The 2018 World Congress on
Advances in Civil, Environmental, & Materials Research (ACEM18)
Songdo Convensia, Incheon, Korea, August 27 - 31, 2018

A novel surrogate-based back-analysis approach for geotechnical applications

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ABSTRACT

This paper proposes an efficient back-analysis technique that is used in conjunction with numerical methods such as finite element or finite difference methods. The proposed technique combines particle swarm optimization and surrogate modeling with kriging interpolant. This combination allows efficient back-analysis with reduced number of forward analysis and thus less computation effort.

1. INTRODUCTION

In geotechnical numerical simulations, one of the most challenging task is to identify representative material parameters for chosen constitutive models. It is well known conventional constitutive models such as linear-elastic model with Mohr-Coulomb yield criterion cannot accurately model soil behaviors. Yet advanced constitutive models need several parameters that may not obtained easily through conventional laboratory tests. Furthermore, parameters obtained from laboratory tests may be not presentative to in-situ condition due to the disturbance during sampling. Back analysis technique offers a viable alternative to determine model parameters that may be more representative than parameters obtained via laboratory tests.

Back analyses using numerical simulations such as finite element method (FEM) or finite difference method (FDM) have become popular in recent years due to advances in the computing capability. Nonetheless, it is still a time-consuming task due to hundreds and thousands of numerical simulations or forward analyses are performed during the back analysis procedure illustrated in Fig. 1. One straightforward solution to the lengthy process is applying parallel and distributed computing technique to conduct forward analyses in parallel. However, authors believe novel algorithms should be employed to reduce the needed number of forward analyses. This reduction can be achieved using surrogate-based back analyses technique.

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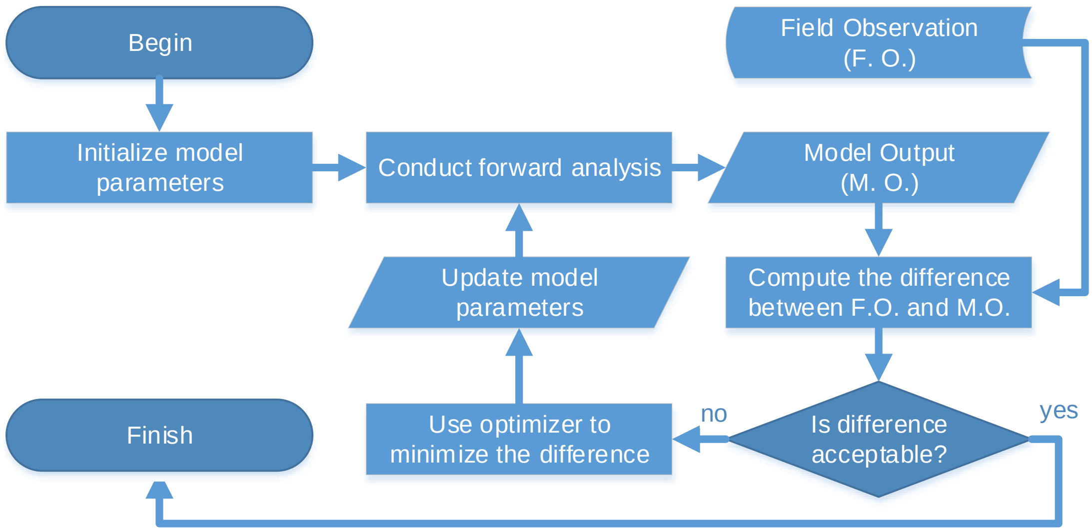

Fig. 1 Typical flowchart for back analyses with numerical simulations

Typical surrogate-based back analysis calibrates surrogate models using certain number of forward analysis. Afterwards, the calibrated surrogate model then replaces forward analysis step in Fig. 1. In this study, authors propose a new back-analysis scheme that introduces interaction between a) the surrogate model and b) the optimizer. In the past, these two components in surrogate-based back-analysis are considered or developed separately. Authors believe introducing interactions between the two can lead to new opportunities and better efficiency. In subsequent sections, related researches using surrogate-based back-analysis technique are first introduced. We then present the newly developed approach. Afterwards, preliminary evaluations of the proposed scheme are presented. Finally, discussions and remarks are given to the newly proposed back-analysis scheme.

2. FORMER STUDIES USING SURROGATE-BASED BACK-ANALYSIS

Surrogate-based back-analysis was proposed in the past to avoid carrying out forward analyses using FEM (Khaledi, 2014) or FDM (You, 2014). Surrogate models can be formulated using polynomial interpolation, radial basis function (Khaledi, 2014), neural network (You, 2014), etc. They often rely on a calibration or training stage to calibrate the surrogate model to behave like the forward analysis to be replaced. Afterwards, back analysis steps illustrated in Fig. 1 are then conducted using the trained surrogate model and a chosen optimizer. How to efficiently calibrate or train the surrogate models that can replace FEM or FDM, however, is rarely discussed.

