

## **Nonlinear analysis of RC shear walls by vector form intrinsic finite element method**

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### **ABSTRACT**

It is always hard to reach accurate solutions for the numerical simulations of nonlinear and large deflection by traditional finite element approaches especially when analyzing reinforced concrete shear walls. Vector form intrinsic finite element (VFIFE) method based on vector mechanics is a novel method to solve nonlinear and discontinuous mechanical behavior of structures or elements. Basde on VFIFE method, a plate element subprogram is developed to provide more effective processing method for the nonlinear analysis of reinforced concrete shear walls. Two main loops, point value loop and element cycle are included in this plate element program to adapt the simulation work for nonlinear analysis of walls, slabs, shells, membranes, etc. Three typical examples including box, large deflection elastomer and experimental shear walls were selected to test and verify this program. Comparison indicates that the simulation results meet the theory results and the experimental results closely even in large deflection stage. Theoretical studies and test results demonstrate that this subprogram has great potential in dealing with nonlinear problems such as large deformation, large displacement, elastic-plasticity, collision, collapse, et al, so to have great importance in theoretical analysis and engineering applications.

### **1. INTRODUCTION**

Technological swift development in civil engineering requires the effective simulation analysis of linear, nonlinear, and even the collapse of structures. The widely applied DBSD (displacement based seismic design) method is actually based on accurate simulation on structure deformation in the whole process (Thomsen and Wallace, 2004).

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While traditional structural mechanics is based on the generalized continuum mechanics theory, it is difficult in solving the nonlinear and large deformation analysis problems, especially for reinforced concrete shear walls. In the nonlinear analysis of reinforced concrete shear walls, elements are often modeled with macro-element, like spring, lever, fiber model or finite element solid model (Zhang, Lu 2010a). Constitutive models like orthotropic model, nonlinear elastic model, plasticity model, endochronic plastic models, fracture mechanic models and micro-models are also put forward to solve the nonlinear finite element analysis of reinforced shear walls (ASCE 1982; ASCE 1994). Few of the models, however, have been used to simulate the nonlinear load-displacement behavior of different types of structural element under various stress combinations to permit an assessment of their general applicability (Ayoub, Filippou 1998). The basement of continuum mechanics theory and the complication of force-deformation mechanism of shear walls make the simulation work confronted great challenge in nonlinear stage, especially after the shear wall yield (Zhang, Lu 2010b).

A new method is now developed to solve the strong nonlinear behavior numerical simulation problem of reinforced concrete. This method is based on vector theory and nonlinear finite element theory. Vector theory is first proposed by Professor Ting (Ting, Shih and Wang 2004) and developed by Duan and Wu (Ting, Duan and Wu 2012). Current research indicates that this theory overcomes the difficulties of traditional continuum mechanics and has its unique advantages in solving nonlinear or discontinuous problems such as large deformation, large deflection, elastic-plasticity, collision, collapse, etc.

Vector Form Intrinsic Finite Element (VFIFE) is this kind of method combining vector mechanics and finite element theory. And it presents a simple and systematic solving process for structure analysis. In the past 10 years, programs based on VFIFE theory have been developed by some researchers, attempting to find more accurate solutions on the nonlinear behavior of all kinds of structures or elements (Wang and Wang 2006, Wu, Wang, et al. 2007, Ting, Duan and Wu 2012). Effective solution and adaption make VFIFE program family become an effective supplement of traditional finite element approach.

This paper mainly aims at the nonlinear behavior simulation of reinforced concrete shear walls. Based on the main program (Ting, Duan and Wu 2012), a two-dimensional panel element subprogram of VFIFE is developed attempting to solve the complicated nonlinear problems of reinforced concrete shear walls. Three examples including typical examples and experiments were selected to test the stability and efficiency of this subroutine. Test result demonstrates that the simulated results of the two-dimensional panel element subprogram of VFIFE have good agreement to the accurate results or experiment results.

## **2. A brief introduction of VFIFE**

The interaction between particles is described by applying the concepts of particles' system in physics and traditional finite element theory in VFIFE theory (Erik 2005). And the motion equations are set up according to the Newton's second law. For this reason, this theory has natural advantages in solving nonlinear or discontinuous mechanics

behavior such as large deformation, large deflection, elastic-plastic deformation, collision, collapse etc.

In vector theory, structures are supposed to be constructed by series of particles and connections, and the interaction between the particles and connections are assumed as generalized behavior. In detail, relative assumption is list below:

- Element can be changed both in geometry and position at the same time and the deformation and motion are not separated.
- Different forms of structures adopt the same framework.
- The connection between elements may include many points, and it can be a rigid body or a flexible body.
- Load and motion path have no restriction in path independence and stationary state.

Based on the above assumptions, three new key concepts were proposed on the vector mechanics, that is, point value description, path element and virtually backward motion (Ting, Duan and Wu 2012).

### 2.1 Point value description

The so-called point value description is that structures are seen as particles in space, which connected with each other with elements. Connection elements between the connect point of components of structures can be constructed. No mass assign to the element, that is different from traditional finite element method. The particles can move under external force and the motion of particles is then strained by the elements around them. The motion of particles under both internal and external forces satisfies Newton's second law, which is

$$\mathbf{M} \frac{d^2\mathbf{x}}{dt^2} = \mathbf{P}_{\text{ext}} + \mathbf{P}_{\text{int}} \quad (1)$$

where  $\mathbf{M}$  represents mass matrix,  $\mathbf{P}_{\text{ext}}$  represents external force vector and  $\mathbf{P}_{\text{int}}$  is internal force vector (Ting, Wu, Wang et al 2006).

