Viscous Effects in the Prediction of the Motions of SWATH Autonomous Vehicles in Waves

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ABSTRACT

The accurate prediction of motion in waves of a marine vehicle is essential to assess the maximum sea state vs. operational requirements. This is particularly true for small crafts as Autonomous Surface Vessels (ASV). We consider two different methods to prediction motions of the SWATH-ASV: an inviscid strip theory initially developed at MIT for catamarans and then adapted for SWATHs and a hybrid strip theory, based on the numerical solution of the radiation forces by an unsteady viscous, non-linear free surface flow solver. Motion predictions obtained by the viscous method are critically discussed against those obtained with the potential flow based strip theory. Effects of viscosity are analyzed by comparison of sectional added mass and damping calculated at different frequencies and for different sections, three dimensional hydrodynamic radiation forces and RAOs. A number of Important conclusions can be drawn from this study: influence of viscosity is definitely non negligible for SWATH types of vessels like the one presented; the hybrid strip theory method with fully non-linear, viscous free surface calculation of the radiation forces appears to be a very valuable tool to improve the accuracy of traditional strip theories, without the burden of long computational times requested by fully viscous time domain three dimensional simulations.

1. INTRODUCTION

A new family of Autonomous Surface Vehicles (ASVs) has been designed to achieve low motions in waves (enhance operability at sea) and reduced powering requirement at cruise speed to maximize endurance or achieve high speeds (Brizzolara & Chryssostomidis, 2013). The family is based on an unconventional SWATH hull which has been optimized with respect to these two objectives resulting in a twin

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canted struts arrangement and a particular underwater hull shape having two maximum area sections and an intermediate minimum area section, opportuneely positioned along the length to minimize the drag.

A design example of the vehicle of the family is presented in Figure 1: it is a small (7m long) Unmanned Surface Vehicle (USV) designed (Brizzolara et al., 2011) to serve a network of Autonomous Underwater Vehicles (AUVs), carrying out autonomously (or under control of a remote pilot) several of essential tasks that normally would required a manned ship, such as launching and recovering them, recharging at sea and taking them home (on a mother ship or on a land base).

These types of missions obviously requires a very stable platform in waves, in terms of minimum motions in irregular sea states at almost zero speed, if not a synchronous motion behavior to the AUV that stays partially submerged during the operation of approach to be recovered or recharged. Since this last task appears to be quite impossible as a design goal and too much dependent on the particular AUV considered, the choice has been to minimize the absolute motions of the USV by adopting an unconventional hull design, such as that above introduced and presented in Figure 1. To design and assess the operability in a sea state different seakeeping codes have been developed, ranging from potential flow strip theory methods to three dimensional viscous flow solvers. Their level of accuracy goes hand in hand with the computational time needed to obtain the motion prediction in a given sea state. Graph of Figure 2 shows the order of magnitude of the computational time and the level of accuracy (normalized by the lowest fidelity method) of different seakeeping numerical methods. On the lower end there are potential flow based codes, two dimensional (strip theory based) or three dimensional (panel methods) based on linear theory, in frequency domain. In the case of catamarans and SWATHs (Mansour and Choo, 1973) and (Lee & Curphey, 1977), have been a reference for this study. Viscosity effects can
be introduced in these methods as empirical addition to damping forces (Centeno et al., 2000). A higher fidelity can be achieved with a non-linear time domain approach in case of large amplitude motions (Fang & Her, 1995). In this case some non-linearities related to the viscous corrections can be introduced. Viscous effects also in these cases, though, are still approximated and added with empirical non-linear motion dependent force corrections.

On the upper end of the time scale of the graph in Figure 2, there are the fully viscous three dimensional, free surface transient flow solvers used to derive the Response Amplitude Operators of motions with a series of time domain simulations in monochromatic waves or in irregular waves (for a given sea state and encountering angle). This kind of simulations are indeed possible (Brizzolara & Chryssostomidis, 2013) and offer the highest possible fidelity against model test results, but require large computational time and resources.

