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The maximum corner displacement magnification of one-storey eccentric systems

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ABSTRACT

This paper gives an insight into the dynamic and seismic behavior of one-storey eccentric systems, with particular attention devoted to provide a physically based formulation of the maximum corner displacement magnification, which involves three contributions (translational response, torsional response and their combination). It is shown that the largest magnifications, which mainly occur for the class of torsionally-flexible systems, are due to the translational contribution which is caused by the shift of the fundamental period of the eccentric system with respect to that of the equivalent not-eccentric system. A simplified method for the estimation of the maximum corner displacement under seismic excitation, based on the physical properties of the eccentric system, is finally proposed.

1. INTRODUCTION

Since the late 1970s, it is known that structures characterized by non coincident centre of mass and centre of stiffness, commonly defined as eccentric (or asymmetric) systems, when subjected to dynamic excitation develop a coupled lateral-torsional response that may considerably increase their local peak response, such as the corner displacements (Kan and Chopra 1977; Hejal and Chopra 1987; Lu and Hall 1992; Naeim and Kelly 1999; Tso 1990; Rutenberg 1992). In order to effectively apply the performance-based design approach to seismic design, there is a growing need for code-oriented methodologies aimed at predicting deformation parameters (Priestley et al. 2007). Thus, the estimation of the displacement demand at different locations, especially for eccentric structures, appears a fundamental issue. Furthermore, the knowledge of the torsional behavior of eccentric systems may also be useful to improve the capabilities of the design approaches based on pushover analyses to predict the seismic behavior of such systems (Perus and Fajfar 2005).

Since the early 1990s, Nagarajaiah et al. (1993), investigating the torsional coupling behavior of base-isolated structures, observed that, for the specific class of torsionally-stiff asymmetric structures, the maximum centre mass displacement can be well approximated by the maximum displacement of the equivalent not-eccentric system. Also the corner displacement magnification exhibited by torsionally-stiff systems can be reasonably estimated. In previous research works (Trombetti 1994; Trombetti and Conte 2005; Silvestri et al. 2008a,b; Trombetti et al. 2008), the authors identified a structural parameter, called "alpha", strictly related to the attitude of one-storey asymmetric systems to develop rotational responses and, based on this parameter, proposed a simplified procedure, called "Alpha-method", for the estimation of the maximum torsional response. The original formulation of the "Alpha-method" is based on the assumption of equal maximum displacement response between the eccentric system and the equivalent not-eccentric system and thus it is able to provide good predictions for the case of torsionally-stiff eccentric systems.

In the more recent years, scientific papers on single-storey eccentric systems have been mainly focusing on the inelastic behavior of lateral-resisting elements and the effects of bi-directional excitation (De Stefano and Pintucchi 2008). In particular, inelastic behavior aroused a great interest. However, despite of extensive research efforts, the complexity of inelastic seismic response and the large number of parameters influencing the behavior of irregular buildings (as already highlighted in thework by Goel and Chopra 1990), as compared to their elastic counterparts, lead to a lack of general and universally accepted conclusions (De Stefano and Pintucchi 2008). In this respect, the work by Perus and Fajfar (2005) is of particular interest since it provides general behavior trends non-linear and corresponding linear eccentric systems. The results are, however, limited to the class of torsionally-stiff systems, even if in a companion paper (Marušic and Fajfar 2005) some remarks are also given for the class of torsionally-flexible systems. Also, works dealing with considerations on design criteria for in plan asymmetric buildings accounting for the ductile behavior of the structural elements can be found in literature (e.g. see Priestley et al. 2007). The design considerations contained in the book by Priestley et al. (2007) are based on the works by Paulay (2001), Castillo (2004) and Beyer (2007), which again are limited to the class of torsionally-stiff systems.

For these reasons, given that a comprehensive understanding is still not available for the class of torsionally-flexible asymmetric systems, in this paper, with the aim of investigating the response of such systems with a physically-based approach, a systematic work is carried out on linear single-storey systems. The physically-based approach is grounded on the system dynamic properties, takes advantage from the free vibration response analysis and searches for closed-form expressions of the displacement response parameters.

Thus, the objective of the present paper is to provide new insight into the linear dynamic response of one-storey eccentric systems, with the specific purpose of extending the "Alpha method" to all classes of eccentric systems.

2. PROBLEM FORMULATION

Let us consider the one-storey eccentric structure (i.e. a system characterized by noncoincident centre of mass, CM, and centre of stiffness, CK) displayed in Fig. 1 (the origin of the reference system is located at CM). The system has a rectangular plan with sides lengths equal to l and b. The two corner sides which are located farther from the stiffness centre are indicated to as F (hereafter also referred to as flexible sides), while the opposite two corner sides are indicated to as S (hereafter also referred to as stiff sides). It is assumed that the diaphragm is infinitely rigid in its own plane, and that the lateral-resisting elements (e.g. columns, shear walls,…) are massless and axially inextensible. The self-torsional stiffness (k_θ) of each lateral-resisting element is also neglected. Under these assumptions, the system has three degrees of freedom, which can be attached at the centre mass: (i) longitudinal centre mass displacement, $u_{y,CM}$; (ii) transversal centre mass displacement, $u_{x,CM}$, (iii) centre mass rotation, $u_{\theta,CM}$, which coincides with the floor rotation, u_θ . It is supposed that the system is subjected to a one-way dynamic excitation (e.g. free vibrations or seismic input) along the longitudinal direction (namely, the y-direction).