### 2.2 Path element

In the theory of VFIFE, the essential of the motion is a series of time segments. Similarly, in vector mechanics, the whole process of motion is divided into a series of short-enough time segments, called path element. In a path element, elements can be assigned continuous deformation and rigid motion at the same time. By controlling the length of path elements, the structural deformation should be small enough to satisfy the little deflection assumption of material mechanics (Yan and Wu 2013). The motion of particles is continuous within each path element, while the amounts and properties of elements can only change at the join points of path elements to simulate fracture or collapse.

### 2.3 Virtual backward motion

For calculating the internal forces, a virtual backward motion to eliminate rigid displacement and pure deformation of the element is supposed in the vector mechanics. As shown in Fig.1, firstly, in order to eliminate rigid translation displacement, element at  $t_b$  is translated to element at  $t_a$ , and this procedure should confirm their geometry

centers overlap together. Then the triangular element is rotated with an angle of  $-\nabla\theta$  to eliminate rigid rotation displacement in contrast to element at  $t_a$ , which called base frame. Thus, comes out the pure deformation of element from  $t_a$  to  $t_b$ . The equation form is

$$\Delta\eta_i^d = (\Delta\mathbf{u}_i - \Delta\mathbf{u}_A) - \Delta\eta_i^r \quad (2)$$

Where  $\Delta\mathbf{u}_i$  and  $\Delta\mathbf{u}_A$  are the displacements of nodes and reference point from time  $t_a$  to time  $t_b$ , respectively.

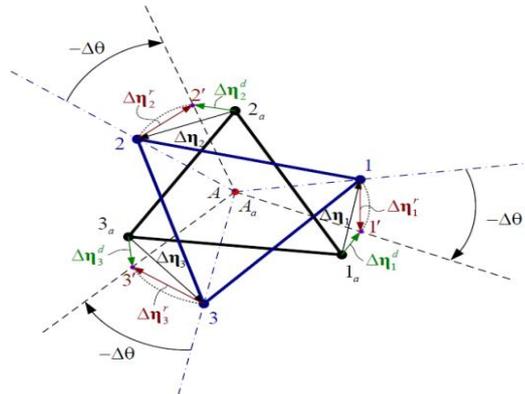


Fig.1 Backward rotation of element

#### 2.4 Motion equation of particles

Vector mechanics adopt the explicit integration method to solve the motion equation of particles. Applying the central differential method, equation (1) can be transformed to the following explicit form

$$\mathbf{X}_{n+1} = h^2 \mathbf{M}^{-1} (\mathbf{P}_{\text{ext}} + \mathbf{P}_{\text{int}}) + 2\mathbf{X}_n - \mathbf{X}_{n-1} \quad (3)$$

Take the damp into consideration, equation (1) can be written as

$$\mathbf{M} \frac{d^2\mathbf{X}}{dt^2} = \mathbf{P}_{\text{ext}} + \mathbf{P}_{\text{int}} + \mathbf{P}_{\text{dmp}} \quad (4)$$

The explicit form is

$$\mathbf{X}_{n+1} = C_1 h^2 \mathbf{M}^{-1} (\mathbf{P}_{\text{ext}} + \mathbf{P}_{\text{int}}) + 2C_1 \mathbf{X}_n - C_2 \mathbf{X}_{n-1} \quad (5)$$

Where

$$C_1 = 1/(1 + 0.5\zeta h) \quad C_2 = C_1(1 - 0.5\zeta h) \quad (6)$$

$\zeta$  is the damping factor.

The calculating process of vector mechanic consists two parts: one is point value calculation, that is, for each point, the current moment position obtains from the earlier moment by differential equations; the other is calculating internal force of the elements with the elemental pure deformation obtained from virtually backward motion. This internal force is returned to the first part to calculate the next moment position of particles. This process will be repeated as a circulation, and point state at each moment can be calculated in this cycle. By this way, the state of the whole structure can be

evaluated. The motion of particles is continuous within each path element, while the amounts and properties of elements can only be changed at the join point of path elements to simulate fracture or collapse.

When vector mechanic is applied to bars system, it can be called vector mechanics of structures, while in shell, membrane and solid problems that is vector mechanic of solids. Vector form intrinsic finite element is the combination of vector mechanic with finite element method. One of the most distinctive features of VFIFE is that, for different structural forms including truss, frame, solid and shell both in plane and space and complicated mechanical behavior including large deformation, space motion, and transformation of material properties and so on, similar concepts and procedures can be applied.

### **3. Program design of vector form finite plate element**

#### *3.1 Introduction of program design of plate element*

Same with traditional finite element method, several types of panel element, such as triangular isoperimetric panel element, Quadrilateral isoperimetric panel element and higher order isoperimetric element are developed based on IFIFE (Wu and Ting 2008). Here, two-dimension panel element is developed to solve plane element problems.

The program of panel element in this article is written in FORTRAN and it includes a main program (250 lines), three internal force subprograms (470 lines) and an external force subprogram (60 lines), 780 lines in total. This program contains two major parts: preprocessor and solver. The preprocessor part includes importation of parametric of module information and preparation of necessary parameters such as time interval for integration and position of particles at time  $t_{-1}$ , which can get from initial conditions. The solver part includes solving and exportation of the results. The solving part includes three parts: point value calculation, elemental internal force calculation and importation of external loading. Point value calculation is to calculate the same position of particles at next moment with the new particle position, and the internal and external force at previous moment. The internal force and external force can be obtained from the internal force subprogram (element circulation) and external force subprogram (importation of external loading), respectively. Element internal force is applied to calculate element pure deformation based on nodal position and then calculate element stress, strain and nodal internal force according to finite element theory and the principle of virtual work. Then the element nodal internal force is assembled to relative particles and the results are returned to nodal circulation for calculating nodal position at the next moment. The importation of external loading, whether it is time variable or not, can be arranged by the external force subprogram also.

