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Role of rotating tidal waves on coupled dynamics of the Earth-Moon system

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Abstract. We theoretically consider a possible influence of periodic oceanic tides on non-periodic changes in the dynamics of the Earth and Moon over a long time scale. A particular emphasis will be placed on the contribution from rotating tidal waves, which rotate along the inner edge of an oceanic basin surrounded by topographic boundary. We formulate the angular momentum and the mechanical energy of the rotating tidal wave in terms of celestial parameters with regard to the Earth and Moon. The obtained formula are used to discuss how the energy dissipation in the rotating tidal wave should be relevant to the secular variation in the Earth's spin rotation and the Earth-Moon distance. We also discuss the applicability of the formula to general oceanic binary planets subject to tidal coupling.

Keywords: ocean tide; tidal energy dissipation; Kelvin wave; celestial mechanics; lunar orbit

1. Introduction

The ocean tide is the rises and falls of the sea level in a periodic manner. Tide plays an important role in the natural world; for instance, the tidal rhythm of marine organisms (Palmer 1973) and the fortnightly-cycle variation in extra strain on geological faults (Ide *et al.* 2016) are natural phenomena that are strongly governed by tidal cycle in the ocean. Tide can also give a marked influence on our daily life, especially on people enjoying marine sports such as surfing, diving, and snorkeling.

From a viewpoint of mechanics, the ocean tide is mainly driven by combination of the two celestial-scale forces that act on sea water (Murray and Dermott 2000): the one is the gravitational force exerted by the Moon, and the other is the centrifugal force associated with the Earth's revolution (i.e., revolution around the common center of gravity of the Moon and the Earth). The resultant force, called tidal force, generates oscillatory flow of sea water at the global scale, called tidal current.

It has been broadly accepted that the friction between tidal current and sea floor (Taylor 1919, Jeffreys 1920) as well as tidal wave scatterings in deep ocean (Egbert and Ray 2000) cause dissipation of the mechanical energy of the Earth-Moon system at a rate of several terawatts. Due to the energy dissipation, the Earth's spin has been slowing down gradually, and the decline in the

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Earth's spin angular momentum is transferred to the increment in the orbital angular momentum of the Moon. This angular momentum transfer is believed to elongate the orbital length radius of the Moon at a rate of 3.82 ± 0.07 cm/year, as was confirmed by the Lunar Laser Ranging experiment based on the reflector installed on the Moon (Samain *et al*. 1998, Murphy *et al*. 2012, Murphy 2013). Accumulating this tiny recession over billions of years, which is the time duration that have passed since the birth of the Moon, attains a celestial length scale comparable to the present Earth-Moon distance ($\cong 3.8 \times 10^6$ km). It is interesting to say that the ocean tide, though quite familiar to our every life, gives a dramatic impact on the secular variation in the celestial-body dynamics in a timescale of billions of years (Burns and Matthews 1986).

In the present work, we consider the contribution from "rotating tidal waves" (or so-called "boundary-trapped surface Kelvin waves") to the secular variation in the coupled dynamic of the Earth-Moon system. It is a special class of tidal waves, rotating along the inner edge of topographic boundary such as a coastline or a submarine basin (Pinet 2014). An important feature of the rotating tidal wave is that it is non-dispersive, i.e., the phase velocity of the wave crests is equal to the group velocity of the wave energy for all frequencies. The wave thus retains its shape as it moves in the alongshore direction over time. Furthermore, most rotating tidal waves in the northern (or southern) hemisphere propagate in a counterclockwise (clockwise) direction, wherein the coastline plays a role of a wave guide. The persistency in the rotation direction implies that those tidal waves may be relevant to the energy dissipation or the angular momentum transfer within the Earth-Moon system, while there has been few theoretical attempts to examine the possibility.

To resolve the problems posed above, we have developed a simplified analytic model that describes both the angular momentum and the mechanical energy of rotating tidal waves trapped in oceanic basins on the Earth. In our argument, the velocity of the sea water consisting the tidal waves was evaluated using the shallow-water wave equation (Pinet 2014); the equation is valid under the condition that both the radius of the basin and the wavelength of the tidal wave are sufficiently longer than the mean depth of the sea. We emphasize that the formulation we have developed can apply to not only the Earth-Moon system dynamics but to other coupled celestial dynamics as long as they hold fluid layer on the surface.

2. Tidal force and tidal potential

The tidal force $\mathbf{F}_\mathrm{T}$ associated with the Moon (with mass M_m), exerting on a body of mass m at position $\mathbf{r}$ measured from the center of the Earth, is given by

$$\mathbf{F}_\mathrm{T} = -GmM_\mathrm{m}\left(\frac{\mathbf{d}}{d^2} - \frac{\mathbf{D}}{D^2}\right). \tag{1}$$

Here G is the gravitational constant, $\mathbf{D}$ is the vector from the center of the Moon to the center of Earth, and $\mathbf{d} \equiv \mathbf{D} + \mathbf{r}$ is the vector from the center of the Moon to the body of mass m (See Fig. 1a). We see from Eq.(1) that the tidal force $\mathbf{F}_\mathrm{T}$ is the vector difference of the gravitational attraction to the Moon, $-(GmM_\mathrm{m}/d^2)\mathbf{d}$, and the centripetal force acting on the center of the Earth, $-(GmM_\mathrm{m}/D^2)\mathbf{D}$.