In between the two types of methods, potential flow based and the 3D fully viscous, there is a gap that we are working to close. There is fact the possibility to create a hybrid frequency domain method (or time domain based on frequency domain results) by calculating the radiation forces only with viscous calculations (in 2D or 3D) and then to provide exciting and restoring forces through a classical linear potential flow approach. Wave induced forces, in fact, are minimally influenced by viscosity.

Being this the goal, we start to present here a numerical study aimed to highlight the relative differences between a classic potential flow strip theory method developed at MIT (Mansour and Choo 1973) and adapted to SWATHs (Chryssostomidis and Patrikalakis, 1984) and a hybrid strip theory where the radiation forces are calculated by a fully viscous flow solver on 2D sections. This with the scope to eliminate the inaccuracies introduced by empirical viscous corrections to potential flow based methods that for SWATHs vessels are relatively very important (Centeno et al. 2010).

![Figure 2 – Accuracy vs. computational time of different numerical methods for the motion prediction of SWATH ship in waves. Considered methods range from 2D potential flow theories to unsteady 3D viscous flow solvers.](image-url)
2. STRIP THEORY FOR SWATH SHIPS

The problem of analyzing motions and loads for catamarans in a random sea state with linear and theory was addressed by Mansour and Choo (1973), the proper assumptions, boundary conditions and equations using were outlined. This work culminated in the development of “CAT-5D”, a strip-theory based code capable of handling such calculations for any given hull geometry, weight distribution and sea characteristics. When the method was first applied to SWATHs, Chryssostomidis and Patrikalakis (1986) passed from a fully analytical integration method over 2D sections of the original CAT-5D code, working well for typical catamarans hull forms, to a more general fully numerical integration scheme needed to avoid the singularities of the fully submerged sections of SWATH type of vessels. A new code has been written in Matlab and it is now integrated into routine design and optimization of SWATH vehicles at MIT Innovative Ship Design lab. Hereinafter we outline the principal theoretical elements of the method.

The simplified equation of motion for a ship, considered as a rigid body, in regular waves, for the six degrees of freedom is

\[ (M_{ij} + A_{ij}) \ddot{\eta}_i + B_{ij} \dot{\eta}_j + C_{ij} \eta_j = F_i(t) \quad i, j = 1, 2, \ldots, 6 \]  

(1)

where \( M_{ij}, A_{ij}, B_{ij} \) and \( C_{ij} \) are 6x6 matrices representing, respectively, the ship’s mass, added-mass, damping and restoring coefficients. \( F(t) \) and \( \eta \) are 6x1 vectors representing, respectively, the forces and motions in the six degrees of freedom, \( \dot{\eta} \) and \( \ddot{\eta} \) are the first and second time derivatives of the motion, in other words, the velocity and acceleration of the vessel.

Equation (1) can be simplified by considering the ship symmetric about its center line, with usual linear assumption of decoupled motions in the horizontal and vertical planes. The simplified equations of linear seakeeping problems in the vertical plane neglecting surge motion influence:

\[ (M + A_{33}) \ddot{\eta}_3 + B_{33} \dot{\eta}_3 + C_{33} \eta_3 + A_{35} \ddot{\eta}_5 + B_{35} \dot{\eta}_5 + C_{35} \eta_5 = F_3 e^{i\omega t} \]  

(2)

\[ A_{53} \ddot{\eta}_3 + B_{53} \dot{\eta}_3 + C_{53} \eta_3 + (I_{55} + A_{55}) \ddot{\eta}_5 + B_{55} \dot{\eta}_5 + C_{55} \eta_5 = F_5 e^{i\omega t} \]  

(3)