Obviously, the calculation process of motion equations of VFIFE is simple. The key point of designing process of program is to calculate the element internal force. Obtaining the element pure deformation through virtually backward motion and then calculating element internal force are the main process of the program. To simplify calculation in this process, we need to choose local coordination system (also called deformation coordinate system) to decrease the freedom degree of element. The nodal internal force components can be obtained with eliminated degree of freedom by solving the equilibrium equations of forces and moments. During the process of

calculating element internal force, element internal force and coordinate system rotate twice totally. One is the virtual rigid rotation and the other is the local coordinate system. Hence, the element nodal internal force in local coordinate system should be transformed to the global coordinate system.

Another key point of program design is the selection of time step. Explicit integration method is adopted by VFIFE to avoid solving stiffness matrix and the corresponding inverse matrix, so it requires a relatively small time step (Ray and Joseph 2003). Explicit integration calculates the internal force always behind the deformation and if the time step is too large, structures may have bizarre deformation or motion due to excessive force before the appearance of reaction of internal force. Thus, the time step value is restrained. Furthermore, the fragmentation and collision problems also need smaller time step. In fact, smaller time step only needs at the moment when there is a sudden change in structure state. Consequently, applying variable time step size can greatly reduce the calculation time without affecting the calculation accuracy.

There are two points in program designing process need to pay attention: one is the selection of local coordinate system. A proper coordinate system can not only reduce element degree of freedom but also avoid strange structural state disabling the calculation process. A good local coordinate system will dramatically increase the stability of the calculation process. The other is the material constitutive relation. Since the internal force is accumulated by increment form, tangent modulus should be adopted about the material elastic modulus.

The VFIFE theory considers the structural behavior in the concept of dynamic mechanics. In order to decrease elapsed time, we can increase the particle mass proportionally or increase damping value when solving static module and pseudo-static module. This method can dramatically decrease time without affecting accuracy of results. The calculation process of VFIFE mechanics is more agree with the actual behavior of a structure and the process of solving structural stiffness matrix in calculating dynamic module can be avoid.

### *3.2 Calculation flow chart of panel element program*

The panel element subroutine introduced in this article is developed based on the VFIFE theory and is a part of previous VFIFE main program. The calculation flow chart is illustrated in Fig.2. Generally speaking, the whole process is divided into three parts: preprocessor, solver and postprocessor. Importation of material information, nodes and elements et al. belongs to preprocessor. The point circulation, from start to finish, belongs to solver. Then go into postprocessor procedure after the solution completed.

Before solving, a finite element model for the structure is constructed. For simple structures, a finite element model can be manually set up directly in the program or developing a program with MATLAB to construct the structures. More powerful modeling software for complex structures is needed. And then the model information (nodes information, elements information, material parameters, boundary conditions and loading method) are imported into this program. After that, it needs to calculate the particle positions at  $t = 1$ . From equation (5), we know that, it needs  $\mathbf{x}(0)$  and  $\mathbf{x}(-1)$  when calculating  $\mathbf{x}(1)$ . Thus,  $\mathbf{x}(-1)$  should be calculated firstly with the initial conditions since it is an unknown value. This is all about the work of preprocessor and then information is transferred to the solver procedure.

The solver part includes two circulations. The outer circulation is associated with time step and the inner circulation includes two sub-circulations and an input that is point value circulation, element circulation and external force input. Point value circulation calls the internal force subprogram and the external force subroutine to obtain force information at time  $t_n$ , and then compute nodal positions at time  $t_{n+1}$  with nodal positions at time  $t_n$ . Element circulation can compute element pure deformation through virtually backward motion with nodal positions at time  $t_n$ , and  $t_{n+1}$  calculated by the point value circulation. Then element stress, strain and element nodal internal force is calculated by finite element theory and principle of virtual work. Element nodal internal forces need to be assembled to particles which connect with the element. After that, the results are returned to point value circulation for computing internal force at the next moment. In element circulation, element should be evaluated whether it gets into plastic deformation or fails at each time point. If the answer is yes, then conduct the corresponding actions. The state of element should be updated before the next element circulation. External loading input can be conducted by the external force subprogram, in which time-history external forces can be input.