From simple trigonometric relationships, with reference to the plan view of the system given in Fig. 1, the longitudinal displacement at corner F (flexible side), $u_{y,F}$, at any generic instant t, is given by:

$$u_{y,F}(t) = u_{y,C_M}(t) - u_\theta(t) \cdot \frac{l}{2} \tag{1}$$

Estimating the corner displacement according to Eq. 1 requires the development of time history analyses. Nevertheless, the practical engineer is often interested in the absolute maximum value, uy,F,max , of the corner displacement response history. If the maximum displacement response and maximum rotational response are known, the maximum absolute corner displacement uy,F,max can be expressed as follows:

$$u_{y,F,\max} = f\left(u_{y,C_M,\max}; u_{\theta,\max}; \frac{l}{2}\right) \tag{2}$$

Equation 2 highlights that the maximum corner displacement depends on the following three contributions:

- translational contribution, as given by the maximum absolute displacement response $u_{y,CM,max}$ of the centre of mass;
- torsional contribution, as given by the product of the maximum absolute rotational response $u_{\theta,max}$ and the lever arm l/2;
- combination between the translational and torsional contributions of above, as indicated by the function f.

It is useful to introduce the following parameters which will be extensively used for all the next developments:

- $\delta = \dfrac{u_{y,CM,\max}}{u_{y,CM,\max,N-E}}$, which indicates the center mass displacement amplification with respect to the equivalent not-eccentric system (N-E);
- $A \cdot \alpha_{u,free} = \rho_m \cdot \dfrac{u_{\theta,\max}}{u_{y,CM,\max}}$, which indicates a rotational parameter (ρ_m is the mass radius of gyration of the system);
- B, which is a parameter of correlation between the maximum displacement and maximum rotational responses;

- $\phi = \dfrac{l}{2\rho_m} = \sqrt{\dfrac{3(l/b)^2}{1+(l/b)^2}}$ which indicates a shape factor of the system.

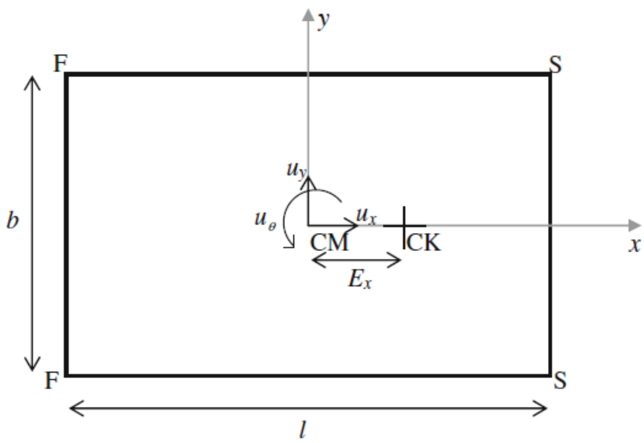


Fig. 1 Plan view of the in-plane eccentric system with the indication of the degrees of freedom

3. THE ECCENTRIC SYSTEM AND ITS DYNAMIC PROPERTIES

Under the following additional assumptions (with respect to those provided in section 2):

- Equal lateral stiffness along the x- and the y-direction (i.e. $k=k_x=k_y$, where k_x and k_y are the translational stiffness along the x- and the y-direction, respectively);
- the rotational response u_θ developed under dynamic excitation is small enough to allow the approximation $u_\theta \cong \sin(u_\theta) \cong \tan(u_\theta)$;
- null longitudinal eccentricity (i.e. $E_y = 0$). It can be demonstrated (Trombetti and Conte 2005) that the one-way eccentric system system exhibits the maximum rotational response in free vibrations with respect the all other eccentric systems (i.e. the same eccentricity E_x but also non null E_y);

the dynamic coupled lateral-torsional response of the system under consideration (Figure 1) is governed by the following set of coupled differential equations of motion (Trombetti and Conte 2005):

$$m\begin{bmatrix} \ddot{u}_x(t) \\ \ddot{u}_y(t) \\ \rho_m \ddot{u}_\theta(t) \end{bmatrix} + \left[C\right]\begin{bmatrix} \dot{u}_x(t) \\ \dot{u}_y(t) \\ \rho_m \dot{u}_\theta(t) \end{bmatrix} + m\omega_L^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & e_x\sqrt{12} \\ 0 & e_x\sqrt{12} & \Omega_\theta^2 + 12e_x^2 \end{bmatrix}\begin{bmatrix} u_x(t) \\ u_y(t) \\ \rho_m u_\theta(t) \end{bmatrix} = \begin{bmatrix} p_x(t) \\ p_y(t) \\ p_\theta(t)/\rho_m \end{bmatrix} \quad (3)$$

In Eq. 3: m is the mass of the system; $e_x=E_x/D_e$ is the relative eccentricity (hereafter it will be simply indicated as e, provided that $e_y=0$); D_e is the equivalent diagonal equal to $12\rho_m$; $\Omega_\theta = \omega_\theta / \omega_L$ is a the frequency ratio (Fajfar et al. 2005) parameter which gives a measure of the system torsional flexibility (ω_L and ω_θ are the uncoupled translational natural frequency of vibration and the uncoupled torsional natural frequency of vibration, defined in a reference system with origin located at CK, respectively); [C] is the damping matrix (classical damping is here assumed).

