System identification of high-rise buildings using shear-bending model and ARX model: Experimental investigation

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ABSTRACT

The system identification is regarded as the most basic technique for structural health monitoring to evaluate structural integrity. Although many system identification techniques that can extract mode information (e.g. mode frequency and mode shape) have been proposed so far, it is also desired to identify physical parameters (e.g. stiffness and damping). As for high-rise buildings subjected to long-period ground motions, the system identification for evaluating only the shear stiffness based on a shear model does not seem to be an appropriate solution to the system identification problem due to the influence of overall bending response.

In this paper, a system identification algorithm using a shear-bending model is developed to identify both shear and bending stiffnesses. In this algorithm, an ARX (Auto-Regressive eXogenous) model corresponding to the transfer function for interstory accelerations is applied for identifying physical parameters. For the experimental verification of the proposed system identification framework, vibration tests for a 3-story steel mini-structure are conducted. The test structure is specifically designed to measure horizontal accelerations including both shear and bending responses. In order to obtain reliable results, system identification theories for two different inputs are investigated; (a) base input motion by a modal shaker, (b) unknown forced input on the top floor.

1. INTRODUCTION

System identification (SI) techniques play an important role in investigating and reducing gaps between the constructed structural systems and their structural design models and in structural health monitoring for damage detection. In the field of SI, there are many achievements (see, for example, World Conferences on Structural...

It is well recognized that the modal-parameter SI and physical-parameter SI are two major branches in SI (Hart and Yao 1977). The former is suitable for identifying the overall mechanical properties of a structural system and exhibits stable characteristics in its implementation. This characteristic may be related to the fact that the modal parameters are system performances representing global properties of a structural system. While the latter is important from different viewpoints, e.g. enhancement of reliability and robustness in active controlled structures (Housner et al. 1997) or base-isolated structures, its development is limited due to the requirement of multiple and accurate measurements or the necessity of complicated manipulation. On the other hand, a mixed approach is often used in which physical parameters are identified from the modal parameters obtained by the modal-parameter SI. However, in view of inverse problem formulation, a sufficient number of modal parameters must be obtained for the unique and accurate identification of the physical parameters. This requirement is usually hard to be satisfied.

In this paper, a shear- bending model is used for reliable identification of high-rise buildings. While a shear building model has been a well-used model for system identification, Kuwabara et al. (2012) and Minami et al. (2012) introduced a shear-bending model and developed system identification theories for high-rise buildings. However their researches are limited mainly to theoretical ones. The corresponding experimental one is developed here.

2. System Identification algorithm for shear and bending stiffnesses

A system identification algorithm for shear and bending stiffnesses of a shear-bending model has been proposed in the reference (Kuwabara et al. 2012 and Minami et al. 2012, 2013). In this section, the proposed system identification algorithms using floor acceleration records for base input and unknown vibration source are briefly explained.

2.1 System Identification algorithm under base input

Consider an \(N\)-story shear-bending model as shown in Fig.1(b). Comparing with the scheme of the system identification algorithm for shear model (Takewaki and Nakamura 2000, Takewaki et al. 2011), the transfer function with respect to absolute horizontal accelerations (also displacements), i.e. the \(j\)th-story acceleration to the \((j-1)\)th-story acceleration can be introduced as

\[
G_j = \frac{\ddot{U}_j + \ddot{U}_{j-1}}{\ddot{U}_j + \ddot{U}_0}
\]  

(1)

where \(\ddot{U}_j\),\(\ddot{U}_{j-1}\) are the Fourier transforms of base and \(j\)th-story floor accelerations \(\ddot{u}_0\),\(\ddot{u}_j\). For evaluating shear and bending stiffnesses, the identification function (IDF) is defined in terms of \(G_j\) as

\[
F_j(\omega) = -\frac{G_{j-1}}{\omega^2 \sum_{i=j}^{N} m_i}
\]  

(2)

By defining the stiffness ratio \(R_j = k_{uj} / k_{sj}\) of the bending to the shear, the shear and bending stiffnesses can be formulated as
can be derived by considering the dynamic equilibrium of a free
body of the shear-bending model as follows:

\[
k_{by} = \frac{R_j + \frac{H_j}{N} \sum_{i=j}^{N} \{m_i(H_i^j - H_{j-1}^i)\}}{\lim_{\omega \to 0^+} \Re\{F_j(\omega)\} - \frac{H_j}{N} \sum_{i=j}^{N} \{m_i(H_i^j - H_{m-1}^i)\}}
\]