The strip theory assumptions permit to reduce the problem to a numerical solution of flow on 2D cross sections. Three dimensional added-mass, damping and restoring coefficients are then calculated through the integration of the two-dimensional ones over the length of the ship, see for instance Lee and Curphey (1977) where no additional viscous corrections have been used. To compute the required 2D coefficients the fluid is assumed to be inviscid, incompressible and irrotational, the environment is infinitely deep and previously undisturbed with negligible surface tension effects. The amplitude of the motions is assumed to be small. With these simplifications the problem reduces to the calculation of a (complex) velocity potential in the form:

\[ \phi^m(y, z, t) = Re \{ \phi^m(y, z) e^{-i\omega t} \} \]  

(4)

\[ \phi^m = \phi^m_v + i \phi^m_s \]  

(5)

where \( m = 2 \) (sway), 3 (heave), 4 (roll) are the three degree of freedom of the transverse plane reduced motion problem.
The perturbation velocity potential flow problem is defined by the following conditions:

Laplace Equation: \( \nabla^2 \phi^m = 0 \quad \text{for} \quad z < 0 \) \hfill (6)

Linearized Free-Surface Condition: \( \frac{\partial^2 \phi^m}{\partial z^2} + g \frac{\partial \phi^m}{\partial x} = 0 \quad \text{for} \quad z = 0 \) \hfill (7)

The Deep Water Boundary Condition: \( \lim_{z \to -\infty} |\nabla \phi^m| = 0 \) \hfill (8)

The Radiation Condition: \( \lim_{y \to \pm \infty} \left( \frac{\partial \phi^m}{\partial y} + iK \phi^m \right) = 0 \) where \( K = \frac{w^2}{g} \) \hfill (9)

The Symmetry Condition: \( \frac{\partial \phi^3}{\partial y}(0, z, t) = 0 \quad \text{and} \quad \phi^2(0, z, t) = \phi^4(0, z, t) = 0 \) \hfill (10)

Linearized Hull Boundary Condition: \( \vec{n} \cdot \nabla \phi^m = u_n^m \) \hfill (11)

where \( \vec{n} \) is the outward unit normal vector on the hull and \( u_n^m \) is the normal component of the normal velocity of the hull due to forced oscillations.

Through Frank source method (1967) based on Green’s functions, the velocity potential can be calculated distributing two dimensional pulsating sources along the hull contour with strength given by:

\[
G_R^m(w, \zeta, t) = \text{Re} \{G_R^m(w, \zeta) e^{-i\omega t}\} \hfill (12)
\]

where \( G_R^m(w, \zeta) = G_{Rc}^m(y, z; \xi, \eta) + iG_{Rf}^m(y, z; \xi, \eta) \) \hfill (13)

\[
G_{Rc}^m = \frac{1}{2\pi} \text{Re} \left\{ \ln \left( \frac{w-\zeta}{w-\xi} \right) + 2\int_0^\infty \frac{e^{-ik(w-\xi)}}{k-k} \, dk \right\} - \left( -1 \right)^m \left\{ \ln \left( \frac{w+\eta}{w+\xi} \right) + 2\int_0^\infty \frac{e^{-ik(w+\xi)}}{k-k} \, dk \right\} \hfill (14)
\]

\[
G_{Rf}^m = \text{Re} \left\{ e^{-ik(w-\xi)} - \left( -1 \right)^m e^{ik(w+\xi)} \right\} \hfill (15)
\]

\( w = y + iz \) is the field point, \( \zeta = \xi + i\eta \) is the source point

The time-independent part of the velocity potential can be calculated through a contour integral of the source singularities of unknown strength along the hull.

\[
\phi^m(y, z) = \oint_{C_x} Q(s) \cdot \frac{d}{ds} G_R^m(y, z; s) \, ds \hfill (16)
\]

where \( C_x \) is the immersed contour at the desired section \( x \).