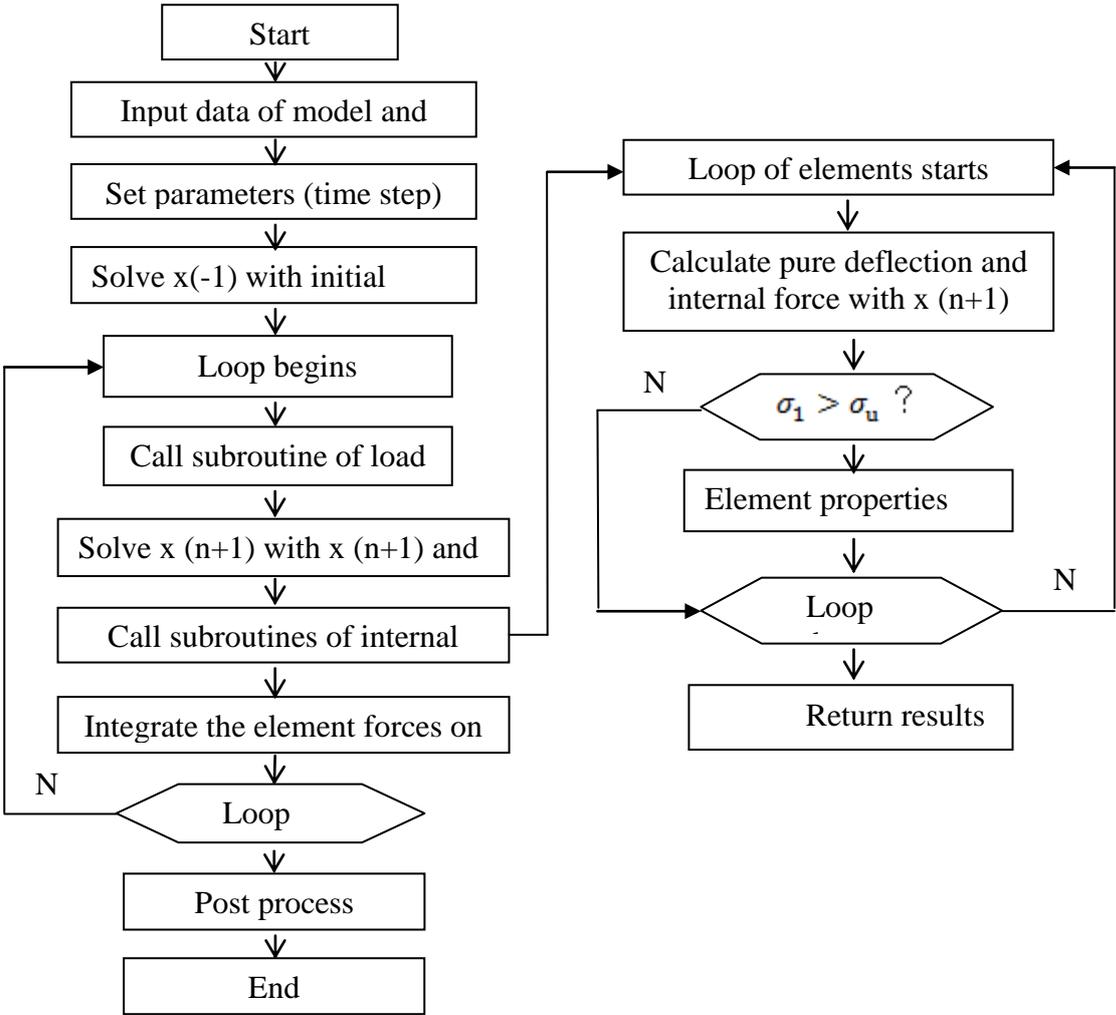


Fig. 2 Calculation flow chart

When calculating the element internal force, local coordinate system can be chosen like this: select one node of the element as origin, the displacement of another node as x-axis and the corresponding y-axis can be determined. Thus the elemental degree of freedom can be decreased to three and we can calculate the corresponding eliminated nodal force components by solving force and moment equilibrium equations. The process of solving element internal force is to calculating element nodal displacement first and conducting virtual backward rigid translation and rotation later. The rigid translation displacement can be determined by either node of the element and rigid rotation degree is an average value of the corner of vertices which can be determined by connecting vertices and geometrical center of the element. There is only small deformation without rigid displacement between element after virtual backward motion and the base frame and this is definitely elemental pure deformation. Translate current coordinate system to local coordinate system, compute element internal force increment by finite element theory and principle of virtual work, transfer internal force increment to global coordinate system and add total internal force at last moment, then obtain the total internal force at current moment. Because there is a virtual backward motion of the element, it needs a forward motion to the actual position and at the same time, element internal force also changes due to the only influence of rigid rotation since the rigid transfer has no effect on both values and directions of the element. The sign of forces should be distinguished because forces of particles and element nodal internal forces are interaction forces.

The determination of the variable elemental properties is tough as well as similar program. This process involves the selection of element failure criteria and the disposal of element failure. Theory of yield surface has its advantages for solid panel element. Accurately, there are different types of internal force subprogram for different types of elements. In this subprogram, elements are connected with each other by particles, the time step should be small enough thus to be timely adjusted when necessary. Furthermore, this subprogram is suit for choosing a ramp load mode for static load. When adding dynamic load, it can be the real time history process, but the initial value should be zero. For nonzero circumstance, it is necessary to set up a slope to gradually loading to the real value.

#### **4. Example verification**

To test the validity of this program, three models including box structure, elastomer and tested reinforced concrete shear walls were simulated by the newly developed subprogram in this paper. The first two are virtual models whose model information and results can get in reference (Ding C.X., Duan Y.F. and Wu D.Y. unpublished) and the third one is a test model of reinforced concrete shear wall in low-reversed cycle load.

##### *4.1 Box structure*

As shown in Fig.3, this box is a virtual perfect model with the size of 20m×20m. And the thickness  $t=1$  m, depth  $d=12$  m, Young modulus  $E=10.0$  N/m<sup>2</sup> and mass density  $\rho=1.0$ kg/m<sup>3</sup>. The force is load at the middle line on the top and bottom of box, which is  $2P$  and the load process is slow. To simplify, since it is symmetrical structure, a quarter

of box structure with boundary condition of perfect sliding constraints on both left end and bottom end of the chosen part. This model included 123 particles and 160 triangle plate elements.

In order to obtain accurate results, the force should be applied slowly with multistep for eliminating the vibration of box, as shown in Fig.4.