Some studies perform thousands of FEM to establish the calibration dataset to calibrate surrogate models, and then use the surrogate model to conduct back analyses (You, 2014). They assume the surrogate model is faithful representing the forward analysis. Some study use lesser initial calibration dataset to train the initial surrogate model, then perform one round of back-analysis. The back analyzed parameter set is then fed to the full forward analysis. Results from the surrogate model and the full analysis are then compared. When the difference between the two is unsatisfactory, the new forward analysis result is added to the calibration dataset and the surrogate model is then refined, and back-analysis is repeated using the updated surrogate model (Pichler et al. 2003).

Optimizer also plays an important role in back analysis. Gradient-less or population-based methods such as genetic algorithm (Levasseur et al. 2008), evolutionary strategy (Hashash et al., 2010), and PSO (particle swarm optimization, Meier 2008) are often used in back analysis. This is because gradient information is often unavailable and these methods can be executed efficiently using parallel computers. These population-based optimizers search optimal solutions by first having a population of candidate solutions. Each of the candidate solution in the population evaluate its fitness value (i.e. how good the solution is). The optimizer then evolves the population into the next generation, achieving better solutions. In the context of back-analysis, each fitness value is evaluated using forward analysis. It is authors' opinion the direct use of population-based optimizers in the context of back-analysis is wasteful. This is because forward analyses are expensive, thus the information obtained from one forward analysis should be reused rather than discarded.

3. THE NEW APPROACH

In this study, we propose a new back-analysis scheme that uses Kriging interpolant (Kleijnen, 2009) as the formulation for the surrogate model, and use PSO (Kennedy and Eberhart, 1995) combined with CGM (Conjugate Gradient Method, Fletcher and Reeves, 1964) as the optimizer. Kriging offers a unique capability that it not only can interpolate quantities at any location based on existing known data, but also can yield a Kriging variance for the interpolated value. The Kriging variance is an indicator how confident the Kriging interpolant is regarding the interpolated value. PSO is a gradient-less population-based optimizer that is easy to implement. In order to accelerate convergence to optimal solutions, we combine CGM to do gradient-based optimization at global best identified by PSO. The gradient information is obtained by using numerical differentiation on the surrogate model.

The overall back-analysis algorithm we propose is summarized in Tab. 1. It must be noted our surrogate model is updated (step 2) in each generation of PSO. The update is owing to new information from particles with large Kriging variance and the particle at global best (step 5c). It must also be noted all expensive forward analysis results are used and reused to calibrate the surrogate model. Finally, the combination between Kriging and PSO enables the use of CGM to accelerate the convergence. Without Kriging, using numerical differentiation (and performing forward analyses) to get gradient information is expensive. Also, initiating CGM with PSO's global best is ideal because CGM may fail to converge if the starting point is far from the optimal.

Tab. 1 The newly proposed back-analysis procedure

1. Initialize PSO population and calculate fitness values for each particle by conducting forward analyses on all particles.
2. Use all available results calculated by forward analyses to calibrate/refine Kriging interpolant.
3. Evolve PSO population by updating each particle's position:
    a. Update particle position by standard PSO.
    b. If the particle is the current global best, use CGM with Kriging and iterative 5 – 10 times.
    c. Otherwise, if the Kriging variance at the updated position is small, mark the particle.
4. Reposition all marked particles (from step 3c.)
5. Update fitness value for all particles:
    a. Evaluate fitness value and Kriging variance at the updated position.
    b. If the Kriging variance is small, the fitness value interpolated by Kriging is accepted.
    c. Otherwise, evaluate fitness value by forward analysis.
6. Repeat step 2 through 5 until satisfactory.

4. PRELIMINAR EVALUTION OF THE NEW APPROACH

To assess the proposed back analysis scheme, we use Bukin function (Jamil and Yang, 2013) defined in Eq. (1). Bukin function in the chosen region illustrated in Fig. 2 resembles the error function reported by Moreira et al. (2013) shown in Fig. 3. The error function is obtained for back analyzing tunnel excavation using linear-elastic constitutive model with Young's modulus E and K_0 to be identified by field measurements of crown downward movement and surface subsidence. The Bukin function gives a valley like shape with several local minima in the valley seen in Fig. 3.

$$\begin{aligned} &f(x,y)=100\sqrt{\left|y-0.01x^2\right|}+0.01\left|x+10\right| \\ &x\in[-15,-5],\, y\in[-3,3] \end{aligned} \tag{1}$$