It is well known (Chopra 1995) that the parameter Ω_θ represents a physical property of the eccentric system, leading to the two following classes of eccentric systems:

- torsionally-stiff systems: $\Omega_\theta \geq 1.0$;
- torsionally-flexible systems: $\Omega_\theta < 1.0$

4. THE EIGENPROBLEM

The solution of the eigenvalues problem governing the undamped free vibrations of the system gives the following closed-form expressions of natural frequencies ω_1, ω_2, ω_3, normalized with respect to the uncoupled longitudinal frequency ω_L and squared (Trombetti and Conte 2005):

$$\begin{gathered}\Omega_1 = \left(\omega_1 / \omega_L\right)^2 = 1/2\left(1+\Omega_\theta{}^2 + 12e^2 - \sqrt{\left(\Omega_\theta{}^2 + 12e^2 - 1\right)^2 + 48e^2}\right) \\ \Omega_2 = \left(\omega_2 / \omega_L\right)^2 = 1 \\ \Omega_3 = \left(\omega_3 / \omega_L\right)^2 = 1/2\left(1+\Omega_\theta{}^2 + 12e^2 + \sqrt{\left(\Omega_\theta{}^2 + 12e^2 - 1\right)^2 + 48e^2}\right)\end{gathered} \tag{4}$$

Figure 2a, which display the normalized natural frequencies versus e and Ω_θ, shows that:

- $\omega_2=\omega_L$. This result derives from the assumption of null longitudinal eccentricity. In this case the dynamic response of the building along the transversal direction (i.e. x-direction) is decoupled from the longitudinal and rotational response;
- ω_1 is generally close to ω_L,
- ω_3 can be quite larger than ω_L.

The solution of the eigenproblem also provides the following expression of the mode shapes (eigenvectors):

$$\{\phi_1\} = \begin{bmatrix} 0 \\ 1 \\ \dfrac{\Omega_1 - 1}{e\sqrt{12}} \end{bmatrix}; \ \{\phi_2\} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}; \ \{\phi_3\} = \begin{bmatrix} 0 \\ 1 \\ \dfrac{\Omega_3 - 1}{e\sqrt{12}} \end{bmatrix} \tag{5}$$

The first and third modes of vibration are coupled modes (i.e. translational component in y-direction coupled with a torsional component), while the second mode is purely translational in x-direction, due to the assumption of null eccentricity in y-direction.

In light of the further developments it is useful provide the expressions of the natural periods of vibration, normalized with respect to the uncoupled lateral period T_L (the period of the equivalent not-eccentric system characterized by the same mass and same lateral stiffness of the eccentric system):

$$\begin{aligned}
\frac{T_1}{T_L} &= \frac{1}{\sqrt{\frac{1}{2}\left(1+\Omega_\theta^{\,2}+12e^2 - \sqrt{\left(\Omega_\theta^{\,2}+12e^2-1\right)^2+48e^2}\right)}} \\
\frac{T_2}{T_L} &= 1 \\
\frac{T_3}{T_L} &= \frac{1}{\sqrt{\frac{1}{2}\left(1+\Omega_\theta^{\,2}+12e^2 + \sqrt{\left(\Omega_\theta^{\,2}+12e^2-1\right)^2+48e^2}\right)}}
\end{aligned} \tag{6}$$

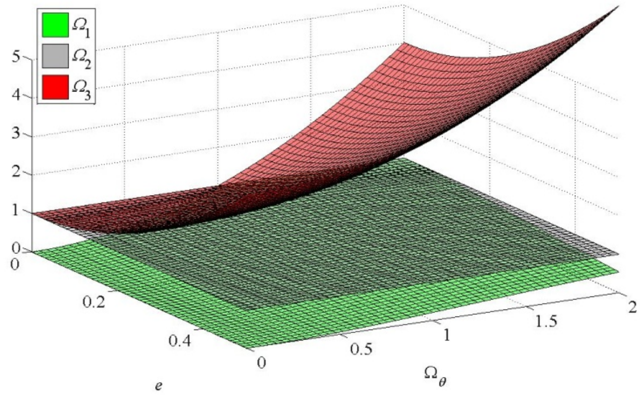
Figure 2b represents the natural periods of vibration, normalized with respect to T_L, versus e and Ω_θ . Inspection of the graph shows that:

- $T_2 = T_L$;
- torsionally-stiff systems are characterized by a fundamental period T_1 close to T_L, and by a third period T_3 quite smaller than T_L;
- torsionally-flexible systems are characterized by a fundamental period T_1 quite larger than T_L (also five times larger) and by a third period T_3 very close to T_L. It will be shown that the significant increase of the fundamental period, which can be defined “fundamental period shifting”, strongly affects the displacement response of torsionally-flexible systems.

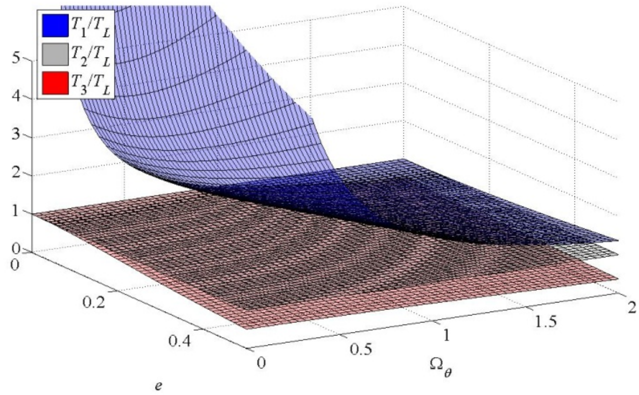
In order to quantify how each mode of vibration contributes to the dynamic response of the system, the closed-form expressions of the modal contribution factors (MCFi, i = 1,2,3, Chopra 1995) activated by a dynamic input characterized by influence vector {0,1,0} (i.e. input only along the y-direction), have been derived (Palermo et al. 2013):

$$\begin{aligned}
MCF_1 &= \frac{1}{1+\left(\frac{\Omega_1-1}{\sqrt{12}}\right)^2} \\
MCF_2 &= 0 \\
MCF_3 &= \frac{1}{1+\left(\frac{\Omega_3-1}{\sqrt{12}}\right)^2}
\end{aligned} \tag{7}$$

Figure 3 displays the modal contribution factors versus e and Ω_θ. The following observations can be deduced:



(a)



(b)

Fig. 2 (a) The graphical representation of the normalized natural frequencies versus e and Ω_θ;
(b) The graphical representation of the normalized natural periods versus e and Ω_θ

