \times 30D$ with the analysis on the flow structures developing around and downstream of the cylinder giving the best results due to the length of the cylinder used of $30D$. For this study a balance between the two domains mentioned above were used to concentrate on the flow around and along the leeward side of the cylinder while focusing the computational power at this location.

2. PRESSURE DISTRIBUTION AROUND THE CABLE MODEL

In order to determine the effect of the flow pattern on the pressure induced at the surface of the cylinder the pressure distributions around the cylinder (upstream and downstream) are shown in Fig. 2. The distributions are shown on a plane parallel to the initial flow and passing through the center of the cylinder for its entire length. As expected for the flow perpendicular to the cylinder, an area of high negative pressure
builds up on the leeward side of the cylinder. The build-up of pressure is a result of the vortices created by the flow around the cylinder before it continues downstream. Due to the three-dimensionality of the flow for this Re the pressure varies along the length of the cylinder. With the cylinder inclined to the flow the pressure surrounding the cylinder decreases, but with a much greater decrease on the windward side of the cylinder in comparison to the leeward side of the cylinder. Also, a rotational variation of the pressure distribution develops along the leeward side of the cylinder. This rotational variation differs for different Re, and angles of inclination and yaw tested. Some formations are closely spaced while others are more spread out along the length of the cylinder.

For flow perpendicular to the cylinder Zdravkovich (1997) showed that the pressure coefficient ($C_P$) distribution will vary with the increase of Re number. Thus, the minimum value of the $C_P$ will decrease, and its position will move around the circumference of the cylinder towards downstream side. For the inclined cylinders investigated in the current research, the relative angle of attack had more influence upon the variation of the values and distribution of $C_P$ than the Re number. Also, the pressure distribution was not symmetric around the cylinder as it would be for the flow perpendicular to the cable due to the elliptic shape of the cross section; the distribution of the $C_P$ results are plotted in Fig. 3.

The pressure profiles for the two Re numbers are used in the current study for the flow perpendicular to the cylinder, inclined to the cylinder to the flow at 60°, and for a
cylinder inclined to the flow by 57.4° and yawed 40° for Re 2.2 × 10^5 to 4.4 × 10^5, as presented in Fig. 3. As can be seen comparing the results for flow perpendicular to the cylinder, a similar change in position along the circumference of the cylinder is seen with the change in Re number as well as a variation of the pressure distribution along the leeward side of the cylinder. The results for the flow inclined to the cylinder at 60° showed less variation in the distribution for the range of Re numbers for both the separation point and along the leeward side of the cylinder. The major difference between the two inclinations of the cylinder is the change along the leeward side of the cylinder for the flow inclined to the cylinder when compared to the flow perpendicular to the cylinder.

a) \( \phi = 0^\circ; \alpha = 0^\circ, \beta = 0^\circ \)  
ii) Re = 2.2 × 10^5;  
iv) Re = 4.4 × 10^5;  
ii) Re = 2.2 × 10^5.

b) \( \phi = 60^\circ; \alpha = 60^\circ, \beta = 0^\circ \)  
iv) Re = 4.4 × 10^5;  
iv) Re = 2.2 × 10^5;  
iv) Re = 4.4 × 10^5.

g) \( \phi = 64.0^\circ; \alpha = 54.7^\circ, \beta = 40^\circ \)  
iv) Re = 2.2 × 10^5;  
iv) Re = 4.4 × 10^5;  
iv) Re = 4.4 × 10^5.

Fig. 3 Pressure distributions around the cylinder

4 CROSS-COHERENCE RESULTS

The spectral coherence was used to examine the relationship between pressure, total velocity, and coefficient of lift. Two signals were compared with one another and presented. The coherence function between two signals \( x(n) \) and \( y(n) \) is defined as (Eq.1):

\[
C_{xy}(\omega) = \frac{|P_{xy}(\omega)|^2}{P_{xx}(\omega)P_{yy}(\omega)}
\]
where, $P_{xy}$ is the cross-spectral density between $x$ and $y$, $P_{xx}$ is the autospectral density of $x$, and $P_{yy}$ is the autospectral density of $y$. The result is a real number between 0 and 1, which shows measures the correlation between $x(n)$ and $y(n)$ at the frequency $\omega$. The function takes sequence $x$ and $y$, and computes the power spectra and CPSD (cross power spectral density). The result is the quotient of the magnitude squared of the CPSD and the product of the power spectra. The result of the coherence function is an estimate of the extent to which $y(t)$ may be predicted from $x(t)$ by an optimum linear least squares function.

