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The Riddle of Determining the True Tensile strength of Normal and High Strength Concrete

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ABSTRACT

Using extensive test data for both normal and high strength (silica fume) concrete, covering a wide range of types and sizes of aggregates (including lightweight aggregate), rates of loading, ages at testing, with cylinder compressive strengths, f_c, of up to 94 MPa, a relationship has been established between the modulus of rupture and cylinder splitting strengths. By using this empirical relationship in conjunction with the previously reported theoretical correction factor of Raoof and Lin, with the latter relating the true uniaxial tensile strength of concrete to the traditional cylinder splitting strength, very simple relationship(s) between modulus of rupture and the true uniaxial tensile strength have been proposed for both normal and high strength concrete. It is now possible to estimate (using very simple formulas, which are amenable to hand calculations using a pocket calculator) the uniaxial tensile strength of concrete, ft, by either using the cylinder splitting or the modulus of rupture tests, with the agreements between the predictions of f_t by these two different traditional indirect tests being practically reasonable. In particular, it is now shown that, over the range $22 < f_c < 94$ MPa, the traditional modulus of rupture, MOR, is always greater than the corresponding true uniaxial tensile strength, f_t, with the ratio MOR/f_t being dependent on the magnitude of the corresponding cylinder compressive strength. As regards the traditional cylinder splitting strength, f'_t, on the other hand, the ratio f_t/f'_t is dependent on the ratio f'_t/f_c, with the true uniaxial tensile strength, f_t, for associated cylinder compressive strengths within the range $22 < f_c < 94$ MPa, always being greater than the traditional cylinder splitting strength, f'_t. The presently reported work is believed to have come some way to resolve the longstanding problem of determining the true tensile strength of concrete which has eluded researchers since the early years of the 20th century.

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1. INTRODUCTION

A survey of available works on the measurement of the uniaxial tensile strength of concrete (Senussi 2004), demonstrated the considerable disunity on the issue of how to obtain an accurate estimate of the uniaxial tensile strength. In particular, it was found that the three popular (traditional) methods for the measurement of the uniaxial tensile strength (i.e. the direct tensile test, the modulus of rupture test and the cylinder splitting test) are all riddled with complex stress conditions (i.e. shortcomings) which have been shown to lead to significant differences between their associated measured values of tensile strength. By comparison of the resulting estimates of tensile strength obtained using each of the three test methods, and considering the particular shortcomings of each test, it was found that, overall, the direct tensile test underestimates the uniaxial tensile strength, while the modulus of rupture results are invariably significantly higher than the corresponding ones based on the cylinder splitting tests with the test data based on the latter method always being higher in magnitude compared with those, from the same concrete batch, based on the direct tension test. Overall, the general view among researchers appeared to be that the cylinder splitting test gives a closer estimate than the direct tensile test. It can also safely be said that the majority of researchers are aware of the shortcomings in these tests, and research on how to improve the estimate has taken the form of either developing new test methods or improving the methods already in use by, for example, developing empirical or theoretical correction factors in order to improve the accuracy of the measured tensile strength based on a particular test.

None of the other (later) innovations, which have been developed to improve the estimate of the concrete uniaxial tensile strength (e.g. the double punch test, the cube or prism splitting test, or the various improvements on the direct tensile test), have actually been proven to be significant improvements in comparison to any of the above mentioned common (traditional) test methods. In addition, the review also showed that (relatively speaking) there have been few detailed studies into developing correction factors to account for the inaccuracies in the traditional test methods. It was, therefore, concluded that, given the present state of uncertainty regarding the determination of the uniaxial tensile strength of concrete, there is a need for further investigation in this area. Further, it was decided that the methodology adopted should be semi-empirical in nature employing an extensive experimental study of the size and detail that, to the present author's knowledge, had previously never been carried out.

In what follows, Raoof and Lin's (1999) correction factor in conjunction with their general methodology will be used, in order to provide the major clues as regards the approach to be taken in analysing the extensive test database compiled by Senussi (2004) and Raoof et al. (2005-2010) with this leading to the development of practical methods (amenable to simple hand calculations using a pocket calculator) for obtaining true (reliable) values of the tensile strength for normal and/or high strength (silica fume) concrete.

2. RAOOF AND LIN'S CORRECTION FACTOR

Back in 1999, Raoof and Lin argued that using their non-linear finite strip programme, it was then possible to have a fresh look at the validity of the old formula, Eq. (1), for the cylinder splitting test, where

$$f_t' = \frac{2P}{\pi D l} \tag{1}$$

In the above, f_t' = the splitting tensile strength of concrete, P= the external load at splitting failure, D = the diameter of the cylinder, and l = the length of the cylinder.

These authors pointed out that Eq. (1) is based on the assumption of plane-stress conditions and is derived using the linear theory of elasticity, although, in actual practice, the cylinder experiences a plane-strain state of stresses and the splitting of concrete along the vertical diameter of the cylinder occurs under combinations of tension and compression (i.e. multi-axial state of stresses) and not under uniaxial tension with the previously reported work by others (e.g. Gedling et al. (1986)) suggesting that, under compression-tension biaxial stress conditions, concrete tensile strength decreases as the compressive stresses increase.

