Transient thermo-mechanical response in biological tissue based on the theory of porous media

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ABSTRACT

Comprehension of heat transfer and related thermo-mechanical interaction in biological tissues is very important to medical applications. Presence of blood and its thermal roles in living tissue such as blood perfusion and convection, make it is quite natural to treat the living tissue as a porous medium. The biological tissue can be categorized as vascular and extra-vascular regions. With this motivation, the thermo-mechanical response of porous biological tissue exposed to an instantaneous thermal shock with considering the thermal osmosis effect is investigated. The governing equations are established in the generalized thermo-hydro-elastic theory and solved by time-domain finite element method. The effects of porosity and the coefficient of thermal osmosis on the response are illustrated graphically. Comparison are presented with the corresponding partially thermo-hydro-elastic model and thermoelastic model to investigate the influence of coupling effect on the elastic medium.

1. INTRODUCTION

Analysis of heat transfer in biological tissue is a complicated physiological process including heat conduction in tissue, convective between blood and vessel and blood perfusion. The biological tissue consist of cell and microvascular bed with the blood flow through many vessels. Thus it can be treated as a porous media and divided into vascular region and extravascular region. The studies about heat transfer across porous media have been drawn much attention for many decades. Darcy (1896) first proposed a linear relationship between the flow velocity and the pressure gradient across the porous medium (Darcy model). Then Darcy model has been widely applied in biomedical and engineering fields. For examples, the effects of Darcy number, Reynolds number and porosity on the velocity and temperature distributions are

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investigated by Khaled and Vafai (2003). Kou et al. (2003) analyzed the influence of directional blood flow and heating schemes on the temperature profile during thermal therapy. Rattanadecho and Keangin (2013) studied heat transfer and blood flow in two-layered porous liver tissue during microwave ablation. Nevertheless, the influence of temperature gradient on fluid flow (thermal osmosis effect) is not considered in classical Darcy model. Srivastava and Avasthi (1975) and Zhou et al. (1998) have shown that thermal osmosis has an important effect on thermal responses of fluid saturated porous media.

It is noted that even a small change of heat-induced stress can suppress immune response, alter production of hormones and protein denaturation (Lau et al., 1995). The blood flow also has an important effect on the deformation. However, there is little study on the thermo-hydro-mechanical interaction at high temperature, even though it is related to the thermal damage of tissue such as tissue shrinkage. Keangin et al. (2011) investigated the temperature distribution in liver tissue and compared the results of pure bioheat transfer and the model with deformation. They showed that the model with deformation is more accurate to describe the physical characteristics of liver cancer in thermotherapy than pure bioheat transfer analysis. But the quasi-static uncoupled thermo-mechanical model is used in their deformation model, in other words, they ignored the influence of mechanical behavior on the temperature distribution.

Based on classical Fourier’s heat conduction law, Biot (1956) firstly formulated the coupled thermoelastic theory to eliminate the paradox inherent in the classical uncoupled theory that elastic deformation has no effect on the temperature. It is well known that Fourier’s heat conduction law predict an infinite speed of propagation for thermal wave. However, heat travels at a finite speed in particular heat treatment conditions, i.e. high-power with short durations and cryogenic temperature, or heat conduction in media with non-homogeneous inner structure. To eliminate such paradox, the two generalized thermoelastic theories established by Lord and Shulman (1967) and Green and Lindsay (1972), which predict the finite speeds of thermal wave, have been widely used in investigating transient thermal shock problems. Biot (1977) proposed a thermo-elastodynamics theory for fluid saturated porous media, which based on classical Darcy model and Fourier’s heat conduction law for fluid flow and heat flow. The linear and nonlinear coupled thermo-hydro-elastodynamics response for a saturated poroelastic medium were investigated by Liu et al. (2007, 2009, 2010). Liu et al. (2010) presented the general equations of a saturated porous media by using the generalized thermoviscoelastic theories. The generalized porous thermoelastic theories with relaxation times for a saturated porous medium are given by Liu et al. (2011). They investigated a two-dimensional thermoelastic problem which is subjected to a time-dependent thermal/mechanical source in the context of the generalized porous thermoelastic theories.