\( Q(s) = Q_c + iQ_s = \text{Source Density} \) \hfill (17)

the source density is determined by imposing the Kinematic Boundary Condition.

\[
\left( \vec{n} \cdot V \right) \left\{ \oint_{C_x} Q_c(s) G_R^m(y, z; s) \, ds - \oint_{C_x} Q_s(s) G_{Rf}^m(y, z; s) \, ds \right\} = A \cdot \omega \cdot n^m \hfill (18)
\]

\[
\left( \vec{n} \cdot V \right) \left\{ \oint_{C_x} Q_s(s) G_R^m(y, z; s) \, ds - \oint_{C_x} Q_c(s) G_{Rf}^m(y, z; s) \, ds \right\} = 0 \hfill (19)
\]

These integrals can be numerically solved when the hull is subdivided in \( N \) straight line segments and the sources of unknown strength distributed over each segment. Then above integral equations (18) (19) become:
\[
\sum_{i=1}^{N} Q_{ij} I_{ij}^m - \sum_{j=1}^{N} Q_{ij} J_{ij}^m = A \omega \eta_{i}^m
\]
(20)
\[
\sum_{j=1}^{N} Q_{ij} I_{ij}^m - \sum_{j=1}^{N} Q_{sj} J_{sj}^m = A \omega \eta_{j}^m
\]
(21)

where

\[
I_{ij}^m = \left( \vec{n} \cdot \nabla \right) \Phi_{ij} G_{ij}^m \, ds
\]
(y = y_i and z = z_i)
(22)
\[
J_{ij}^m = \left( \vec{n} \cdot \nabla \right) \Phi_{ij} G_{ij}^m \, ds
\]
(y = y_i and z = z_i)
(23)
\[
\vec{n} \text{ at } C_i = (\sin(a_i) - \cos(a_i))
\]
\[
a_i = \tan^{-1} \frac{(\eta_i - \eta_{i+1})}{(\xi_{i+1} - \xi_i)}
\]
(24)

The potential is then numerically approximated through the equation:

\[
\phi_{ij}(y_i, z_i) = \sum_{j=1}^{N} (Q_{ij} I_{ij}^m - Q_{sj} J_{sj}^m) + i \sum_{j=1}^{N} (Q_{ij} J_{ij}^m - Q_{sj} I_{sj}^m)
\]
(25)

where:

\[
I_{ij}^m = \left( \Phi_{ij} G_{ij}^m (y_i, z_i; s) \right) \, ds
\]
\[
J_{ij}^m = \left( \Phi_{ij} G_{ij}^m (y_i, z_i; s) \right) \, ds
\]
(26)

The total force or moment, \( F^m \), is obtained by integrating the pressure given by the linearized Bernoulli equation. The 2D added-mass and damping can be obtained from:

\[
F^m = -a_{mm} \bar{l}(t) - b_{mm} \bar{l}(t) = \left[ -2 \rho \omega \Phi_{cx} \Phi_c(y_i, z_i) \cdot \vec{n} \, ds \right] \sin(\omega t) + \left[ 2 \rho \omega \Phi_{sx} \Phi_s(y_i, z_i) \cdot \vec{n} \, ds \right] \cos(\omega t)
\]
(27)

which leads to:

\[
a_{mm} = \frac{-2 \rho}{A \omega} \Phi_{cx} \Phi_c \cdot \vec{n} \, ds
\]
\[
b_{mm} = \frac{-2 \rho}{A} \Phi_{sx} \Phi_s \cdot \vec{n} \, ds
\]
(28)

The kernel and potential integrals \( I_{ij}^m, J_{ij}^m, I_{ij}^m \) and \( J_{ij}^m \) are evaluated in a similar manner as given in Frank (1967).

The three dimensional coefficients, for the zero speed problem considered in this study, are calculated by integration along the hull length as from Table 1.

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<tr>
<th>( A_{33} )</th>
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<tr>
<td>( \int a_{33} , dx )</td>
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<th>( A_{35} = A_{53} )</th>
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<th>( A_{55} )</th>
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<tr>
<td>( \int x^2 , a_{33} , dx )</td>
<td>( \int x^2 , b_{33} , dx )</td>
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</table>

According standard linear theory the responses are calculated solving the system (2) of two equations in two unknown (\( \eta_3 \) and \( \eta_5 \)) when correctly projected in frequency domain (Lee and Curphey, 1977).