The non-linear isoparametric finite strip model of Lin et al. (1997) that Raoof and Lin (1999) used for examining the applicability of Equation (1) as regards determination of the concrete uniaxial tensile strength, employed a rather efficient tangent constitutive model under plane-strain conditions for concrete and a triaxial concrete failure envelope (Ottoson 1979, Hseih et al. 1979 and Lin and Raoof 1993a) while employing the concept of smeared cracking. Their formulations for the isoparametric finite strip elements extended the range of applications of the finite strip technique (as previously reported by others) to cases involving elements with non-rectangular and/or curved boundaries and, hence, enabled them to analyse the cylinder splitting problem with reasonable effort. The generality of their computer programme had been previously verified against test data reported by others as regards various characteristics of deep RC beams with simply supported and also fixed end conditions as well as continuous deep RC beams with changes of depth over the supports (Lin and Raoof 1993b, Lin et al. 1997, and Lin and Raoof 1995), and in all cases the correlations between the results based on their computer programme and such test data had been very encouraging. In view of the central role that the findings of Raoof and Lin (1999) play in the developments reported in the following chapters, their results will be briefly mentioned here: this will, then, enable the interested reader to properly understand (and appreciate) the underlying reasons for the work reported in what follows. In an initial study, Raoof and Lin (1999) used the same size and concrete material properties for the cylinder as previously adopted by Chen and Suzuki (1980) who used a non-linear finite element method for this same problem, whereby the diameter of the cylinder, d, was assumed to be equal to 152.4 mm, with the cylinder laid horizontally in-between the compressive loading platens of the testing machine. The externally applied transverse compressive load was, then, reasonably assumed to be uniformly

distributed over a width of 12.7 mm (equal to the width of the strip of packing material (= d/12)), Figure 1.

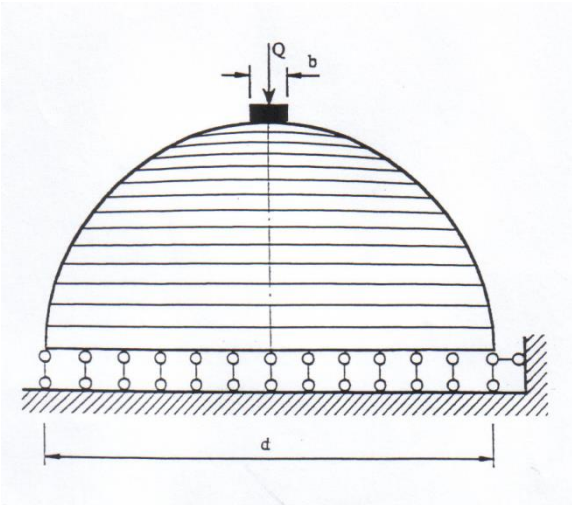

Fig. 1 Sub-division of the cylinder into finite strips-after Raoof and Lin (1999)

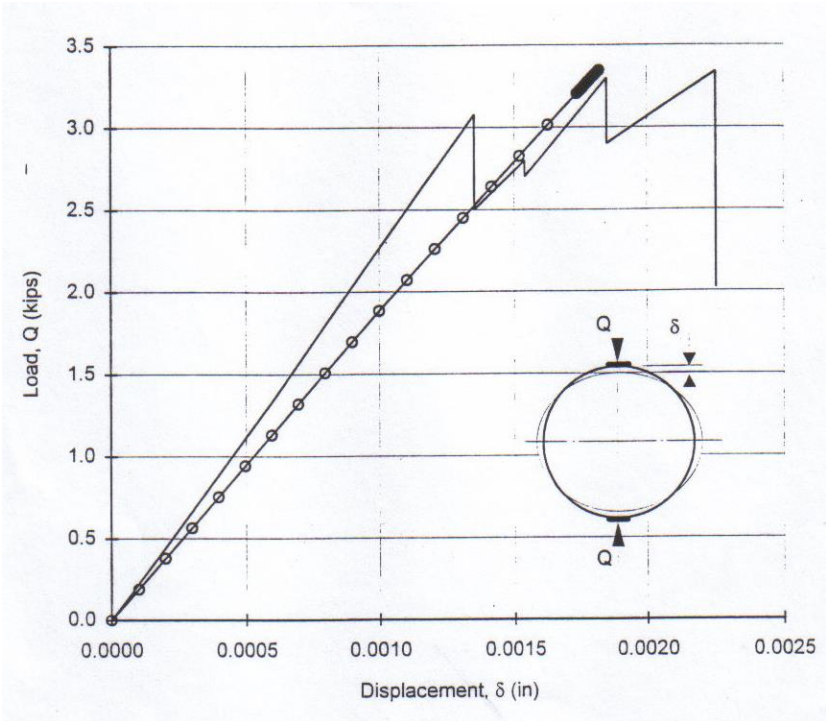

Fig. 2 Plot of Q versus δ compared with that after Chen and Suzuki (1980)-after Raoof and Lin (1999).