The coherence and cross-coherence were calculated between the lift coefficient, the pressure coefficient along the leeward side of the cylinder, and the total velocity along the leeward side of the cylinder. To assist in the descriptions of the results, the coherence between the lift, pressure coefficient, and leeward velocity will be loosely categorized as either ordered or chaotic (Van Atta and Gharib, 1987). An ordered result will be characterized by a power spectrum dominated by a narrow primary spectral peak, and associated side band peaks. A chaotic result will be characterized by a relatively broad band power spectrum. The three cases investigated for the flow characteristics around a cylinder inclined to the flow include the case of the cylinder perpendicular to the flow, a second case of flow inclined to the flow by $60^\circ$ ($\phi = 60^\circ; \alpha = 60^\circ, \beta = 0^\circ$), and the third case is for a cylinder inclined to the flow by $57.4^\circ$ and yawed $40^\circ$ ($\phi = 64^\circ; \alpha = 57.4^\circ, \beta = 40^\circ$). For each case the coherence was calculated between the coefficient of lift ($C_L$) and pressure coefficient along the leeward side of the cylinder, between the coefficient of lift and the total velocity fluctuation along the leeward side of the cylinder, and between the pressure coefficient and the total velocity along the leeward side of the cylinder. Most of the results presented can be categorized as an ordered signal with narrow peaks throughout the analyzed range. The exception to this are the coherences between the coefficient of lift and the total velocity fluctuation along the leeward side of the cylinder for $Re = 4.4 \times 10^5$ (Fig. 4). These coherences can be categorized as a chaotic signal with a broader band in the power spectrum.

$\phi = 0^\circ; \alpha = 0^\circ, \beta = 0^\circ, Re = 2.2 \times 10^5$;

$\phi = 0^\circ; \alpha = 0^\circ, \beta = 0^\circ, Re = 4.4 \times 10^5$;
Fig. 4 Coherence distributions for $C_L$-Pressure, $C_L$-Total velocity and Pressure-Total velocity
5 CROSS-BICOHERENCE RESULTS

Bicoherence is a squared normalized version of the bispectrum. The bispectrum is a statistic used to search for nonlinear interactions. The cross-bicorrelation measures the relationship between three sets of variables (Swami et al., 1998) in the time domain, and gives an indication of the persistence of that interaction. The cross-bispectrum density is the measure in the frequency domain of such interaction. Three data sets are segmented into possibly overlapping records. The mean from each record is removed, and then the time-domain window is applied. The cross-bispectrum is computed for the kth record as (Eq. 2)

\[ B_{wx,y,k}(m,n) = W_k(m)X_k(n)Y_k^*(m+n) \]

(2)

where, \( W_k \) is the FFT for the kth segment of \( w \), \( X_k \) is the FFT for the kth segment of \( x \), and \( Y_k \) is the FFT for the kth segment of \( y \). The spectra are computed as (Eq. 3)

\[ P_{w,k}(m) = |W_k(m)|^2 \]
\[ P_{x,k}(m) = |X_k(m)|^2 \]
\[ P_{y,k}(m) = |Y_k(m)|^2 \]

(3)

The cross-bicoherence is estimated by averaging the spectral and cross-bispectral estimates across records. The cross-bicoherence is defined as (Swami et al., 1998)

\[ \text{bic}_x(y,z)(f_1, f_2) = \frac{S_{xyz}(f_1, f_2)}{\sqrt{S_{xx}(f_1 + f_2)S_{yy}(f_1)S_{zz}(f_2)}} \]

(4)

where, \( S_{xyz}(f_1, f_2) \) is the averaged estimate of the cross-bispectrum, \( S_x(f) \) is the averaged estimate of the power spectra of \( x \), \( S_y(f) \) is the averaged estimate of the power spectra of \( y \), and \( S_z(f) \) is the averaged estimate of the power spectra of \( z \).

The resulting spectrum of the cross-bicoherence ranges from 0 to 1, for comparisons between two variables. The bicoherence functions are presented as three-dimensional plots, one axis for each frequency \( f_1 \) and \( f_2 \) and a magnitude axis (Van Diver and Jong, 1987). To enhance visualization and interpretation of the cross-bicoherence the results are shown in contour plots (Silva et al., 2005). The result is an estimate of the cross-bicoherence of the three variables. Therefore, this is an nfft-by-nfft array, with the origin at the center, and the axes pointing down and to the right (Swami et al., 1998).