(3a,b)

\[
k_{s_j} = k_{by}/R_j
\]

Assuming that the damping coefficient is in proportion to the stiffness, the relationship between \(G_j\) and \(G_{j-1}\) can be derived by considering the dynamic equilibrium of a free
body of the shear-bending model as follows:

\[
G_{j-1} = \left[1 + \frac{H_{j-1}}{N} \left(1 - G_j\right)\right]^{-1} \left[1 + \frac{\omega^2}{k_{s_j} + i\omega c_s} \sum_{i=j}^{N} m_i G_G(i) \sum_{k=j}^{N} G_G(j-1) - G_G(j) + \frac{\omega^2}{k_{by} + i\omega c_y} \frac{H_j}{N} \sum_{i=j}^{N} m_i G_G(i) \sum_{k=j}^{N} G_G(j-1) - G_G(j)\right]
\]

(4)

where \(H_{j-1}, k_{s_j}, k_{by}, c_{s_j}, c_{by}\) are the structural parameters of the shear-bending model as shown in Fig.1. In addition, \(G_G(j) = \prod_{k=1}^{j} G_k\). From Eq.(4), it can be observed that the transfer function \(G_{j-1}\) is formulated in terms of the transfer functions from the \(j\)th through the \(N\)th-story.

Fig.2(a) shows the flowchart of the proposed system identification method using the floor accelerations under base input. Regarding the stiffness ratios \(R\) as variable
parameters, the stiffnesses are identified so that the transfer function evaluated by
Eq.(4) as the function of \(R\) approximately coincides with that computed by using an
ARX model. The SQP (Sequential Quadratic Programming) is applied to find the optimal stiffness ratio \(R\) so as to minimize the objective function \(J\) defined by

\[
J = \sum_{j=1}^{N-1} \frac{\sum_\omega \left[G_j(\omega,R) - ARX G_j(\omega)\right]}{\sum_\omega \left[ARX G_j(\omega)\right]}
\]

(5)

2.2 System Identification algorithm under unknown vibration source

Consider the same building model again excited by an unknown vibration source
\(f(t)\) at the top floor as shown in Fig.1(c). Let \(V_j\) denote the Fourier transform of
the interstory drift \(v_j = u_j - u_{j-1}\). The transfer function \(g_j\) of the interstory drift ratio can be evaluated from the records of floor accelerations as

\[
g_j(\omega) = \frac{V_j(\omega)}{U_j(\omega)} = \frac{U_{j+1}(\omega) - U_j(\omega)}{U_j(\omega) - U_{j-1}(\omega)} \approx \frac{\dot{U}_{j+1}(\omega) - \dot{U}_j(\omega)}{\dot{U}_j(\omega) - \dot{U}_{j-1}(\omega)} \approx \frac{\ddot{U}_{j+1}(\omega) - \ddot{U}_j(\omega)}{\ddot{U}_j(\omega) - \ddot{U}_{j-1}(\omega)}
\]

(6)
From the estimation of the recurrence relation between the shear stiffness ratio \( r_{sj} = k_{sj+1} / k_{sj} \) through the investigation of a few-degrees-of-freedom shear-bending model, the shear stiffness ratio \( r_{sj} \) can be derived (Minami et al. 2013) as the function of the stiffness ratio \( R_j \) and \( g_j(\omega) \) as follows:

\[
\frac{J}{\prod_{i=1}^{j} A_i - H_j \sum_{i=1}^{j} \left( \prod_{k=0}^{i-1} A_k \prod_{k=1}^{j-i} B_k \right) \left( R_{j+1} + H_{j+1} H'_{j+1} \right)}
\]

where \( H'_j = \sum_{i} H_i \cdot \) This recurrence relationship was also proved by the mathematical induction in the reference (Minami et al. 2013). From Eq.(7), by regarding the stiffness ratio \( R_i (i = 1, 2, \cdots, N) \) as the variables, it can be observed that the shear and bending stiffnesses of all the stories can be derived recurrently by specifying the first story shear stiffness.

Fig.2(b) shows the flowchart of the system identification method for an unknown vibration source. Considering any story stiffness as a leading parameter, the stiffness can be identified so that the fundamental natural circular frequency computed by the eigenvalue analysis is equal to that obtained as the reference value from another record (e.g. microtremor measurement). Then, the stiffness ratios \( R \) can be determined by the SQP method to minimize the objective function \( J \), i.e. the difference of second and third natural circular frequencies. The objective function used in this system identification method is defined by

\[
J = \left| \bar{\omega}^{(2)} - \omega^{(2)} \right| / \omega^{(2)} + \left| \bar{\omega}^{(3)} - \omega^{(3)} \right| / \omega^{(3)}
\]

where \( \bar{\omega}^{(j)} \) and \( \omega^{(j)} \) denote the \( j \)th natural circular frequency computed by the eigenvalue analysis of the identified model and reference one.