The force-displacement curves that the program gives out are shown in Fig.7 and Fig.8. Fig.7 shows the load and the displacement response of B point of the box. The calculated curve is close to the theory line with trend of convergence in the end of the curve. Fig.8 shows the deflection of structure under different force with  $P1 = PL^2/EI$  equaling to 0, 2 and 4, which are in accordance with the theory results. Comparison of the calculated results with the theoretical result indicates efficiency of this calculation.

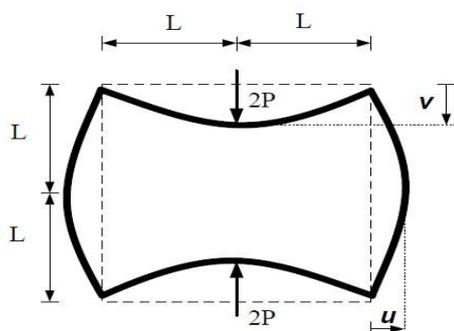


Fig.3 Box structure

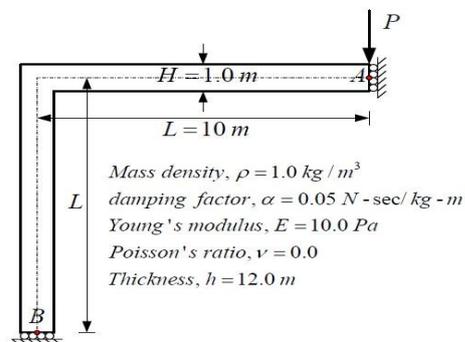


Fig.4 Analysis model

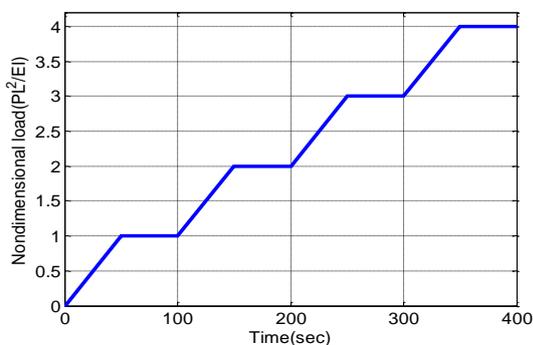


Fig.5 Load history

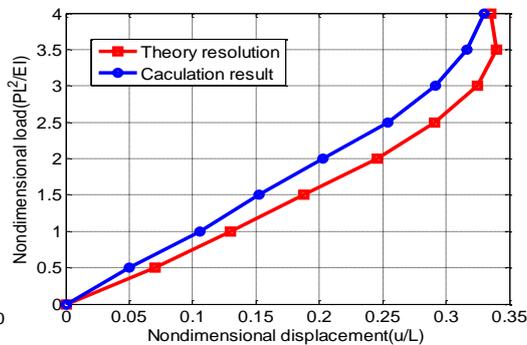


Fig.6 Force-displacement curve at B point

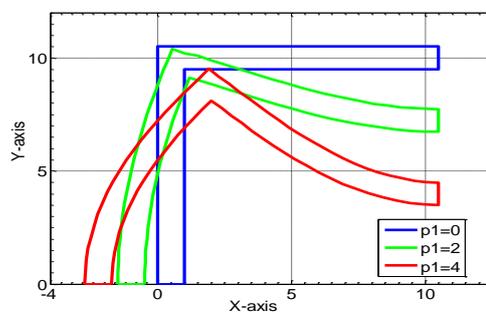


Fig.7 Deflection of structure under different load ( $P1 = PL^2/EI$ )

#### 4.2 Elastic body

Fig.8 (a) shows a rectangle elastomer linked with steel plate at the each end of the elastomer. This elastomer is then squeezed by two plates with same constant speed, 0.09m/s. In addition, the dimensions and material properties of the elastomer are specified in Fig.8 (a). To calculate by the subprogram, the elastic body is divided into 20 parts in each side, and the particle located at each cross section, as shown in Fig.8 (b). Thus the total numbers of particles and elements modeled for calculate is 441 and 400. The plates are modeled as rigid surface with no deflection, and the time step is 0.001 second.

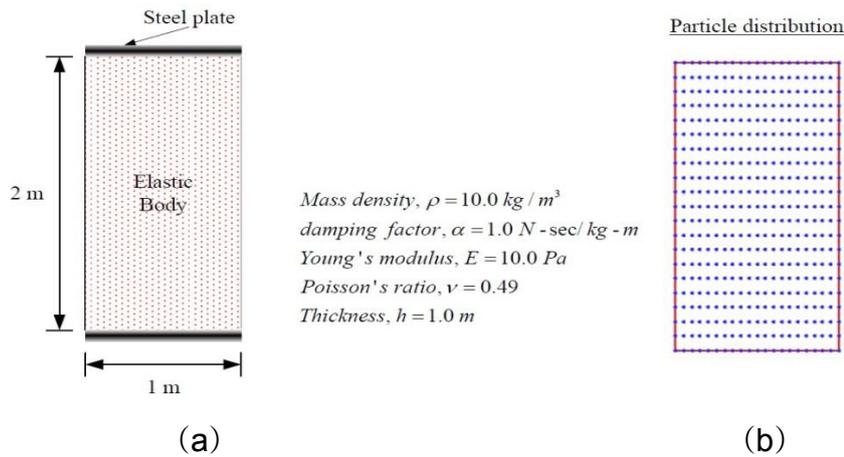


Fig.8 A rectangle elastic body

Fig.9 shows the shape of elastic body in different time as the subprogram gives out. The X-axis is length of the elastomer and the Y-axis is the height of the elastomer. Fig.9 (a), (b), (c) and (d) are the deformation appearance at the time 1s, 4s, 6s and 8s respectively. From the calculated results, the elastic body near the rigid plate is squeezed most seriously and is extruded to outside at the 1st second and then the elastic body is further compressed and the middle of the elastic body protrude out at the 4th second. Next, the elastomer is compressed like balloon, the particles extrude outside dramatically. The situation is same as balloon under compression.