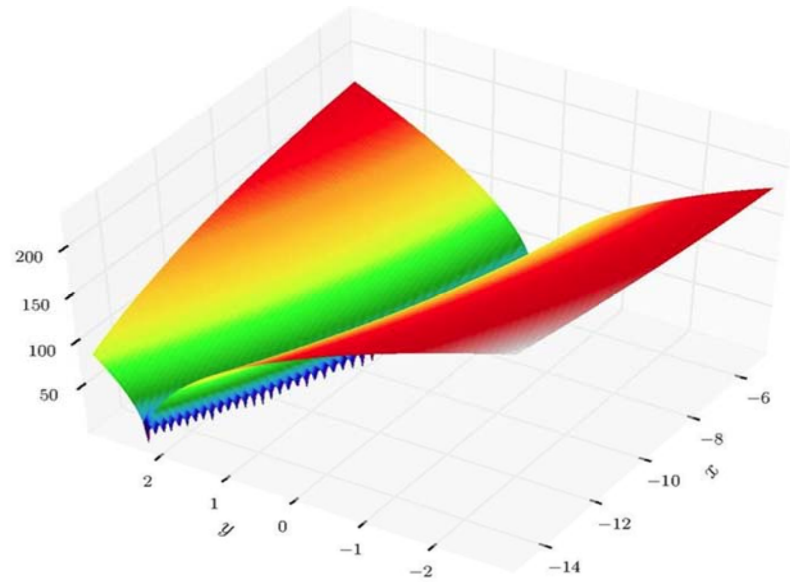

Fig. 2. Bukin function

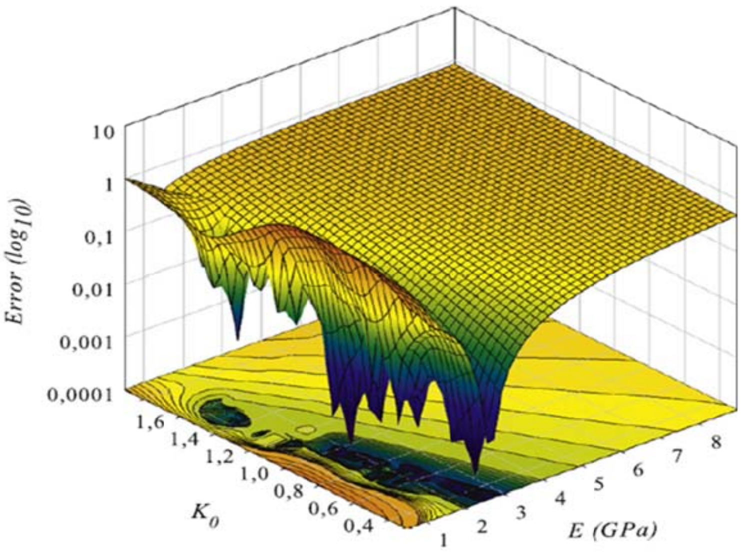

Fig. 3. Error function for a tunnel excavation with two field measurements (Moreira et al., 2013)

We evaluate the new approach with 30 particles and 20 generations. Since our new approach has a threshold for Kriging variance in order to determine whether to accept surrogate model's response or to perform full forward analysis. The evaluation tries three levels of variance threshold: small, medium, and large. Standard PSO without using surrogate model is also performed three times for comparison. Best and worst PSO results (based on the final solution found) are also reported in our evaluation.

Fig. 4 shows the convergence history, which is the population best solution in each generation. The true solution 0 is at the bottom border of the charts. It must be kept in mind that PSO is a stochastic search method with some randomness during its search. It is seen in Fig. 4 that PSO and the new approach show similar performance in a broad sense. Almost all methods converge to the true solution except NEW-MV – the new approach with medium variance threshold. However, authors believe the true solution can be obtained if more generations are allowed.

Fig. 5 shows the number of forward analyses used in each generation. Without using surrogate model, the number of forward analyses is 30 in each generation because each particle in the population needs to evaluate its fitness value. It is seen the new approach can indeed reduce the number of forward analysis, and the reduction is determined by the Kriging variance threshold. Small variance threshold (NEW-SV) means we only accept Kriging results with high confidence. It is seen there is nearly no reduction in the number of forward analyses. For both medium and large variance threshold, Fig. 5 shows a downward trend with fewer and fewer number of forward analyses. This is because each generation introduces more samples to refine the Kriging surrogate model. Thus the Kriging surrogate model yields more and more accurate responses and reduces the need to use the original forward analysis.

5. CONCLUDING REMARKS

A new surrogate-based back-analysis approach is proposed in this study. The new approach uses PSO as the optimizer and Kriging interpolant as the surrogate model. Kriging interpolant is used to make use of all forward analysis results, and the Kriging variance is used to evaluate the surrogate response in order to reduce the use of full forward analysis.