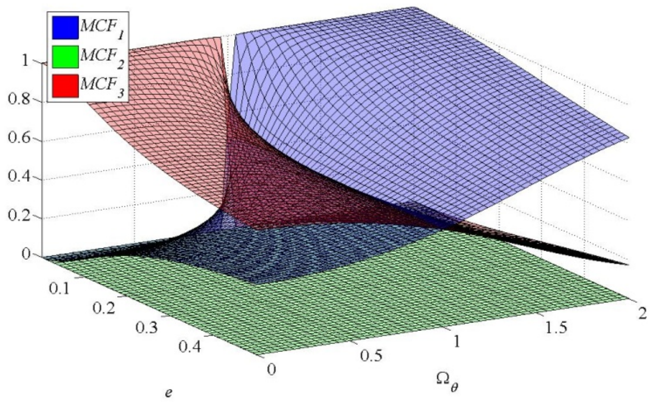


Fig. 3 Modal contribution factors versus e and Ω_θ

- $MCF_2 = 0$. This result derives from the assumptions of null eccentricity in the y-direction and influence vector along the y-direction;
- torsionally-stiff systems are principally governed by the first mode of vibration T_1 that, as showed in the previous section, is close to the second period of vibration,

T$_2$, which in turn is equal to the uncoupled lateral period, T_L;

- torsionally-flexible systems with small eccentricity (e < 0.1) are mainly governed by the third mode of vibration that is approximately equal to T_L; torsionally-flexible systems with high eccentricity (e > 0.3) are substantially governed by the first mode of vibration that may be considerably higher than T_L; for torsionally-flexible systems characterized by eccentricity e comprised between 0.1 and 0.3 both the first, T_1, and third, T_3, natural periods of vibration contribute to the dynamic response of the system.

5. THE MAXIMUM DISPLACEMENT AT THE CENTER OF MASS

In a recent paper (Palermo et al. 2013) a closed-form expression of the center mass displacement magnification, δ, as a function of e and Ω_θ has been derived:

$$\delta = \frac{u_{y,CM,\max}}{u_{y,CM,\max,N-E}} \cong 12e^2 \sqrt{\frac{1}{\Omega_1\left[12e^2+\left(\Omega_1-1\right)^2\right]^2}+\frac{1}{\Omega_3\left[12e^2+\left(\Omega_3-1\right)^2\right]^2}} \tag{8}$$

Coefficient δ allows to express the maximum longitudinal displacement at the center mass as a function of the equivalent not-eccentric longitudinal displacement:

$$u_{y,CM,\max} = \delta \cdot u_{y,CM,\max,N-E} \tag{9}$$

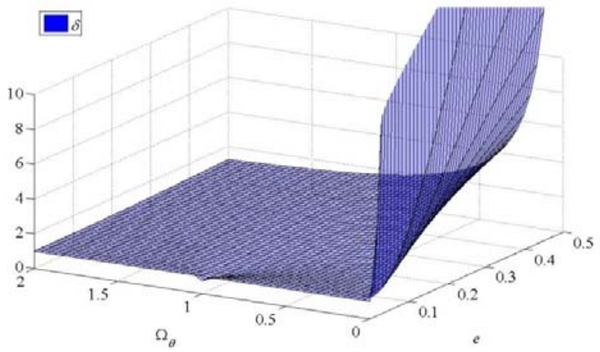


Fig. 4 The graphical representation of δ versus e and Ω_θ

The displacement magnification coefficient δ is represented in Figure 4 as a function of e and Ω_θ. The graph reveals that:

- for a wide region of e and Ω_θ, δ is close to one;
- for high values of eccentricity e coupled with low values of Ω_θ, the displacement amplification δ can achieve values larger than 5.

Based on their attitude of developing center mass displacement magnifications, the eccentric systems may be classified into 4 classes:

- CLASS 1: torsionally-stiff systems ($\Omega_\theta \geq 1.0$). The displacement response of these systems is governed by the first natural period of vibration T_1, close to T_L. In this case, the center mass displacement can be approximated by the displacement of the equivalent not-eccentric system. An example of such systems is given by frame structures with perimeter shear walls.
- CLASS 2: systems characterized by $\Omega_\theta \cong 1.0$. The displacement response of these systems results from both the contributions of the first and third mode of vibration. In detail: (i) for low values of eccentricity (e < 0.2), the center mass displacement is slightly lower than the displacement of the equivalent not-eccentric system (in this case, provided that the three periods are very close to each other, it is known that the SRSS combination may underestimate the response); (ii) for high values of eccentricity (e > 0.2), the center mass displacement is slightly higher than the corresponding displacement of the equivalent not-eccentric system. An example of these systems is given by frame structures.
- CLASS 3: low-eccentric torsionally-flexible systems (Ω_θ < 1.0 and e < 0.1). The displacement response of these systems is mainly governed by the third mode of vibration that is approximately equal to the uncoupled lateral period T_L. For this class the center mass displacement can be approximated by the displacement of the equivalent not-eccentric system. An example of these systems is given by structures with slightly asymmetric interior stiff cores.
- CLASS 4: high-eccentric torsionally-flexible systems ($\Omega\theta$ < 1.0 and e > 0.3). The displacement response of these systems is mainly governed by the first mode of vibration T_1 that may be considerably higher than the uncoupled lateral period T_L. Clearly, for this category of buildings, the assumption of equal center mass displacement between the eccentric system and its equivalent not-eccentric system is not conservative. An example of these systems is given by frame structures with highly asymmetric interior stiff cores.

6. THE MAXIMUM ROTATIONAL RESPONSE

In a previous research works (Trombetti and Conte 2005), the authors identified a rotational parameter called “alpha”, governing the maximum rotational response of eccentric systems:

$$\alpha \overset{def}{=} \rho_m \cdot \frac{u_{\theta,\max}}{u_{y,CM,\max}} \tag{10}$$

The alpha parameter has a closed-form solution in free vibration (Trombetti and Conte) and can be indicted as $\alpha_{u,free}$ (Figure 5).