3. VISCOUS SOLUTION OF THE RADIATION PROBLEM

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We supply here the basic details of the Navier-Stokes solver used for the calculation of the viscous added mass and damping of the 2D sections. The method has been first applied and successfully validated for simple singular sections (Bonfiglio et al. 2012) then for twin sections and recently also for SWATH like sections (Bonfiglio et al., 2013) with very good correlation with experimental results.

The conservation law for mass and momentum applied to a Newtonian incompressible fluid leads to the Navier-Stokes equations, that written in a Cartesian reference frame in a non-conservative form:

\[
\frac{\partial (\rho u_i)}{\partial x_i} = 0
\]  

\[
\frac{\partial u_i}{\partial t} + \frac{\partial (u_i u_j)}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \nu \frac{\partial u_i}{\partial x_j} \right) - \frac{1}{\rho} \frac{\partial p}{\partial x_i} + g_i,
\]  

The system is a set of partial differential non linear equations in the unknowns of pressure and velocity components.

A fully non-linear viscous method is proposed to predict the velocity and pressure field through the numerical solution of the Navier-Stokes equations, performed exploiting a finite volume technique in which a collocated arrangement is used to store the values of the unknown dependent variables: pressure and velocities are evaluated for the same control volume at the centroids of each cell.

The structure of the momentum equations (30) suggests to determin the velocity components of the flow. Instead, using the continuity equation (29) that contains only the velocity terms, the pressure is found by the solution of a Poisson equation obtained from the divergence of the momentum equation for an incompressible fluid:

\[
\frac{\partial}{\partial x_i} \left( \frac{\partial p}{\partial x_i} \right) = - \frac{\partial}{\partial x_i} \left[ \frac{\partial (\rho u_i u_i)}{\partial x_i} \right]
\]  

The numerical solution of this equation together with the solution of the velocity equation (momentum equation) is performed in OpenFOAM through the PISO scheme (Pressure Implicit with Splitting of Operators) introduced by Issa (1985).

Since the integrands which compare in the conservation equation are not known over all CV or over all CV faces, approximations and interpolations are required in order to evaluate values in other point of CV faces. The time derivative is numerically solved with a Euler implicit scheme, while the interpolations to determine face values, are performed through the central differential scheme. Spatial derivatives, required for the gradients that appear in the velocity and pressure equations, are evaluated using a linear central differential scheme. The same scheme is used for the interpolation of the viscosity \( \nu \) in the diffusive term \( \nabla (\nu \nabla U) \), where \( \nabla U \) is interpolated through an explicit scheme with non-orthogonal correction (generally used for all surface normal gradients). Convection terms, that depend also on the scalar volume fraction of water in each cell (a scalar representing the quantity of water in each cell), are interpolated using a vanLeer scheme, which is specifically designed for bounded flow fields.

The sharp free surface between air and water is well captured by the volume of fluid approach that permits to solve the Navier-Stokes equations with respect to a single fluid mixture whose density and viscosity depend on the local concentration the two
phases. The concentration is defined by an additional scalar, the volume fraction or indicator function $\alpha$, which is normalized from 0 to 1 all over the domain.

The indicator function can be considered as a time dependent scalar field that is found through the solution of the following conservation and transport equations:

$$\frac{d\alpha}{dt} = 0 \quad \Rightarrow \quad \frac{\partial \alpha}{\partial t} + \nabla \cdot \alpha \mathbf{U} = 0$$

(32)

Particular attention in the discretization and solution of the transport equation is required to fulfill the boundness criteria of $\alpha$ and to preserve the sharpness of the interface. For this purpose, in the present study, the method proposed by Berberovic et al. (2009) is used, that consists in the addition of a correction sharpening term in the transport equation:

$$\frac{\partial \alpha}{\partial t} + \nabla \cdot \alpha \mathbf{U} + \nabla \cdot [U_R \alpha (1-\alpha)] = 0$$

(33)

where $U_R$ is the relative velocity between air and water and the compression term $\nabla [U_R \alpha (1-\alpha)]$ is introduced in order to preserve the sharpness of the free surface. A series of unsteady viscous free surface simulations have been performed.