Because we have yet to integrate the new approach with FEM or FDM, we evaluated our approach with Bukin benchmark function often used in optimization. The evaluation suggests the solution quality is maintained and the use of forward analysis is reduced. This result promises reducing solution time for back-analyses with FEM or FDM because typical forward analyses with FEM or FDM takes several minutes to complete in geotechnical engineering applications.

We are currently working on integrating the proposed approach with Plaxis software package, so that the new approach can be applied in practical engineering applications.

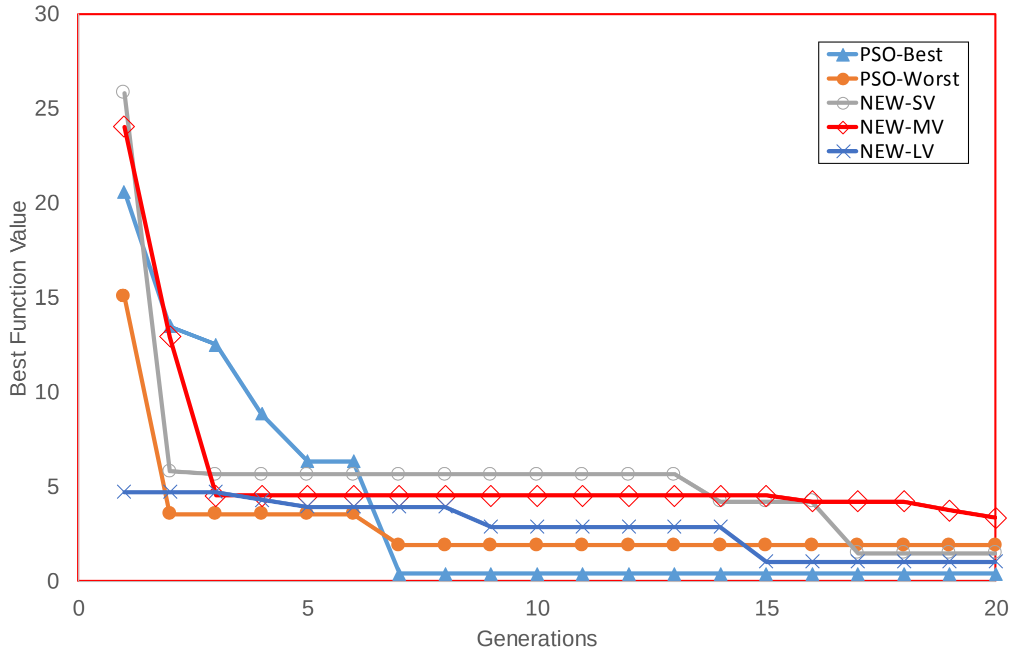

Fig. 4 Convergence History

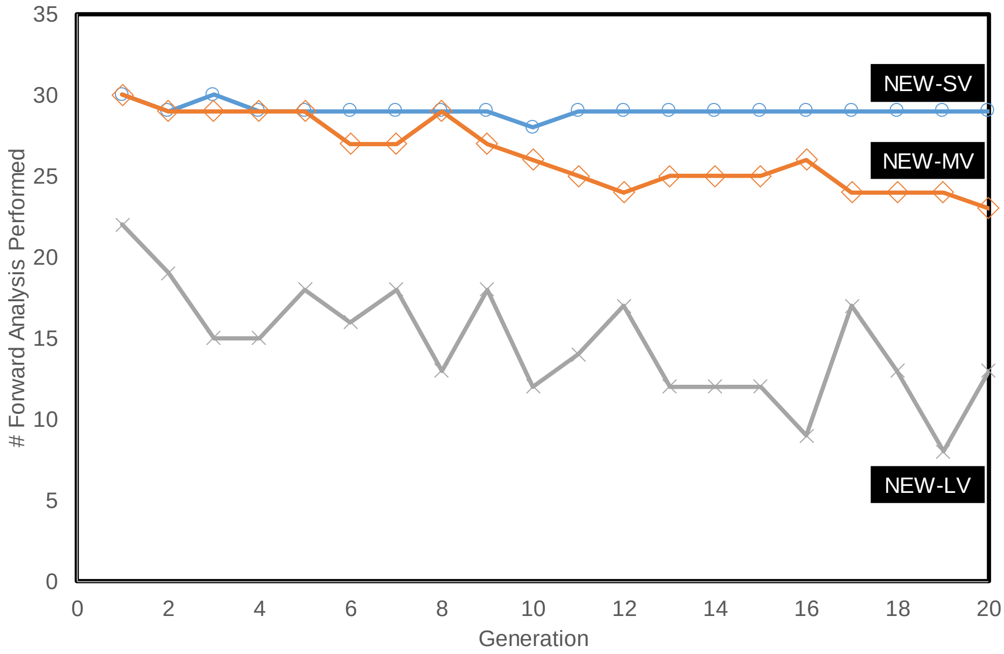

Fig. 5 The number of full forward analyses performed in each generation

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