In the case of damped systems subjected to seismic excitation, the alpha parameter is indicated as:

$$\alpha_{d,eqke} \overset{def}{=} \rho_m \cdot \left. \frac{u_{\theta,\max}}{u_{y,CM,\max}} \right|_{d,eqke} \tag{11}$$

By posing:

$$A \overset{def}{=} \frac{\alpha_{d,eqke}}{\alpha_{u,free}} \tag{12}$$

the maximum rotational response experienced by a damped eccentric system under seismic excitation can be expressed by the following simple relationship:

$$u_{\theta,\max} = A \cdot \frac{\alpha_{u,free}}{\rho_m} \cdot u_{y,CM,\max} \tag{13}$$

Parameter A has been calibrated through an extensive numerical analysis. In detail a simple one-storey eccentric system with equal sides length (i.e. l=b, leading to a shape factor ϕ = 1.22) has been subjected to linear time-history analyses. The base input is composed of an ensemble of 1000 natural recorded earthquakes. Each accelerogram is applied along the y-direction. A number of 2028000 time-history analyses have been conducted. The ranges of the parameters adopted for the numerical analysis are collected in Table 1. A fixed damping ratio equal to 0.05 has been adopted.

Figure 6 provides the mean values, μ_A, and the coefficient of variations, COV_A, of parameter A versus e and Ω_θ, for selected values of T_L.

Inspection of the graph leads to the following observations:

- $_A$ are between 0.5 and 5.0
- For torsionally-stiff systems μ_A limited to 1.0.
- For torsionally-flexible systems μ_A may be quite larger than 1.0. In detail: high torsionally-flexible systems with small eccentricity (e < 0.1) and low uncoupled lateral period T_L ($T_L \leq 0.4\,\text{sec}$) are characterized by the largest μ_A (μ_A =3.0-5.0);
- COVA is between 0.06 and 0.90. In detail: torsionally-stiff systems exhibit a small coefficient of variation (COV_A < 0.20) while large values of COVA are observed for high torsionally-flexible systems with small eccentricity (e < 0.1) and low uncoupled lateral period T_L ($T_L \leq 0.4\,\text{sec}$)
- The uncoupled lateral period T_L has not a clear influence on parameter A.

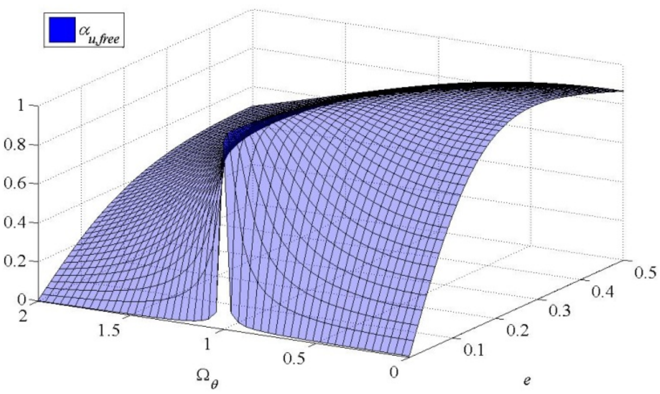
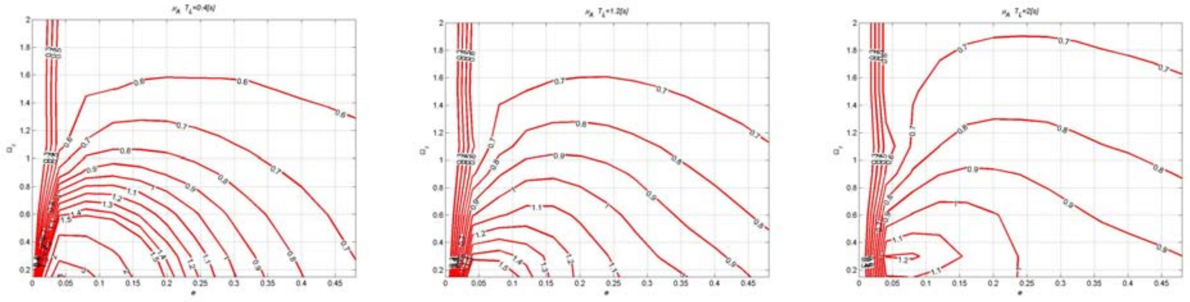
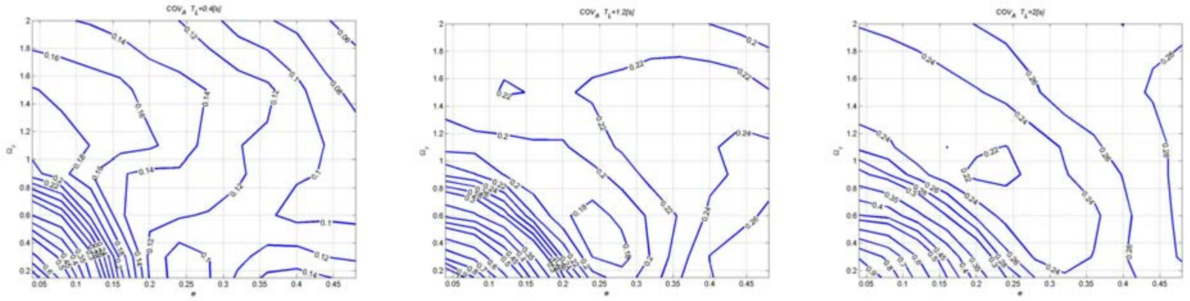


Fig. 5 The graphical representation of $\alpha_{u,free}$ versus e and Ω_θ



(a)



(b)