In this work, we investigate the transient coupled thermo-mechanical response of porous biological tissue with considering thermal osmosis effect in the context of the generalized thermo-hydro-mechanical model. The governing equations are established in the generalized porous thermoelastic theory and solved by time-domain finite element method. The effects of porosity and thermal osmosis on the response are illustrated graphically.
2. PROBLEM FORMULATION

2.1 Basic equations

It is assumed that biological tissue is uniform with linear, homogeneous and isotropic thermoelastic properties in the present work. The metabolic heat generated by the biological tissues can be neglected compared with the high-intensity laser. Neglecting body force, the thermo-hydro-elastic equations with considering thermal osmosis effect can be expressed as:

\[ q_i = -k \theta_j + \rho_c c_i \omega_i \theta \]  
\[ \omega_i = -\kappa \left( \rho_j + \left( \frac{\rho_b}{n} \right) \omega_i + \rho_j \dot{u}_j \right) - D_j \theta_j \]  
\[ q_{ij} = -\rho T_0 \dot{S} + Q_{\text{ext}} \]  
\[ p = M \left( \alpha \dot{u}_i + \rho_b \omega_i \right) \]  
\[ \sigma_{ij} = \rho \dot{u}_i + \rho_b \omega_i \]  
\[ \sigma_y = 2 \mu \epsilon_{ik} \delta_{ij} - \gamma \left( \theta + \tau_i \dot{\theta} \right) \delta_{ij} - \alpha p \delta_{ij} \]  
\[ \rho S = \frac{\rho c}{T_0} \left( \theta + \tau_2 \dot{\theta} \right) + \gamma \epsilon_{ii} + \gamma_b \xi_{kk} \]  
\[ \epsilon_y = \frac{1}{2} \left( u_{i,j} + u_{j,i} \right) \]  
\[ \xi_{kk} = -\omega_{ii} \]  

where \( q_i, k, \rho, c, \theta \) are heat flux, effect thermal conductivity, effect density, effect specific heat and temperature increment, \( \theta = T - T_0, |\theta / T_0| << 1 \) , \( T \) is the temperature, \( T_0 \) is the reference temperature; 
\( k = nk_b + (1-n) k_t, \rho c = n \rho_b c_b + (1-n) \rho_c \) and \( k_b, k_t, \rho_b, \rho_c, c_b, c_t \) are the thermal conductivity of blood and tissue, mass density of blood and tissue, specific heat of blood and tissue, respectively; \( S \) is entropy density; \( Q_{\text{ext}} \) is the external heat source supplied by laser; \( \sigma_{ij}, u_i, w_i \) and \( p \) are the stress tensor, tissue displacement vector, blood displacement with respect to the tissue and excess pore blood pressure respectively; \( \alpha \) is the Biot coefficient and \( \alpha = 1 - K / K_t \), \( K, K_t, K_b \) are the bulk modulus of porous biological tissue, tissue grains and blood; and 
\( 1 / M = n / K_b + (1-n) / K_t \), \( \kappa = k_t / \left( \rho_b g \right), k_t \) is the intrinsic permeability, \( g \) is the gravitational acceleration; \( \gamma_b = K_b a_b, \gamma = K a_u, a_u = n a_b + (1-n) a_t, a_b, a_t \) are the linear thermal expansion coefficient of blood and tissue; 
\( \lambda = 3K \nu / \left( 1+\nu \right), \mu = 3K \left( 1-2\nu \right) / 2 / \left( 1+\nu \right) \), they are Lame’s constants, \( \nu \) is Poisson’s coefficient; \( \epsilon_{ij} \) is the strain tensor and \( \epsilon_{ik} = u_{ij} \); \( \xi_{kk} \) is the variation of fluid content; 
\( \delta_{ij} \) is Kronecker delta function. \( \tau_1, \tau_2 \) are the relaxation time factors; \( D_j \) is a phenomenological coefficient associated with the influence of thermal gradient on the blood flux (thermo osmosis effect), it can be directly measured in experiments. Besides,
super-dot refers to the derivative with respect to time; comma followed by sub-index denotes the corresponding partial differentiation.