![Fig.1 Shear-bending model](image-url)
3. Experimental verification

Since it is difficult to obtain the true value of shear and bending stiffnesses for existing structures, an experimental verification of the proposed system identification methodology was conducted using a scaled structural model. By comparing shear and bending stiffnesses identified by the proposed methodologies with those evaluated by the static loading test, the reliability of the proposed system identification methodologies is investigated.

3.1 Design of test structure

The test structure was designed so as to have certain amounts of remarkable shear and bending properties. Fig.3 shows the schematic diagram of the test structure; floor plates, column plates, spacer blocks (SUS304, 2B) and angle bars supporting column (steel). The vibration of this test structure along with $x$ axis was investigated as shown in Fig.3. As a preliminary experiment, vibration tests of a 3-story test structure composed of one column plate in each story were conducted. It was confirmed that bending deformation is predominant. In order to obtain an appropriate ratio of shear and bending responses, the test structure was re-designed to have two column plates and adjustable spacer blocks. In this paper, the inner distance between two column plates was set to 10 mm. The basement of the test structure was made by a relatively thick stainless plate (14mm) which was also used as a base of a modal shaker.
3.2 Static loading test

For evaluating the shear and bending stiffnesses of the test structure as the referenced values, static loading tests were conducted. Fig.4 shows the photo of the static loading test setup. The static loading was conducted at the top floor only due to a difficulty of setup of the loading system. The rotational angle $\theta_i$ ($i = 1, 2, 3$) of the floor plate was evaluated by measuring vertical displacements of the two points in the same floor mass plate and the horizontal displacement was measured by conventional displacement transducers (CDP-50, Tokyo Sokki). As for the horizontal displacement measurement, since the influence of stiffness of the displacement sensor itself is not clear, a laser displacement meter (LB-300, Keyence) was also used instead of conventional transducers. Fig.5 illustrates the loading location and displacement sensors positions.

The estimated shear and bending stiffnesses $k_{si}$, $k_{bi}$ ($i = 1, 2, 3$) from the static loading test were evaluated by the equilibrium equations including measured horizontal displacements and rotational angles of the floor plates as

$$k_{si} = \frac{\sum_{i=1}^{3} f_i}{u_i - u_{i-1} - H_i \theta_i}, \quad k_{bi} = \frac{\sum_{i=1}^{3} \left( f_i \sum_{m=1}^{j} H_i \right)}{\theta_i - \theta_{i-1}} \quad (10a, b)$$

where $f_i$ is the static load at the $i$th floor. Fig.6 shows the evaluated shear and bending stiffnesses using Eqs.(10) from the static loading test; the solid line was derived from the record of the conventional displacement transducers (both horizontal and vertical), while the red or blue marker was derived from the record of combination of different displacement sensors (horizontal: laser, vertical: transducers). Since the vertical displacements were relatively small, it seems difficult to evaluate true value of shear and bending stiffnesses. For instance, the bending stiffnesses of the first and
third stories attain quite large values when the interstory rotation angle $\theta_i - \theta_{i-1}$ is small. This is because, even though the static loading at the top story floor keeps increasing, the interstory rotation angle $\theta_i - \theta_{i-1}$ varies from the negative value in the low-level deformation to the positive value in the high-level deformation. This phenomenon may be caused by the loading condition or unique characteristic of the test structure. The estimated bound stiffnesses are summarized in Table 1.

![Fig.4 Photo; Static loading test](image)

![Fig.5 Loading and measurement position](image)

![Fig.6 Evaluation of shear and bending stiffnesses in static loading test](image)

<table>
<thead>
<tr>
<th></th>
<th>Shear stiffness [N/m]</th>
<th>Bending stiffness [Nm/rad]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lower</td>
<td>Upper</td>
</tr>
<tr>
<td>first story</td>
<td>$3.79 \times 10^3$</td>
<td>$7.11 \times 10^3$</td>
</tr>
<tr>
<td>second story</td>
<td>$4.99 \times 10^3$</td>
<td>$6.73 \times 10^3$</td>
</tr>
<tr>
<td>third story</td>
<td>$5.67 \times 10^3$</td>
<td>$5.73 \times 10^3$</td>
</tr>
</tbody>
</table>
3.3 System Identification of shear and bending stiffnesses based on vibration test
3.3.1 Shaking system for base input
A modal shaker (encapsulated type shaker, YZ-203, Asahi Manufacturing) was mainly used as the excitation source for the base input. A modal shaker was attached to the base thick plate on which the test structure is also installed. In order to excite enough vibration into the test structure, the base plate for the shaker and the test structure was placed on conventional office table which has a relatively weak horizontal stiffness. In this shaking test, the base thick plate can be regarded as the ground level for the test structure. Fig.7. shows the photo of the setup of the test structure with the shaking system.