Fig. 6 (a) level curves of μ_A versus e and Ω_θ; (b) level curves of COV_A versus e and Ω_θ

7. ON THE CORRELATION BETWEEN THE TRASLATIONAL AND ROTATIONAL RESPONSE

The solution of the equations of motion of the studied eccentric system, in the case of undamped free vibrations from a given initial displacement a along the y-direction, is given by (Trombetti and Conte 2005, Palermo et al. 2013):

$$\begin{aligned} u_{y,CM}(t) &= a \cdot \left(MCF_1 \cos(\omega_1 t) + MCF_3 \cos(\omega_3 t) \right) \\ u_{x,CM}(t) &= 0 \\ u_{\theta,CM}(t) &= \frac{a}{\rho_m} \cdot \frac{\alpha_u}{2} \cdot \left(\cos(\omega_1 t) - \cos(\omega_3 t) \right) \end{aligned} \tag{14}$$

In the case of seismic excitation and damped condition, the analysis of correlation between the displacement response and rotational response has been developed through extensive numerical analysis. The parameter and the seismic input are the same as those used for the study of the rotational response.

Provided that the main objective of the study is to evaluate the maximum corner side displacement response a parameter of correlation B, which gives a measure of the correlation between the maximum displacement response and the maximum rotational response, is introduced:

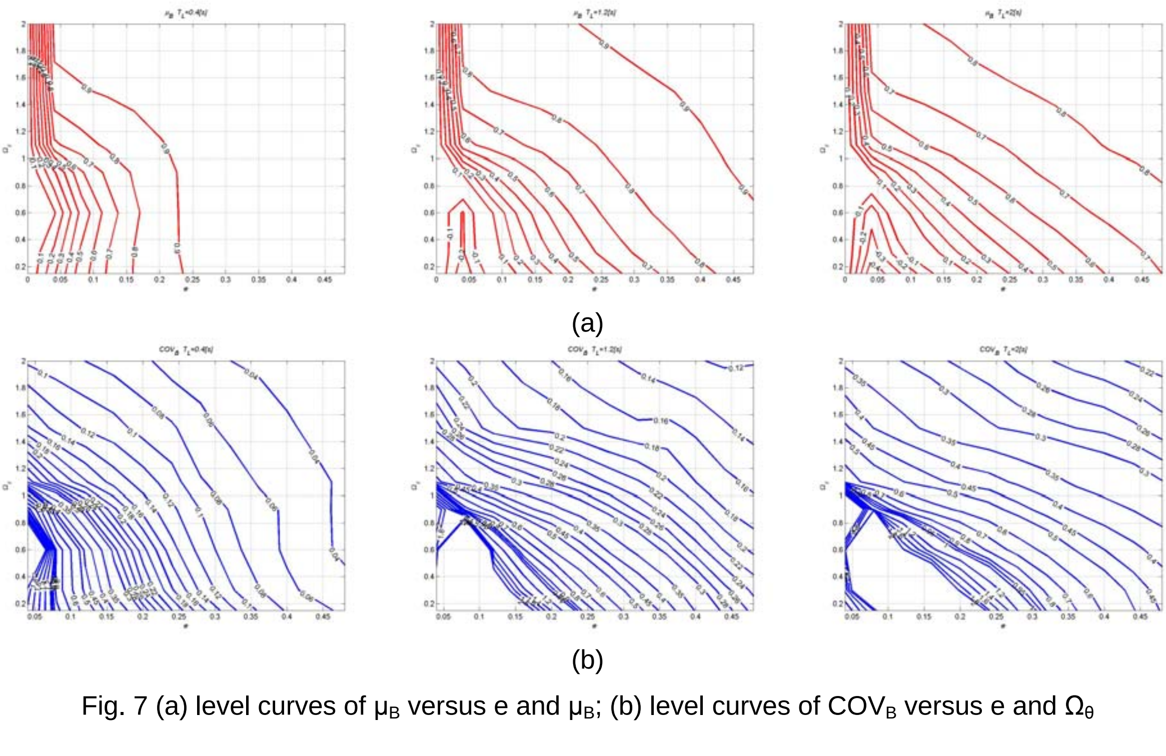
$$B \overset{def}{=} \frac{u_{y,F,\max} - u_{y,CM,\max}}{u_{\theta,\max} \cdot l / 2} \tag{15}$$

The correlation parameter B allows to estimate the maximum corner side displacement as a function of the maximum displacement response and maximum rotational response:

$$u_{y,B,\max} = u_{y,CM,\max} + B \cdot u_{\theta,\max} \cdot l/2 \tag{16}$$

Figure 7 provides the mean values, $_{B}$, and the coefficient of variations, COV_B, of parameter B versus e and Ω_θ, for selected values of T_L.
Inspection of the graph leads to the following observations:

- μ_B are between -0.3 and 0.90. Negative values of B correspond to the cases of a maximum corner side displacement at the flexible side (i.e. point F) lower than the center mass displacement. In these cases the maximum corner displacement is that of the so-called stiff side (i.e. the closer side to the center of stiffness, defined to as)
- The uncoupled longitudinal period T_L strongly affects μ_B. In detail: for low T_L (T_L<0.4 sec) μ_B increases as e increases and are practically equal to 1.0 for eccentricities larger than 0.25. For higher T_L (T_L>0.4 sec) μ_B is more variable and generally decrease as T_L increases. Furthermore $_{B}$ is generally larger for torsionally-stiff systems.
- COV_B is between 0.02 and 2.0 and it is significantly influenced by T_L, e and $\Omega\theta$. In detail: value of COV_B >1.0 correspond to negative μ_B; COV_B significantly increases as T_L increases; systems with small eccentricities are characterized by smaller COV_B; torsionally-flexible systems are characterized by highest COV_B;



(a)

(b)

Fig. 7 (a) level curves of μ_B versus e and μ_B; (b) level curves of COV_B versus e and Ω_θ

8. A SIMPLIFIED METHOD FOR THE EASTIMATION OF THE MAXIMUM CORNER SIDE DISPLACEMENT: THE "ALPHA" METHOD

In a previous research work (Trombetti and Conte 2005), the authors proposed a simplified method, called "Alpha-method", for the prediction of the maximum rotational response of eccentric systems. The original formulation of the method was developed studying the dynamic behaviour of torsionally-stiff system assuming that the maximum center mass displacement of the eccentric system should be reasonably approximated by the corresponding displacement of the equivalent not-eccentric system.

