Active control of damaged composite structure for the recovery of dynamic characteristics

*Jung Woo Sohn\textsuperscript{1) and Heung Soo Kim\textsuperscript{2)}

\textsuperscript{1)} Department of Mechanical Design Engineering, Kumoh National Institute of Technology, Gumi, Gyeongbuk, 730-701, Korea
\textsuperscript{2)} Department of Mechanical, Robotics and Energy Engineering, Dongguk University, Seoul, 100-715, Korea
heungsoo@dgu.edu

ABSTRACT

In this paper, active control algorithm was adopted in order to recover dynamic characteristics of damaged composite structure and control performances were numerically investigated. Finite element model for delaminated composite structure was constructed by using improved layerwise theory and then piezoelectric actuator and sensor models were included in the model. After implementation of active control algorithm to the system model, dynamic characteristics and structural performances of the delaminated composite structure were evaluated.

1. INTRODUCTION

In order to reduce carbon dioxide emission and to increase energy efficiency, many research works have been carried out. Study on light weight structure is one of main part of these researches. Composite structure, which has inherent high strength and stiffness to weight ratio, can be widely used in light structure. However, laminated composite structure had not been used as primary structure since it has complex failure modes. Delamination is one of prevalent damage phenomenon. As a result of delamination of composite structure, structural stiffness is decreased and then natural frequencies are also decreased. Kim (2003) investigated change of dynamic characteristics of delaminated composite structure by using improved layerwise theory. Change of natural frequencies might cause increase of structural vibration and degradation of structural performances.

In this work, dynamic characteristics of the damaged composite structure are recovered by using piezoelectric actuator and active control algorithm. As a first step, finite element model for delaminated composite structure is established based on improved layerwise theory. After inclusion of piezoelectric constitutive equation, state space control model is constructed. Active control algorithm is implemented to the system model and then, dynamic characteristics and structural performances of the...
delaminated composite structure are evaluated.

2. SYSTEM MODELING

The schematic diagram of the proposed laminated composite structure is shown in Fig. 1. Symmetrically layered cross ply laminate ([0/90]4s) is considered as host structure and one piezoelectric actuator is perfectly bonded on the surface of the composite structure. The dimensions of the composite structure are 30cm, 5cm and 0.2cm for length, width and thickness, respectively. Single delamination is located at the mid-plane with size of 10cm. After finite element modeling of the delaminated composite structure based on improved layerwise theory and high order electric potential for piezoelectric actuator, the governing equation can be expressed in matrix form as follows (Kim 2007).

\[
\begin{align*}
\mathbf{M} \ddot{\mathbf{d}}_u + \mathbf{C} \dot{\mathbf{d}}_u + \mathbf{K}_{uu} \mathbf{d}_u + \mathbf{K}_{u\phi} \mathbf{d}_\phi &= \mathbf{F}_u \\
\mathbf{K}_{u\phi} \mathbf{d}_u + \mathbf{K}_{\phi\phi} \mathbf{d}_\phi &= \mathbf{F}_\phi
\end{align*}
\]  

(1)

where \( \mathbf{d}_u \) and \( \mathbf{d}_\phi \) denotes the displacement of element and electrical displacement, respectively. The matrix \( \mathbf{M} \) is the structural mass matrix and \( \mathbf{C} \) is the structural damping matrix. The matrices \( \mathbf{K}_{u\phi} \) and \( \mathbf{K}_{\phi\phi} \) are stiffness matrices due to piezoelectric-mechanical coupling effects. The matrices \( \mathbf{K}_{uu} \) and \( \mathbf{K}_{\phi\phi} \) are stiffness matrices resulting from mechanical and electrical fields, respectively. The vectors \( \mathbf{F}_u \) and \( \mathbf{F}_\phi \) are force vectors due to mechanical and electrical fields, respectively. The natural frequencies and corresponding mode shapes of the proposed structure without delamination are presented in Fig. 2.