Once the time history of the radiation force component (in the selected direction) is obtained from each simulation, at each given oscillation frequency (and degree of freedom: heave in this case), the results are projected from time domain to frequency domain applying the Fourier analysis which is used to calculate the real and the imaginary part of the force for the $k$th harmonic:

$$\Re F_{z_1}(k \omega) = \int_0^T F_{z_1}(t) \cos(2 \pi k t/\delta) dt$$

$$\Im F_{z_2}(k \omega) = \int_0^T F_{z_2}(t) \sin(2 \pi k t/\delta) dt$$

(34)

in which $\delta$ represents the forced oscillation frequency.

According a classical linear seakeeping theory the added mass and damping coefficient are obtained from the 1st harmonic obtained by the Fourier analysis, i.e. the force component characterized by the same frequency of the motion. Considering the sinusoidal motion amplitude of $\xi_k$ imposed in the transient simulation, sectional added mass and damping coefficient are expressed as follow:

$$a_{z_2}(\omega_k) = \frac{\Re F_{z_1}(\omega_k)}{\omega_k^2 \xi_k}$$

$$b_{z_3}(\omega_k) = \frac{\Im F_{z_2}(\omega_k)}{\omega_k \xi_k}$$

(35)

4. RESULTS
Following the strip theory approach, the 3D geometry of the SWATH has been represented with 11 sections along the length that are represented in Figure 3. The sections have been selected considering a non-uniform spacing to minimize the number without compromising accuracy (sensitivity analysis tests were done).

As anticipated, radiation forces have been estimated at each section with the two different approaches: potential flow (Frank’s close fit method with 50 points along the section) and the viscous method based on the solution of the transient N-S equation for the oscillating sections in calm water. The numerical simulations have been performed for a scaled model in scale 1:6 to the actual vessel (i.e.: submerged hull length L=1m). The amplitude of the motion has been set to 0.025·L  ≡ 0.0127·T, where T=draft.

Only vertical plane motions are discussed in this paper, i.e. only heave and pitch motions related coefficients. This means that only the problem of the 11 vertical oscillating sections has been considered and solved with the viscous free surface N-S solver. For SWATH vessels in general and in particular for the twin canted struts version considered in this study, viscous effects are important not only for the transverse motions but also for heave and pitch. In fact, for these last motions, wave damping is relatively low or comparable to the viscous forces when far from the resonant frequencies (piston effect).

The nature of viscous effects can be evinced from the snapshots of Figure 4 that presents the vorticity field for three stations along the hull length having different typologies: a thin strut (st.1), a larger strut (st.4) and a fully submerged section (st.6). Vorticity generated by separation of the flow along the struts due to the vertical oscillating motion or in the alternate wake of the elliptic section are noted and principally contribute to the viscous damping and added mass which is neglected by the presented potential flow based method. In case of the first two sections a frequency close to the resonance of the piston effect has been chosen to highlight the complex interaction between vorticity and free surface: vortexes shed by the struts are convected to the free surface and interact with wave induced flow field, enhancing or reducing their intensities. Especially in the case of section 4 a strong non-linear created by the hull portion above the design waterline is noted: this effect is again neglected by traditional linearized seakeeping methods.
The relative importance of viscosity is quite evident when the sectional added mass and damping by the two different methods are compared, as in graphs of Figure 5 which plots their values calculated along the length of the vessel for three different frequencies, among those considered in the study: low, medium and high.

As expected, the added mass of the fully submerged sections is not influenced by viscous effects: the predicted value of $a_{33}$ for the three midship sections is the same for the two methods. Instead, for the sections with struts (four at bow and four at stern), $a_{33}$ increases at relatively low circular frequencies ($\lambda \geq 3L$), while it decreases at relatively high circular frequencies ($\lambda \leq 2L$).