The results described in this paper provided a more comprehensive understanding of the dynamic behaviour of eccentric system, especially for the class of torsionally-flexible systems whose dynamic behaviour is significant affected by the so called fundamental period shifting phenomenon. In light of this result, the original formulation of the "Alpha-method", whose effectiveness was proved for the class of torsionally-stiff systems, is here extended to all classes of eccentric systems (i.e. torsionally-flexible and torsionally-stiff systems), removing the "equal center mass displacement" assumption:

$$u_{y,F,\max} = u_{y,C_M,\max,N-E} \cdot \left[\delta \cdot \left(1 + A \cdot B \cdot \alpha_{u,free} \cdot \phi\right)\right] = u_{y,C_M,\max,N-E} \cdot M_{N-E} \tag{17}$$

Or:

$$u_{y,F,\max} = \left(\delta \cdot u_{y,C_M,\max,N-E}\right) \cdot \left(1 + A \cdot B \cdot \alpha_{u,free} \cdot \phi\right) = u_{y,C_M,\max} \cdot M_{CM} \tag{18}$$

M_{N-E} is the corner displacement magnification coefficient (with respect to the displacement of the equivalent not-eccentric system, which, in the case of seismic analysis, can be reasonably approximated by $S_d(T_L)$):

$$M_{N-E} = \delta \cdot \left(1 + A \cdot B \cdot \alpha_{u,free} \cdot \phi\right) \tag{19}$$

M_{CM} is the corner displacement magnification coefficient (with respect to the center mass displacement, which can be reasonably approximated by δ $S_d(T_L)$):

$$M_{CM} = 1 + A \cdot B \cdot \alpha_{u,free} \cdot \phi \tag{20}$$

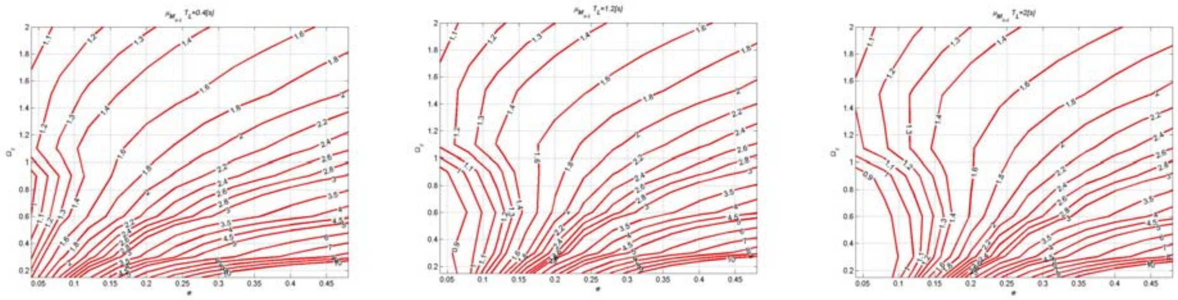
Figure 8 provides the level curves representation of the mean values, μ_{MN-E} and MCM, of the magnification coefficients as a function of e and Ω_θ.

Inspection of Figure 8 allows the following observations:

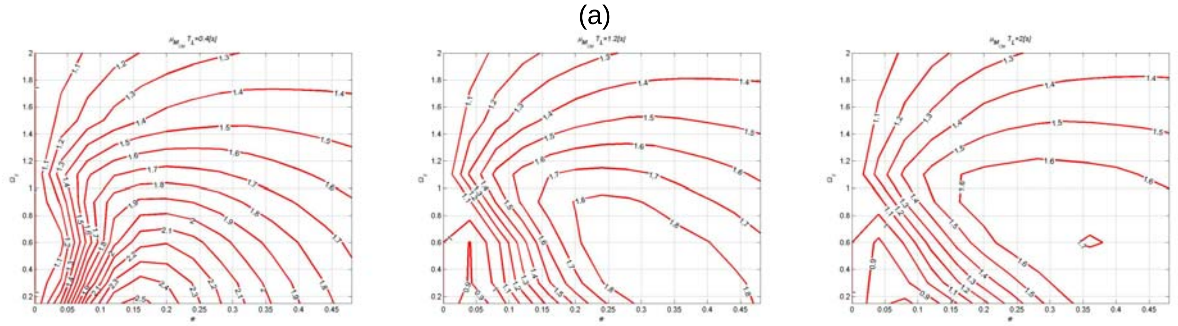
- the maximum corner displacement can be expressed as a function of the maximum displacement response of the equivalent not-eccentric system, obtained as the maximum deformation of a SDOF oscillator of undamped natural period T_L, damping ratio ξ, and mass equal to the total mass of the structure (e.g., use of displacement response spectrum (Chopra 2001));
- the formulation of the "Alpha-method" requires the introduction of two parameters, named A and B, which characterize the damped response under

seismic input, in general depending on T_L, e, ξ ,ϕ and Ω_θ.