Substituting Eqs. (1) and (7) into Eq. (3), we can obtain:

\[ k \frac{\partial \theta}{\partial t} + Q_{ext} = \rho c \left( \frac{\partial \theta}{\partial t} + \tau \frac{\partial \theta}{\partial \tau} \right) + \gamma T_0 \delta_{kk} + \left( \gamma - \rho \gamma \right) \frac{\partial \psi}{\partial t} + \rho \gamma \psi \frac{\partial \theta}{\partial t} \]  
(10)

Substituting Eqs. (4) and (9) into Eq. (2), yields:

\[ \frac{1}{\kappa} \frac{\partial \psi}{\partial t} + \frac{p_0}{n} \frac{\partial \psi}{\partial \tau} + \rho_b \frac{\partial \psi}{\partial \tau} + \frac{M \alpha u_{ji} + M \alpha w_{j,ji}}{\frac{\partial \theta}{\partial t} + \theta} - \frac{D_1 \theta}{\partial t} - Ma' \left( \theta + \tau \frac{\partial \theta}{\partial \tau} \right) \]  
(11)

Substituting Eq. (4) into Eq. (6), we obtain:

\[ \sigma_{ij} = 2 \mu \dot{\epsilon}_{ij} + \left( \lambda + \alpha^2 M \right) \dot{\epsilon}_{ik} \delta_{ij} - \left( \gamma + \alpha \alpha M \right) \left( \theta + \tau \frac{\partial \theta}{\partial \tau} \right) \delta_{ij} - \alpha M \delta_{ik} \dot{\delta}_{ij} \]  
(12)

Combining Eqs (5), (8), (9) and (12), we have

\[ \rho u_{ii} + \rho_b \psi_{ii} = \left( \lambda + \mu + \alpha^2 M \right) u_{ij,ij} + \mu u_{ij,ij} + \alpha M w_{j,ij} - \left( \gamma + \alpha \alpha M \right) \left( \theta + \tau \frac{\partial \theta}{\partial \tau} \right) \]  
(13)

If the thermo osmosis effect is neglected \( (D_1 = 0) \), Eq. (2) can be reduced to Darcy model used in Liu (2007). The fluid flux equation of partially coupled thermo-hydro-mechanical model (PTHM) is obtained. If the blood flow is not considered, the above-mentioned equations will be reduced to the generalized thermo-mechanical model (TM).

2.2 Problem description

We consider a cylindrical porous biological tissue model shown in Fig. 1 and the center parts of biological tissue surface is subjected to laser irradiation. The radius of biological tissue is \( R_0 \), the radius of laser beam is \( r_0 \). Due to the geometry of the structure and loading conditions, only the 1/4 domain region is used for calculating in the numerical evaluation. Thus the considered functions will depend on the space variables \( r, z \) and time \( t \).

![Fig. 1 Schematic diagram of the cylindrical biological tissue](image)

The initial conditions are:

\[ \theta = u_r = u_z = w_r = w_z = \dot{\theta} = \dot{u}_r = \dot{u}_z = \dot{w}_r = \dot{w}_z = 0 \]  
(14)

The boundary conditions can be expressed as:

\begin{align*}
& \text{r} = 0: \ u_z = w_z = 0, \quad \theta = 0; \quad \text{r} = R_0: \ u_z = u_y = w_z = w_y = 0, \quad q = 0; \\
& \text{z} = 0: \quad q = 0; \quad \text{z} = L: \ u_z = u_y = w_z = w_y = 0, \quad q = 0
\end{align*}
(15)

which indicates the biological surface is traction free and subjected to laser irradiation. Comparing to the high-intensity laser, the surface convection is assumed negligible in this study. The external heat source function is:

\[ Q_{ext} = \frac{1 - R}{z_0} e^{-z_0 z} H \left( r, r - r_0 \right) Q_0 \]  
(16)
where $R$ is reflection coefficient; $z_0$ is the depth of the laser irradiation; $Q_o$ is the laser intensity.