![Fig.7 Test structure with shaking system](image)

3.3.2 Sweep testing
So as to evaluate the modal frequencies of the test structure, the sweep testing was conducted first. The target frequency band of the sweep base input was set from 2[Hz] to 30[Hz]. The minimum frequency control range of the shaker was limited by 2[Hz] and a target base acceleration was controlled as 0.2[m/s²].

Fig.8 shows the transfer function of each story acceleration to the base acceleration for a record which was calculated by the ensemble average of power-spectrum. Considering the control frequency band of the sweep input, the transfer functions are smoothly obtained between the target frequencies as shown in Fig.8. The mode frequencies of the test structure which was identified from the peak of the transfer functions are given in Table 2.

![Fig.8 Transfer function of horizontal acceleration (sweep vibration test)](image)
(a) Third story/base, (b) Second story/base, (c) First story/base
### Table 2 Natural frequencies and transfer function amplitudes of test structure

<table>
<thead>
<tr>
<th>mode</th>
<th>Natural frequency [Hz]</th>
<th>2F/base</th>
<th>3F/base</th>
<th>4F/base</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.76</td>
<td>16.36</td>
<td>29.68</td>
<td>38.16</td>
</tr>
<tr>
<td>2</td>
<td>9.06</td>
<td>47.32</td>
<td>19.33</td>
<td>42.63</td>
</tr>
<tr>
<td>3</td>
<td>12.72</td>
<td>18.06</td>
<td>25.65</td>
<td>17.65</td>
</tr>
</tbody>
</table>

#### 3.3.3. System identification of shear and bending stiffnesses using base input

The system identification method explained in Section 2.1 was verified by using floor accelerations under base inputs; (a) White noise, (b) El Centro NS (1940). For comparing the influence of deformation amplitude dependency caused by geometrical nonlinearity of the test structure, several cases with different input motion’s amplitudes were investigated. Fig. 9 shows the comparison of the transfer functions (Eq. 1) of the interstory drift derived by the ARX model with that calculated by the raw data. The IDF can be described in a manner as shown in Figs. 10. Compared with the IDF by the raw data in Figs. 10, it can be confirmed that the IDFs using the transfer function derived by the ARX model are obtained smoothly even in the low-frequency range.

The identified shear and bending stiffnesses for various amplitudes of the base input are summarized in Figs. 11, where the initial stiffness ratio was $R = 1$. In Figs. 11, the estimated bounds of those stiffnesses as the reference values (Table 1) are also indicated. From Figs. 11, the variability of the identified shear stiffnesses seems to be low, while the values of the bending stiffnesses vary with relatively high variability. Similar results were observed for the base input of the white noise. Fig. 12 shows the comparison of the simulated time-history acceleration of the bending-shear model, which was derived by the identified stiffnesses with damping ratio $h = 0.005$, with that of recorded data at the top story. As seen in Fig. 12, the structural response under base input can be simulated within allowable accuracy by the proposed SI method.
Fig. 11 Shear and bending stiffnesses identified in various amplitudes of the base input (El Centro NS), (a) Shear stiffness, (b) Bending stiffness

Fig. 12 Comparison of the simulated time histories of the floor acceleration at the top story with that of actual records (El Centro NS 10%, h=0.005)

3.3.4. System identification of shear and bending stiffnesses using unknown vibration source in structure

The system identification method explained in section 2.2 was also verified by using the floor accelerations under an unknown vibration source. The test structure was excited at the top floor, by man-power. The accelerations on the base plate and each floor were recorded during about 60 seconds. The recorded time history accelerations are shown in Figs. 13. As mentioned in section 2.2, in applying the proposed method under an unknown vibration source, it is undesirable to cause vibration at the basement. Therefore, the base plate was placed on the floor. The root mean square of the accelerations at the basement was \(4.69 \times 10^{-3} \text{[m/s}^2]\) which was relatively smaller than \(2.10 \text{[m/s}^2]\) at the third story.