- two displacement magnifications coefficients are proposed. First, MN-E allows to estimate the maximum corner displacement based on a simple evaluation of the lateral uncoupled period T_L. For practical applications, T_L can be estimated once the mass and lateral stiffness of the system are known, without the need of models which explicitly account for the torsional flexibility. Second, MCM allows to estimate the maximum corner displacement based on the evaluation of the maximum displacement at the center mass, thus requiring a model which accounts for the effective eccentricities of the system.
- $\mu_{N\text{-}E}$ is between 0.8 and 10.0. In detail: for torsionally-flexible systems $\mu_{N\text{-}E}$ exhibits a great variation (0.8-10) and the largest values are observed for high torsionally-flexible, high eccentric systems. It is worth to know that the main contribution of the magnification coefficient is given by the so called “fundamental period shifting” effect; for torsionally-stiff systems $\mu_{N\text{-}E}$ is between 1.0 and 2.8 with the largest values for $\Omega_\theta \cong 1.0$ and high eccentricities.
- μ_{CM} is between 0.8 and 3.0 and is significantly affected by the uncoupled period T_L. In detail: for torsionally-stiff systems μ_{CM} is limited to 1.8 and the maximum values are observed for $\Omega_\theta \cong 1.0$ and high eccentricities; torsionally-flexible systems with low uncoupled periods (T_L< 0.4) exhibit the highest magnifications for relative small eccentricities (0.05 < e < 0.10), while in the case of higher uncoupled periods T_L the largest magnifications are observed at high eccentricities.



(a)



(b)

Fig. 8 (a) level curves of $\mu_{M\text{-}NE}$ versus e and Ω_θ; (b) level curves of $\mu_{M\text{-}CM}$ versus e and Ω_θ

It can be noted that the proposed approach, due to its physically-base derivation, can be used also for the estimation of the maximum displacement at the stiff side, which may be, as in the case of wall structures the stiffest wall yields at the smallest displacement, the critical one for design purpose, even though smaller with respect to that at the flexible side, provided that the stiffest walls may yield at quite small displacement (Priestley et al., 2007). With reference of the eccentric system of Figure 1, the displacement at the corner S (the stiff side) at any instant t, is given by:

$$u_{y,S}(t) = u_{y,C_M}(t) + u_{\theta}(t) \cdot \frac{l}{2} \tag{21}$$

By simply adapting all the results which have been previously described for the case of stiff side, it is easy to show that the maximum displacement at the stiff side may be estimated using the following equation:

$$u_{y,F,\max} = u_{y,C_M,\max,N-E} \cdot \left[\delta \cdot \left(1 \ - \ A \cdot B \cdot \alpha_{u,free} \cdot \phi \right) \right] \tag{22}$$

It should be pointed out that while the formulation proposed for the estimation of the maximum corner side magnification may be considered, in general, as a conservative estimation of the corresponding inelastic magnification, the formulation for the evaluation of the maximum displacement at the stiff side (Eq. 22) may yield in un-conservative estimates if the elastic limit of the structural elements are exceeded.

9. CONCLUSIONS

This paper provides a new insight into the dynamic behavior of one-storey eccentric systems, aimed at increasing the knowledge about this class of structures, as well as providing simple tools for their seismic design. For the specific case of undamped eccentric systems in free vibrations, closed-form expressions for an upper bound and a lower bound of the maximum longitudinal corner displacement have been derived. Based on these results, a simplified approach for the seismic design of eccentric systems, originally proposed by the author for the evaluation of the torsional response of torsionally-stiff eccentric systems, has been revised accounting for all classes of eccentric systems.

The main conclusions which are derived from the study can be summarised as follows:

- The key parameters governing the dynamic behaviour of eccentric systems are: the uncoupled lateral period, T_L; the eccentricity, e; the torsional flexibility parameter, Ω_θ; the shape factor, ϕ; and the damping ratio, ξ.
- A physically based approach for the evaluation of the maximum corner displacement magnifications has been proposed, starting from the analytical study of dynamic response of the system in free vibration. On the base of these results a simple formula for the evaluation of the maximum corner displacement magnification, under seismic excitation, has been derived. The formula is composed of three fundamental contributions: (i) the translational contribution, depending on δ parameter; (ii) the torsional contribution, depending on the αu,

free parameter and on the shape factor ϕ; (iii) the combination of the two translational and torsional responses.

- The translational contribution can be strongly affected by the "fundamental period shifting" effect, i.e. the shift of the fundamental period of vibration of the eccentric system with respect to that of the equivalent not-eccentric system. The period shifting mainly affects the class of torsionally-flexible systems. For this class of eccentric systems, the estimation of the center mass displacement by response spectrum analysis using the uncoupled period of vibration of the equivalent not-eccentric system may lead to very unconservative results (the displacement amplifications can be larger than 5).
- The torsional contribution is given by two components: (i) the pure rotational response measured by the parameter αu,free that is bounded to one for all classes of eccentric systems; (ii) the "lever arm" effect depending on the plan shape of the structures, measured by ϕ.
- The time correlation between the displacement and the rotational response is dependent on the eccentricity and torsional flexibility of the systems. In detail: (i) torsionally-stiff systems exhibit a high correlation between the two maximum responses; (ii) torsionally-flexible systems show a correlation strongly dependent on the eccentricity.
- Two displacement magnifications coefficients are proposed, namely MN-E and MCM. The first, MN-E allows to estimate the maximum corner displacement based on a simple evaluation of the lateral uncoupled period T_L. For practical applications, T_L can be estimated once the mass and lateral stiffness of the system are known, without the need of models which explicitly account for the torsional flexibility. For torsionally-stiff systems, MN-E values are around 1.1 to 2.0; for torsionally-flexible systems, MN-E values can assume very high values depending on the structural parameters (maximum values around 10). The second, MCM allows to estimate the maximum corner displacement based on a the evaluation of the maximum displacement at the center mass, thus requiring a model which accounts for the effective eccentricities of the system. For torsionally-stiff systems, MCM values are around 1.1 to 1.5; for torsionally-flexible systems, MCM values are around 1.1 to 2.5.

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