For simplicity, non-dimensional variables are used in the following:

$$ (r^*, z^*, t_0^*) = \frac{1}{t}(r, z, t) ; \quad \left( r^*, t^*, z^* \right) = \frac{v}{t}(t, r, z) \quad \left( u^i, u^2, w_i, w_2 \right) = \frac{1}{T}(u_i, u_2, w_i, w_2) ;$$

$$ \beta^* = \frac{\theta}{T_0} ; \quad \sigma^* = \frac{\sigma}{\lambda + 2\mu} ; \quad p^* = \frac{\alpha \rho p}{\lambda + 2\mu} ; \quad Q_{\text{ext}}^* = \frac{Q_{\text{ext}}}{kT_0} ; \quad v = \sqrt{1/(\lambda + 2\mu) / \rho} $$

where $l$ and $v$ are the characteristic length and velocity, respectively. The asterisk of the non-dimensional variables is dropped in the following for the sake of brevity.

3. **FINITE ELEMENT FORMULATIONS**

Time-domain finite element method is used in this work to solve coupled thermo-hydro-mechanical equations (Tian and Shen, 2005). In order to get the finite element equations conveniently, Eqs. (4), (6) and (7) can be written in matrix form:

$$ \{ p \} = [D_T]\{ \xi \} - [D_T]\{ \xi \} + [a_D]\{ \theta + \tau, \theta \} \quad \text{ (17)} $$

$$ \{ \sigma \} = [C_T]\{ \xi \} - [C_T]\{ \xi \} + [a_D]\{ \theta + \tau, \theta \} - [D_0]\{ \xi \} \quad \text{ (18)} $$

$$ \rho_S = \{ \gamma \}^T \{ \xi \} + \{ \gamma \}^T \{ \xi \} + c(\theta + \tau, \theta) \quad \text{ (19)} $$

The basic constitutive variables are temperature and displacements for tissue and blood. According to finite element method, the geometrical domain can be subdivided into a finite number of regions or elements. In each element, the displacements and temperature can be expressed as three sets of shape functions:

$$ \{ u^0 \} = [N_3]\{ u^e \} , \quad \{ w^0 \} = [N_2]\{ w^e \} , \quad \{ \theta \} = [N_3]\{ \theta^0 \} \quad \text{ (20)} $$

The strain and temperature gradient may be expressed as:

$$ \{ \varepsilon \} = [B_1]\{ u^0 \} , \quad \{ \xi \} = [B_2]\{ w^0 \} , \quad \{ \theta \} = [B_3]\{ \theta^0 \} \quad \text{ (21)} $$

where $[N_1], [N_2]$ and $[N_3]$ are the shape function, $[B_1], [B_2]$ and $[B_3]$ are derived from $[N_1], [N_2]$ and $[N_3]$, $\theta^0 = \theta$. If the body force is neglected, the principle of virtual displacement for the generalized thermo-hydro-elasticity yields,

$$ \int_V \left[ \delta \{ \varepsilon \}^T \{ \sigma \} + \delta \{ \theta \}^T \{ q \} + \delta \{ \xi \}^T \{ p \} - \delta \{ \theta \} \rho T_0 \{ \dot{S} \} \right] dV = -\int_V \delta \{ u \}^T \rho \{ \dot{u} \} dV - \int_V \delta \{ \dot{w} \}^T \rho_b \{ \dot{w} \} dV + \int_{A_v} \delta \{ u \}^T \{ \dot{T}_1 \} dA + \int_{A_v} \delta \{ \dot{w} \}^T \{ \dot{T}_2 \} dA + \int_{A_v} \delta \theta \dot{q} dA \quad \text{ (22)} $$

where $\{ \dot{T}_1 \}, \{ \dot{T}_2 \}$ and $\ddot{q}$ represent the traction and heat flux acted on surface $A_v$, $A_p$ and $A_q$ respectively.