Fig. 13 Floor accelerations (man-power shaking test at the third story) (a) third story, (b) second story, (c) first story
Figs. 14 shows the comparison of the transfer function of the interstory drift ratio (Eq. (6)) derived by an ARX model with that by raw records. Since this transfer function described by the ARX model is used to identify shear and bending stiffnesses, an appropriate selection of the band pass filter is quite important in this method. The reliability of the proposed SI method will be discussed elsewhere. The identified structural properties, where the initial stiffness ratios were assumed as $R = \{1\}$, are summarized in Table 2. Lower and upper frequencies of the band-path filter for the optimally identified stiffnesses was 19 [rad/s] and 94 [rad/s], respectively.

### Table 3 Identified stiffnesses and modal frequencies of test structure

<table>
<thead>
<tr>
<th>Story</th>
<th>Shear stiffness $k_s$ [N/m]</th>
<th>Bending stiffness $k_b$ [Nm/rad]</th>
<th>Stiffness ratio $R$</th>
<th>Mode</th>
<th>Natural circular frequency $\omega$ [rad/s]</th>
<th>Error ratio of $\omega$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>5700</td>
<td>2565</td>
<td>0.45</td>
<td>1</td>
<td>17.34</td>
<td>-0.13</td>
</tr>
<tr>
<td>Second</td>
<td>4928</td>
<td>14178</td>
<td>2.88</td>
<td>2</td>
<td>56.91</td>
<td>0.00</td>
</tr>
<tr>
<td>Third</td>
<td>7494</td>
<td>7494</td>
<td>1.00</td>
<td>3</td>
<td>80.24</td>
<td>0.37</td>
</tr>
</tbody>
</table>

3.4. Summary of identification of shear and bending stiffnesses

The comparison of the identified shear and bending stiffnesses with the estimated bound of those stiffnesses derived by the static loading test is summarized in Table 4. In Table 4, the average and standard deviation of the stiffnesses identified by the SI method for the base input include the results for all the records of both El Centro NS and white noise inputs. Meanwhile, those for an unknown vibration source were evaluated by a series of measurements.

As for the shear stiffness, by comparing the results identified by the SI method for the base input with those for an unknown vibration source, acceptable values compatible with the results by the static loading test have been obtained. On the other hand, as for the bending stiffnesses, it is concluded that the results by the SI method for an unknown vibration source are worse in view of the variability of the bending stiffness at the second story than that for the base input. The reason of this inaccuracy may be due to the difference of the objective function $J$. Since a difference between the second and third mode frequencies of the identified shear-bending model and those of the reference value is minimized in the SI method for an unknown vibration source, the
variability of the natural frequencies higher than the third mode frequency caused by the change of bending stiffnesses is not necessarily included.

Table 4 Summary of the test results

<table>
<thead>
<tr>
<th>SI method</th>
<th>Story</th>
<th>Shear stiffness (k_s) [N/m]</th>
<th>Bending stiffness (k_b) [Nm/rad]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Average Standard deviation</td>
<td>Reference Low Upper</td>
</tr>
<tr>
<td>Base Input</td>
<td>1</td>
<td>5021.3 370.7</td>
<td>3790 7110</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>5580.2 421.6</td>
<td>4990 6730</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>5492.8 930.6</td>
<td>5670 5730</td>
</tr>
<tr>
<td>Unknown Inner Vibration Source</td>
<td>1</td>
<td>5650.0 70.7</td>
<td>3790 7110</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>4995.0 94.7</td>
<td>4990 6730</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>7324.5 239.7</td>
<td>5670 5730</td>
</tr>
</tbody>
</table>

4. CONCLUSIONS
The following conclusions have been derived.

(1) The system identification theories using a shear model and a shear-bending model by the same research group were reviewed in detail. The theories use the base acceleration input or forced input in a building model.

(2) The system identification results by the static loading, the base acceleration input and the forced input in a building model have been compared for mutual confirmation of the reliability of these identification theories.

(3) The estimated shear and bending stiffnesses as the reference value were derived by the static loading test. Because the difficulty exists in the loading system setup and measurement of the vertical displacements, lower and upper bounds of those stiffnesses were obtained.

(4) The identified shear stiffnesses of the test structure by the proposed SI methods were acceptable compared with those derived by the static loading test. The standard deviation of the identified story stiffness becomes larger in the upper story.

(5) The identified bending stiffnesses of the test structure by the SI method for the base input were compatible with the reference values. Those derived by the SI method for an unknown vibration source were not appropriate because of a large variability in the second story.

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