Thus, the cross-bicorrelation measures the relationship between three sets of variables in the time domain and gives an indication of the persistence of their interaction. The cross-bispectrum density is the measure of the frequency domain of such interaction. The result is a contour plot of the magnitude of the estimated cross-bicoherence (Swami et al., 1998).
The results of the cross-bicoherence between the coefficient of lift, the pressure coefficient along the leeward side of the cylinder, and the velocity fluctuation along the leeward side of the cylinder for the three cases are presented in the lower part of the Fig. 5 below. As shown in the colour scale, the darker the colour the less cross-coherence is present between the variables, and the lighter the color the more intensive coherence is between the variables for the St numbers. Most cases show little to low cross-coherence between the three variables. For the three cases presented in this section, for flow perpendicular to the cylinder, for a cylinder inclined to the flow by 60°, and a cylinder inclined to the flow by 54.7° and yaw of 40°, the magnitude of the cross-coherence generally decreased with the increase in Re. The exception to this observation is the case of the cylinder inclined to the flow by 54.7° and yaw of 40°, the cross-coherence across the tested Re remains consistent except for the case of Re = 4.4 × 10^5 where a much lower pattern in the results was noticed. The other case is for the flow perpendicular to the cylinder at Re = 1.1 × 10^5, where peaks in the cross-coherence occur at St combinations of approximately 0.1 and 0.5, and 0.2 and 0.3. Moreover, for Re = 4.4 × 10^5 for a cylinder inclined to the flow by 54.7° and yaw of 0° peaks were noted for the cross-coherence, but an unstructured pattern in the results was noticed. The results were categorized under three cases, low non-linear interaction, intermediate non-linear interaction, and high non-linear interaction, as illustrated by the cross-coherence. The case φ = 60°; α = 60°, and β = 0°, for Re = 4.4 × 10^5 showed low non-linear interaction between the coefficient of lift, the pressure coefficient along the leeward side of the cylinder. Cases considered with intermediate non-linear interaction were: φ = 60°; α = 60°, and β = 0° for Re = 2.2 × 10^5, and φ = 57.1°; α = 54.7°, and β = 20° for Re = 2.2 × 10^5. Finally, the case considered with high non-linear interaction was φ = 54.7°; α = 54.7°, and β = 0°, for Re = 4.4 × 10^5.

φ = 60°; α = 60°, β = 0°, Re = 2.2 × 10^5;  
φ = 54.7°; α = 54.7°, β = 0°, Re = 4.4 × 10^5;
3. CONCLUSIONS

The current study focused on numerically investigating the flow behavior around the inclined and yawed cylinders, aiming at clarifying the occurrence and properties of the axial flow attached on the leeward side of the cylinder, and the effect this could have on dry inclined cable galloping phenomenon. Several cases of inclined ($\alpha$) and yawed ($\beta$) circular cylinders with angles between 0° and 60°, and 0° and 40° respectively. The coherence and cross-coherence has been calculated between the coefficient of lift, the pressure coefficient along the leeward side of the cylinder, and the velocity fluctuation along the leeward side of the cylinder. The cross-bicorrelation which measures the relationship between three sets of variables in the time domain and gives an indication of the persistence of that interaction was also reported. The cross-bispectrum density is the measure in the frequency domain of such interaction. The three cases highlighted above, namely, $\alpha = 60^\circ$, $\beta = 0^\circ$, $\phi = 60^\circ$; $\alpha = 0^\circ$, $\beta = 0^\circ$, $\phi = 0^\circ$, and $\alpha = 54.7^\circ$, $\beta = 40^\circ$; $\phi = 64^\circ$, for $\text{Re} = 2.2$ and $4.4 \times 10^5$, were used for the fast Fourier analysis with respect to
the coefficient of drag, coefficient of lift, pressure along the leeward point of the cylinder, and the velocity along the leeward side of the cylinder. Coherence and cross-coherence between three of these variables were determined for the same cases. The cross-coherence results for the same three cases showed a consistent distribution, indicating a linear interaction among the variables. The case \( \alpha = 60^\circ, \beta = 0^\circ, \phi = 60^\circ \), for Re of \( 2.2 \times 10^5 \) also shows indications of aerodynamic instability in this study. The cause of instability is due to the large change in the lift coefficient variation at \( \phi \) close to 60\(^\circ\), for all cases investigated in this study, when using the local derivatives. More investigations are required for clarifying the effect of cable inclination angle (\( \alpha \)) and yaw angle (\( \beta \)) on this large variation of lift coefficient for this relative angle of attack (\( \phi \)).

REFERENCES