Noting that the coupled blood convection item in Eq. (1) is neglected to obtain the finite element governing equations. Then substituting Eqs. (17)-(21) into Eq. (22), we obtain:

$$ \sum_i \left[ M_{i1}^e \quad 0 \quad 0 \quad \{ \ddot{u}^0 \} \quad 0 \quad 0 \quad C_{i3}^e \quad \{ \ddot{w}^0 \} \quad K_{i1}^e \quad -K_{i2}^e \quad -K_{i3}^e \quad \{ u^e \} \quad T_{i1}^e \right] = \left[ 0 \quad M_{i2}^e \quad 0 \quad \{ \ddot{w}^0 \} \quad 0 \quad 0 \quad C_{i3}^e \quad \{ \ddot{u}^0 \} \quad -K_{i1}^e \quad K_{i2}^e \quad K_{i3}^e \quad \{ w^e \} \quad T_{i2}^e \right] $$

$$ \sum_i \left[ 0 \quad 0 \quad M_{i3}^e \quad \{ \ddot{u}^0 \} \quad 0 \quad 0 \quad C_{i3}^e \quad \{ \ddot{u}^0 \} \quad \theta^0 \quad \{ \theta^0 \} \quad \theta^0 \right] \quad \text{ (23)} $$
where \( n_e \) is the number of elements. The coefficients and load vectors in Eq. (23)
given in the following equation:

\[
M_{11} = \int_v [N_i]^T \rho [N_i] \, dV, \quad M_{22} = \int_v [N_2]^T \rho_b [N_2] \, dV, \quad M_{33} = T_0 c \tau_2 \int_v [N_3]^T [N_3] \, dV,
\]

\[
C_{11} = \int_v \tau_1 [B_1]^T [\gamma + a_n D_n] [N_i] \, dV, \quad C_{23} = \int_v [B_2]^T [a_n D_n] [B_1] \, dV,
\]

\[
C_{31} = T_0 \int_v [\gamma_1]^T [B_1] [N_3] \, dV, \quad C_{32} = T_0 \int_v [\gamma_2]^T [B_2] [N_3] \, dV, \quad C_{33} = T_0 c \int_v [N_3]^T [N_3] \, dV,
\]

\[
K_{11} = \int_v [B_1]^T [C] [B_1] \, dV, \quad K_{12} = \int_v [B_1]^T [D_1] [B_2] \, dV, \quad K_{13} = \int_v [B_1]^T [\gamma + a_n D_n] [N_3] \, dV,
\]

\[
K_{22} = \int_v [B_2]^T [D_2] [B_2] \, dV, \quad K_{23} = \int_v [B_2]^T [D_0] [B_3] \, dV, \quad K_{33} = \int_v [B_3]^T [a_n D_n] [N_3] \, dV,
\]

\[
K_{31} = \int_v [k] [N_3] \, dV, \quad T_m^e = \int_{\partial v} [N_1]^T \{ \vec{T} \} \, dA, \quad T_r^e = \int_{\partial v} [N_2]^T \{ \vec{T} \} \, dA, \quad T_0^e = -\int_{\partial v} [N_3]^T \{ \vec{q} \} \, dA
\]

(24)

Here, \([M], [C]\) and \([K]\) are the mass, damping and stiffness matrices, respectively.

\([T_m^e, T_r^e, T_0^e]^T\) are the load vectors associated with the boundary conditions. Thus far, finite element formulations of generalized thermo-hydro-mechanical problem are given. Then the finite element equations (Eq. (23)) can be solved directly in time domain with the initial and boundary conditions.

4. **NUMERICAL RESULTS AND DISCUSSIONS**

In the calculation, the material parameters of biological tissue are shown in Table 1 (Keangin et al. 2011, Vyas et al. 2016). The non-dimensional length are \( L = R = 5.0 \), \( z_0 = 0.2 \), \( r_0 = 1.0 \), respectively. The non-dimensional time \( t = 0.12 \), \( \tau_1 = \tau_2 = 0.04 \) are used in the following numerical examples.