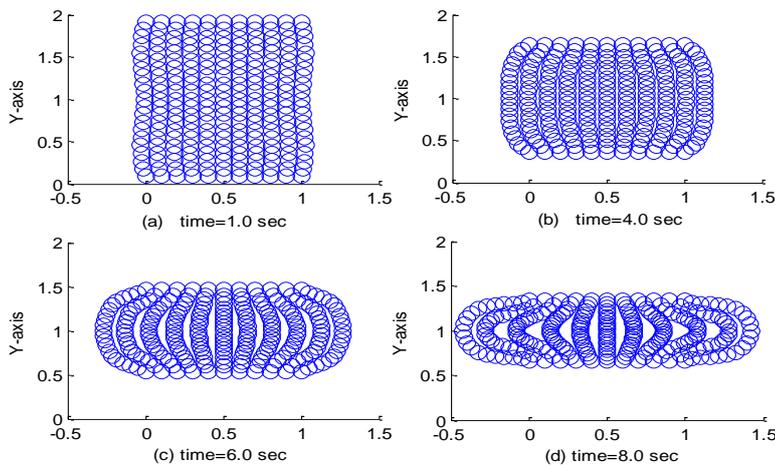


Fig.9 Deformation of elastic body

The results are the same with the results from Ref. (Ting, Duan and Wu 2012). It suggests that the VFIFE plane element can solve the large deflection problems of two dimensions entities perfectly. Compared with similar calculation method, VFIFE can avoid the limitations of traditional methods in dealing with large deflection problems with high computational efficiency and accuracy.

#### 4.3 RC shear walls

To further test the efficiency of program, the low-reversed cyclical experiment on RC shear walls is also simulated and the results was compared with the experimental curves. To simplify the calculation, this paper employ the pushover analysis method to simulate the experimental situation as some studies indicated that the force-displacement skeleton curves of low-reversed cyclical experiment of RC shear walls are almost the same with that of monotonic pushover (Dong 2007).

##### 4.3.1 Experiment parameters of RC shear walls

This experiment is a part of variable parameter low-reversed cyclic experiment of RC shear walls conducted in state key laboratory of disaster in civil engineering at Tongji University. All the experiment data were disclosed in the Pacific Earthquake Engineering Research Center.

The tested specimen SW6-3, SW4-4 and SW1-1 is with the same size. Top beam is located on the top of the shear wall to serves as a loading beam and bottom beam on the bottom is used to fix the shear wall. The steel bars in shear walls stretch into both the top and bottom beams and beams. Top beam, bottom beam and core wall were casted at the same time, which can guarantee the integrity of the wall. The concrete of SW6-3 and SW4-4 is designed as C40, and SW1-1 is C30. Embedded columns at left and right side were constructed in the specimens, and distribution steel in both horizontal and vertical directions were constructed in the inner part of the shear wall. All the steel bars were designed according to the current Chinese code. The dimensions and reinforcement are shown in Fig.10. Table 1 lists the detailed experiment parameters and Table 2 illustrates the parameters of steel bars. The prism compressive strength of C40 concrete is 30.7MPa, which obtained from the compressive strength test of 100mm×100mm×300mm prism and the characteristic value of axial pressed concrete uses 25.7Mpa ( $30.7 \times 0.95 \times 0.88 = 25.7$ ). The compressive strength experiment specimen of C30 adopts 150 mm×150 mm ×150 mm cube and the cube compressive strength is 19.7MPa, the characteristic value of axial pressed concrete uses 13.17MPa ( $19.7 \times 0.76 \times 0.88 = 13.17$ ).

Table 1 Test parameters of specimens

Number of specimen	Height of cross-section (mm)	Aspect ratio	Strength of concrete	Length of restrained area(mm)	Axial load ratio	Boundary longitudinal bar	Boundary stirrup
SW6-3	2000×1000×125	2	C40	200	0.6	6B10	A6@60
SW4-4	2000×1000×125	2	C40	200	0.2	6B12	A6@80
SW1-1	2000×1000×125	2	C30	200	0.1	6B10	A6@80

Table 2 Material parameters of steel

Steel type	A4	A6	B10	B12
Actual diameter(mm)	3.91	6.54	9.74	12.55
Yield strength(MPa)	348	392	379	325
Ultimate strength(MPa)	409	479	554	195
Elasticity modulus (kN/mm <sup>2</sup> )	198.8	200.6	181.2	169.0