![Fig. 1 Schematic diagram of the proposed composite structure](image-url)
3. CONTROL RESULTS

State space control model can be derived from the governing equation expressed in Eq. (1) as follow

$$\dot{x} = Ax + Bu$$  \hspace{1cm} (2)

where $A$, $B$ and $u$ denote system matrix, input matrix and control input, respectively. By assuming that all state variables are measurable and are available for feedback and control input is unconstrained, pole placement algorithm is adopted. The control input $u$ is determined by the base on Ackermann’s formula and can be described as follows.

$$u = -Kx$$  \hspace{1cm} (3)

$$K = [0 \ 0 \ 0 \ \cdots \ \cdots 1][B \ AB \ A^2B \ \cdots \ \cdots A^{n-1}B]^{-1}\phi(A)$$  \hspace{1cm} (4)

$$|sI - (A - BK)| = (s - \mu_1) \cdots (s - \mu_n) = s^n + \alpha_1s^{n-1} + \alpha_2s^{n-2} + \cdots + \alpha_{n-1}s + \alpha_n = 0$$  \hspace{1cm} (5)

$$\phi(A) = A^n + \alpha_1A^{n-1} + \cdots + \alpha_{n-1}A + \alpha_nI$$  \hspace{1cm} (6)

In order to verify control performances, the transient response of the proposed structure is investigated. After excitation with initial displacements of four modes, control input is applied to the system. As shown in Fig. 1, first and third modes are bending modes and second and forth modes are twisting modes. Then, just two bending modes are controlled and two twisting modes are not controlled by using single piezoelectric actuator located at the center of the width direction. Eigenvalues of the proposed structure for healthy structure, delaminated structure and controlled structure are listed in Table 1. It is observed that eigenvalues of the delaminated structure are
recovered to healthy structure by applying control input in first and third modes. Changes in natural frequencies are also listed in Table 2. It is clearly observed that natural frequencies of the delaminated structure are recovered to healthy structure in first and third modes. Transient responses in time domain and frequency domain are plotted in Fig. 3 and Fig. 4, respectively. The responses of healthy, delaminated and controlled structure are expressed in dashed, dotted and solid line, respectively. As shown in Fig. 3, controlled response is well matched with response of healthy structure by recovering its dynamic characteristics. Some differences between healthy and controlled response are caused by uncontrolled two twisting modes. It is also clearly observed in Fig. 4 that controlled response is well matched with response of healthy structure just in first and third modes.

### Table 1 Changes in eigenvalues

<table>
<thead>
<tr>
<th></th>
<th>Healthy Structure</th>
<th>Delaminated Structure</th>
<th>Controlled Structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.0000 ± 0.0255i</td>
<td>-0.0000 ± 0.0247i</td>
<td>-0.0000 ± 0.0255i</td>
<td></td>
</tr>
<tr>
<td>-0.0003 ± 0.0789i</td>
<td>-0.0003 ± 0.0717i</td>
<td>-0.0003 ± 0.0717i</td>
<td></td>
</tr>
<tr>
<td>-0.0012 ± 0.1580i</td>
<td>-0.0012 ± 0.1553i</td>
<td>-0.0012 ± 0.1580i</td>
<td></td>
</tr>
<tr>
<td>-0.0037 ± 0.2717i</td>
<td>-0.0033 ± 0.2557i</td>
<td>-0.0033 ± 0.2557i</td>
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</tbody>
</table>

### Table 2 Changes in natural frequencies

<table>
<thead>
<tr>
<th></th>
<th>Healthy Structure</th>
<th>Delaminated Structure</th>
<th>Controlled Structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st mode (Bending)</td>
<td>40.6</td>
<td>39.3</td>
<td>40.6</td>
</tr>
<tr>
<td>2nd mode (Twisting)</td>
<td>125.6</td>
<td>114.0</td>
<td>114.0</td>
</tr>
<tr>
<td>3rd mode (Bending)</td>
<td>251.4</td>
<td>247.2</td>
<td>251.4</td>
</tr>
<tr>
<td>4th mode (Twisting)</td>
<td>432.5</td>
<td>407.0</td>
<td>407.0</td>
</tr>
</tbody>
</table>

![Fig. 3 Transient response in time domain](image)

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4. CONCLUSIONS

The dynamic characteristics of the delaminated composite structure were recovered by adopting piezoelectric actuator and active control algorithm. Finite element model for the delaminated composite structure was developed based on improved layerwise theory and piezoelectric actuator and sensor models were also included in the model. By implementing active control algorithm to the system model, dynamic characteristics of the delaminated composite structure were effectively recovered and it was clearly observed by investigation on transient responses of the structure in time and frequency domain.

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