<table>
<thead>
<tr>
<th>Table 1 the material parameters of biological tissue</th>
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<tbody>
<tr>
<td>( K = 6.87 \times 10^6 \text{pa} )</td>
</tr>
<tr>
<td>( \rho_i = 1000 \text{kg/m}^3 )</td>
</tr>
<tr>
<td>( c_i = 4200 \text{J/kg/K} )</td>
</tr>
<tr>
<td>( k_j = 10^{-4} \text{m/s} )</td>
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</table>

In order to investigate the influence of coupling effect on the porous biological tissue, the results of the THM model and PTHM model are given, the results of TM model is also considered to show the effect of blood fluid on the biological tissue. The distributions of non-dimensional temperature, displacement, stress and pressure are shown in Fig. 2-Fig. 7. From Fig. 2, we can see that the temperature profile for the TM model, PTHM model and THM model are almost coincident. This means thermal osmosis effect and porous biological model has little influence on the temperature. Fig. 3 shows the peak of axial displacement \( u_z \) for PTHM model is greater than TM model and THM model. The distributions of axial displacement and radial displacement along \( r \)-axis are plotted in Fig. 4-Fig. 5, respectively. The axial displacement \( u_z \) along \( r \)-axis is always negative. This indicates that the biological tissue surface suffer thermal expansion deformation and moves to the unconstrained direction. It can be seen from
Fig. 5 that the displacement of the origin along the $r$-axis is zero. This is consistent with the symmetry condition. The absolute value of displacement for THM model is a little greater than PTHM model, the magnitude of displacement for TM model is much less than other two models. The stress $\sigma_{zz}$ on the origin is zero (traction free surface) and increases as moving away from the origin in Fig. 6. The absolute value of stress for PTHM model is greater than TM model and THM model. It also can see that the velocity of elastic wave for TM model is slower than other two model. In Fig. 7, the absolute value of pressure for PTHM model is greater than THM model.

![Fig. 2 The temperature distribution of different models along z-axis](image1)

![Fig. 3 The displacement distribution of different models along z-axis](image2)
Fig. 4 The displacement distribution of different models along \( r \)-axis

Fig. 5 The displacement distribution of different models along \( r \)-axis

Fig. 6 The stress distribution of different models along \( z \)-axis
The influences of porosity and thermal osmosis on the studied fields are shown in Fig. 8-Fig. 13. The porosity and thermal osmosis almost have no effect on temperature in Fig. 8. The magnitudes of displacement, stress and pressure decrease with the increasing of porosity $n$. With increasing thermal osmosis coefficient $D_T$, the peak of radial displacement $u_r$ decrease shown in Fig. 11, while it has a little effect on the other considered fields.
Fig. 9 The influences of porosity and thermal osmosis on the displacement along $z$-axis

Fig. 10 The influences of porosity and thermal osmosis on the displacement along $r$-axis

Fig. 11 The influences of porosity and thermal osmosis on the displacement along $r$-axis
5. CONCLUSIONS

Presence of blood and its thermal roles in living tissue such as blood perfusion and convection, make it is quite natural to treat the living tissue as a porous medium. With this motivation, the thermo-mechanical response of porous biological tissue exposed to an instantaneous thermal shock with considering the thermal osmosis effect is investigated. Time domain finite element method is employed to solve this problem. From the numerical results, one may conclude that: the results of displacement and stress for TM model is much smaller than PTHM model and THM model, which indicates it is necessary to regard the biological tissue as a porous model when thermo-mechanical interaction is investigated; the magnitudes of displacement, stress and pressure decrease with the increasing of porosity \( n \); the peak of radial displacement \( u_r \) decrease with the increasing of thermal osmosis coefficient \( D_T \); the porosity and thermal osmosis have no influence on temperature increment.

Fig. 12 The influences of porosity and thermal osmosis on the stress along \( z \)-axis

Fig. 13 The influences of porosity and thermal osmosis on the pressure along \( z \)-axis
REFERENCES