The damping coefficients calculated with the viscous flow solver show the same trend of the inviscid ones, but they are generally higher both for the fully submerged sections, as generally expected, and also for the sections with struts. This is not always the case when the strut is vertical, while the present canted strut design does create additional damping. Around the resonant frequency of the piston mode effect ($\omega = 5.03$), the non-linear effects related to the amplified standing wave in between the two hulls, drastically increase the damping force on the strut sections by about a factor 4 with respect to subcritical or supercritical frequencies: this is evident comparing the end scale of the three graphs for damping.
Figure 4 – Snapshot of the predicted viscous flow field (vorticity) for station 1, 4 and 6 respectively for the heave radiation problem.
Figure 5 - Sectional heave added mass \(\mathbf{a_{33}}\) and damping \(\mathbf{b_{33}}\) calculated with inviscid (LT) and viscous (OF) methods at three different oscillation frequencies plot along ship length.

Figure 6 – Three dimensional (global) added mass and damping calculated with Frank close fit and with Viscous (N-S) flow solver.

When integrated along length according formulae of Table 1, the effects just commented on individual transverse sections become even more evident, as captured in Figure 6: the added mass for the whole hull increases at lower frequencies \((\lambda \geq 3L)\) and decreases at higher frequencies \((\lambda \leq 2L)\); the difference at the peaks amounts to a difference of about 20%. At the same time, the damping is generally increased with relative a difference of about +35÷50% around the peak frequency.

The difference in terms of motion response in regular waves is given in Figure 7, where the R.A.O. of heave and pitch calculated by the two methods are presented. At lower frequencies, a general non-uniform decrease of the predicted heave motion amplitude is noted. The heave motion values predicted with the viscous method are about 15% lower than those predicted by the inviscid strip theory. The difference in the predicted pitch motion is even more important: around the resonance frequency \((\lambda \approx 2.7L)\) the
value predicted by the viscous method is about one half \((1/2!\) of that predicted by the inviscid theory.

![Figure 7 – Response Amplitude Operators of Heave \((\eta_3)\) and Pitch \((\eta_5)\)](image)

5. CONCLUSIONS

A hybrid strip theory method for calculating motions of SWATHs vessels in waves based on fully numerical predictions of viscous radiation forces has been outlined in its principal theoretical and numerical aspects. The method is based on a traditional strip theory method developed at MIT for catamarans and SWATHs where the radiation problem is solved by an unsteady viscous fully non-linear free surface Navier-Stokes solver, built on OpenFoam libraries. Added mass and damping of the hull cross sections are obtained from the time history of the calculated viscous force acting on the oscillating sections by Fourier analysis and used inside the strip theory instead of the inviscid ones. The difference between the hydrodynamic radiation forces predicted by the viscous and the inviscid (Frank’s close fit) methods are important and commented in the paper.

The physical nature and relative importance of viscous effects is critically discussed through the analysis of the predicted viscous flow vorticity field around the different types of sections of the hull.

As a general results, viscosity effects are markedly non-negligible for the prediction of the motions in of the unconventional SWATH considered in this study: the difference in the predicted amplitudes of heave ad pitch motions around the characteristic piston mode resonance frequency of the SWATH is about -20\% and -50\% respectively (viscous against inviscid).

No empirical correction formulae exists or can be developed to accurately predict the effect of these complex non-linear phenomena above briefly described for a general SWATH geometry. From this point of view, the current hybrid (viscous/inviscid) strip theory method offers a unique opportunity to enhance the fidelity of the predicted motions in waves, without the burden of long computational times needed by fully 3D time domain viscous free surface solvers.

Future studies will continue the application, verification and validation of the viscous strip theory method with viscous time domain 3D solvers and possibly also with
experiments on a model of the AUV-SWATH presented in this study, currently under construction at the MIT Innovative Ship (I-Ship) design lab.

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