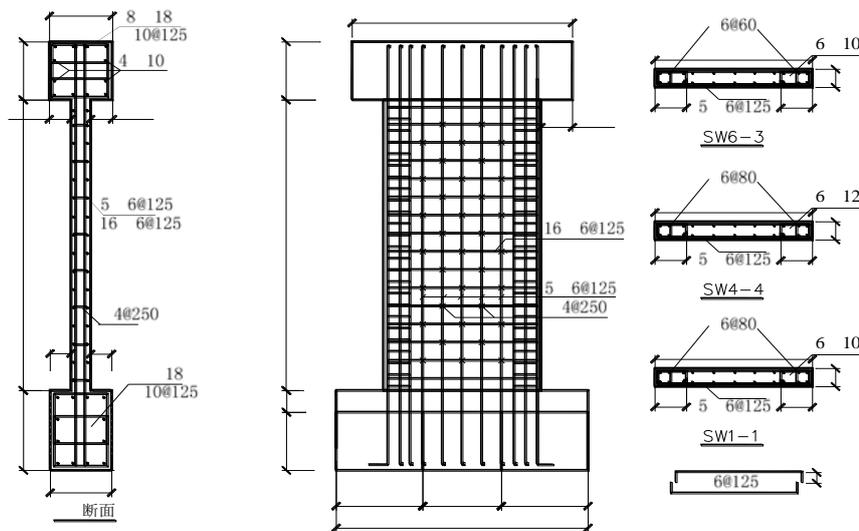


Fig.10 Detail drawing of shear wall reinforcement

#### 4.3.2 Modeling for walls

The up and bottom condition of the shear walls is applied as rigid constraint conditions since there are two rigid beams on the wall. The wall model is divided into two parts: boundary regions (embedded columns) and middle part. To take into account the effect of stirrups in the columns, the concrete of embedded columns are considered as confined concrete. The constrained relationship refers to CEB FIP MC90 model. The inner part of the wall is considered as common concrete. The stress-strain constitutive relationship of normal and constraint concrete adopts nonlinear elastic Darwin-Pecknold increment mode (Zhang, Lu and Lu 2007). In fact, as shown in Fig.11, two lines model is applied to simplify the relationship which can be further optimized and the related parameters are illustrated in Table 3. The constitutive relationship of steel bars adopts elastic-perfectly plastic and the related parameters are same with experiment data.

Table 3 Parameters of concrete

Type of specimen	Type of concrete	Elasticity modulus(kN/mm <sup>2</sup> )	Yield strength(MPa)
SW6-3	Confined concrete	11.35	32.00
	Normal concrete	14.25	27.26

SW4-4	Confined concrete	11.97	30.40
	Normal concrete	14.36	27.86
SW1-1	Confined concrete	16.60	5.27
	Normal concrete	14.66	7.35

When modeling the walls, constraint concrete employed the concept of multi-layer shell element (Miao, Lu and Ye 2006, 2008 and Bazant 2000) and normal concrete adopts smeared reinforced concrete model (Zhang, Lu and Huang. 1990). The finite element model contains 231 particles, 120 quadrilateral plate elements for normal concrete, 80 plate quadrilateral elements for constraint concrete and 204 link elements for steel, as shown in Fig.12, in which the X-axis is the length of the walls and Y-axis is the height and the red areas are confined concrete.

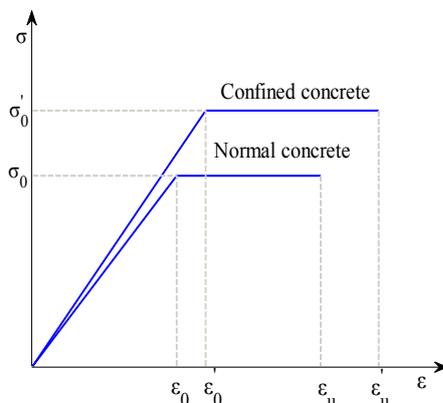


Fig. 11 Stress-strain relation of concrete

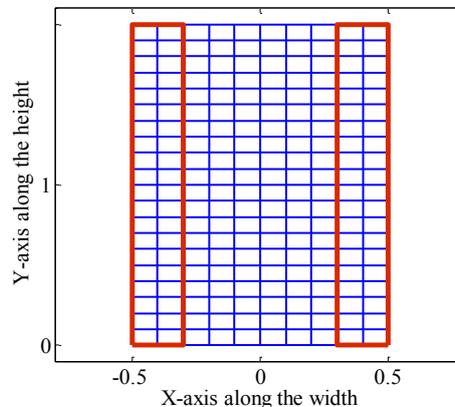


Fig.12 Finite element mesh

#### 4.3.3 Loading process

The loading process is consisted by two stages: applying vertical load and applying horizontal load. At the first stage, the vertical force was loaded to the specified value in 10 seconds according to time step and the horizontal displacement of the top of wall is limited in this phase. Keeping the axis load constant and apply the horizontal load by displacement at the second stage, and the top lateral displacement reaches 20 mm in 50s.

#### 4.3.4 Calculation Results

The force-lateral displacement curves of the simulation and the experiment are both shown in Fig.13. All the calculated curves are very close to the experiment one; except that the peak value of calculation curve of SW6-3, that is a little lower than experiment. In addition, the lateral bearing capacity of calculated one is a little lower than the experiment at the beginning of the curves and the declining stage of calculated curves are not so obvious. In Fig. 13 the yield point is defined as the outside bars begin to yield and the namely ultimate point is that the strain of concrete on the edge reaches ultimate strain 0.0033. Both the experimental and simulated yield and ultimate point of each specimen are drawn in Fig.15. From this figure, the calculated yield points and ultimate points of SW6-3 and SW1-1 are basically close to the experimental ones. The maximum strain and displacement angle at yield point and ultimate point are also

shown in Fig.13 that is also close to the experimental results. There are no yield point and ultimate point drawn in picture (c) as related experiment data of SW4-4 have not collected effectively.

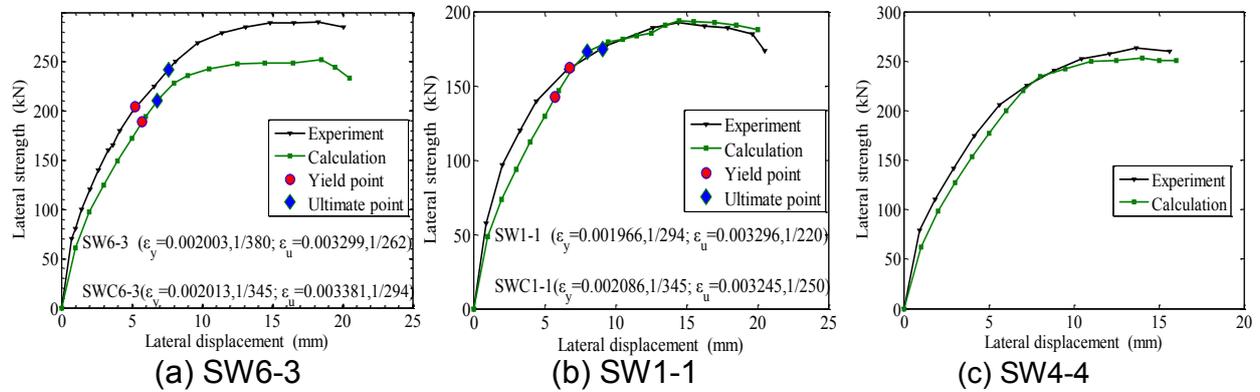


Fig. 13 Lateral strength-top displacement curve of the specimens

#### 4.3.5 Results analysis and discussion

The calculated results are in accordance well with experiment data generally. During modeling the construction, the secant modulus was selected as the concrete elastic modulus in the calculation, which may led to the initial strength of walls lower than actual one. However, the ideal elastic- plastic stress-strain relationship is chosen in the calculation, which is obviously stronger than the real curves that will increase strength of the wall at the latter stage. The analysis indicates that the model and calculation is effective, and more accurate constitutive relation model for concrete is needed to obtain more exact results. Furthermore, the connection slip of steel bar and steel bucking after crack of concrete need to sufficiently take into account.

Lateral strength-displacement curves and yield point and ultimate point shown in Fig. 15 demonstrate the newly developed subprogram based on VFIFE gives out very close result to the experimental result, which is always difficult to solve especially after the force-displacement curve reaches its peak points. While this problem is solved by this program in a clear and convenient way under the theory of VIFIE. Still, this test is a trial on nonlinear analysis on shear walls, some of the modeling and detail output can be further optimized.

## 5. ANALYSIS AND DISCUSSION

This paper developed the panel element as a part of existing main program on the basis of VFIFE. The design progress of this program is clear and the program is concise since the motion equation of structure only needs to follow the basic mechanics principle of Newton's second law. The panel element subroutine has only 780 lines in total which is less than similar programs. In the same way, this program can output the deflection, nodes force and stress-strain of elements to meet the demand of research and design.

Three kinds of models have been tested in the paper, two of which are provided by the reference and the other are selected form experiment. The calculated results are compared with actual results. Compared to traditional finite element method, this

program has unique advantage in calculating large deformation and nonlinear problems. However, the development of the program is still in preliminary stage and also has many problems need to solve, including material constitutive relationship, definition of failure criteria and interrelation between reinforcement and concrete, behavior rules after concrete crack or crushed and enhance or buckling of reinforcement. These problems are things that must be clearly defined for simulating plate structures accurately. However, balance rule of forces is mainly adopted in VFIFE, which makes this program has widely adaption, regardless of elastic stage or strong nonlinear stage. The strong nonlinear and collapse analysis is feasible in this model.

To be exactly, this program is still in its early stage at present and the interaction of pretreatment and post-treatment still needs further development, including design the interface with other procedures. But theoretically, the analysis of the internal force, deformation, stresses and strain of structures can be refined by a reprocessing program to make the results more intuitive. In addition, the program can also be combined with national codes to calculate deformation for different requirements, such as the limit value of elastic displacement angles of shear wall structures is 1/1000 in Chinese code and Elastic-plastic limit displacement angle (life safe stage) is 1/120. And the structures design can be easily optimized and evaluated under different displacement angles demands by combining with this kind of programs.

Therefore, this program is considered to have wide adaptability as the advantages in strong nonlinear analysis and the further development and application of similar numerical calculation program will promote the structural design.

## 6. CONCLUSIONS

To reach a more effective solution to the nonlinear behavior simulation of RC shear walls, two-dimension plane subprogram was newly developed based on VFIFE. Triangle or quadrilateral finite plane element was applied in this subroutine. Numerical simulation tests were conducted on large deflection box, large deformation elastic body and laboratory experimental RC shear wall specimens by this program. And above comparing and analyzing can lead to the following conclusions:

- Since the concepts of vector theory is simple and clear, for the forces of nodes only need to satisfy the balance rule, the design of the VFIFE is more convenient, and so this kind of finite element theory has wide adaptability.
- Large deformation example testes demonstrate the simulation results of the newly developed plane subprogram based on VFIFE is closely to the accurate solution with higher computational efficiency compared with traditional finite element theory.
- Nonlinear Pushover simulation on experimental RC shear walls by the two-dimension plane subprogram gives out closely lateral force-top drift results comparing with experimental results even on the plastic stage. That manifests the good adaption of this program to strong nonlinear solutions on RC walls.

This two-dimension subprogram based on VFIFE can be used to solve some large deformation and nonlinear problems especially on RC shear walls, which is advanced to other approaches. However, it still needs further development due to current limitation on vector finite element programs. Researches should be conducted on

optimizing the plane element program, and three-dimensional plate element program is also need to develop to reach effective solution in three-dimension problems. As above investigated, the advantages of VFIFE make this theory has great potential in solving large deflection, strong nonlinear and discontinuous problems. For these reason, the development of VFIFE is considered have good prospect in simulating the whole process of structures including nonlinear stage and collapse problem, and is also help to promote the deformation-based seismic design theory.

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