Figure 2 presents the plots of normal displacements, δ, versus variations of the associated vertical compressive load, Q, based on both Chen and Suzuki's (1980) finite

element analysis (shown as a continuous line in the figure) and also Raoof and Lin's (1999) approach (shown as a line intersected by small circular points in the figure): both approaches were found to give rather similar predictions of the ultimate load although, unlike Raoof and Lin's (1999) approach which predicted a truly brittle type of failure, the alternative plot of Chen and Suzuki exhibited rather noticeable numerical variations close to failure. In their work, Chen and Suzuki used triangular finite elements with elastoplastic fracture and a displacement-control procedure. Figure 3, on the other hand, presents variations of the horizontal tensile stress, σ_x, along the vertical diameter of the quarter portion of the cylinder for different stages of external loading (i.e. changes in the magnitude of Q) up to failure, with the numerical data based on the non-linear finite strip method.

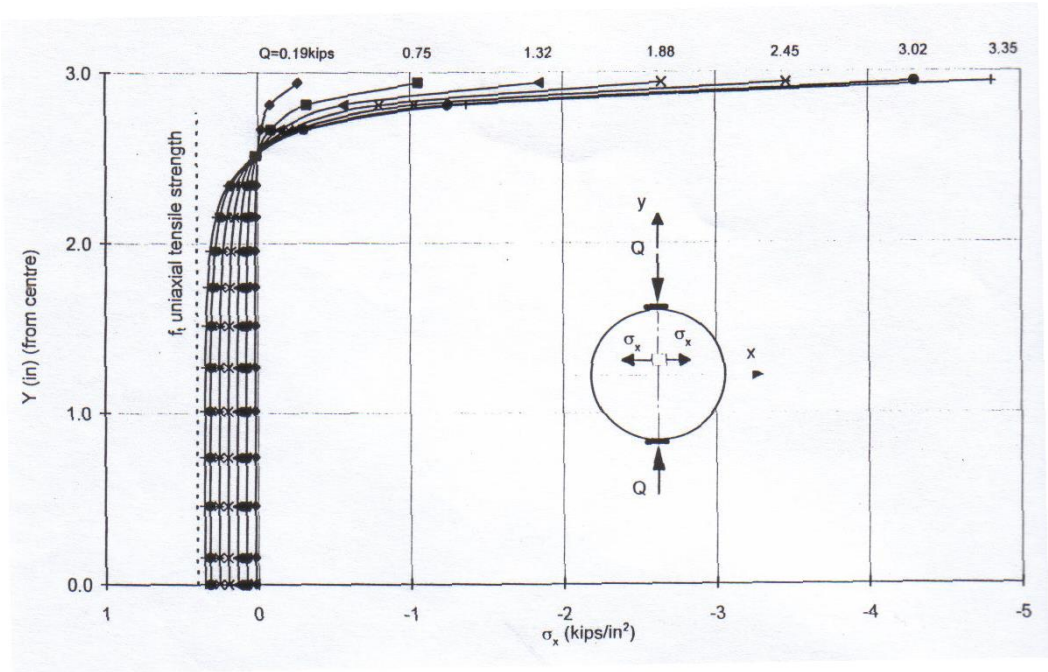

Fig. 3 Patterns of horizontal tensile stresses along the diameter of the cylinder associated with changes in Q up to failure-after Raoof and Lin (1999).

It is most interesting that, as pointed out by Raoof and Lin (1999), the uniaxial tensile strength of concrete along the vertical diameter of the cylinder (shown as a dashed line) is not found to have been reached at failure. Substituting the ultimate load, as predicted by the present numerical (finite strip) method, into Eq. (1) was found to lead to a value (based on the simple traditional formula) of concrete tensile strength, f_t', which was significantly different from the initially assumed true value of uniaxial tensile strength, with the latter used as an initial input into the finite strip computer programme. This, Raoof and Lin (1999) attributed to the fact that the concrete split occurs along the vertical diameter of the cylinder under combinations of tension and compression (i.e. under a multi-axial state of stresses) and not under uniaxial tension, with the ratios of

normal compressive stresses to the horizontal tensile stresses being sufficiently high to make a difference- an important point noted but not quantified by Wright (1955) at the early years of the cylinder splitting test method.

Linear-elastic computer runs using the finite strip method were also carried out for both cases of plane-stress and plane-strain, where the patterns of elastic stress distributions, along the vertical diameter of the cylinder, were obtained: the final numerical results for both the disc (i.e. plane-stress case) and also the cylinder (i.e. plane-strain case) were found to almost exactly match the predictions based on available closed-form elastic distribution solutions for both the horizontal and vertical stresses σ_x and σ_y, respectively, along the vertical diameter for cases of either poisson's ratio v = 0.2 for both the disc and/or the cylinder or when v = 0. In other words, at least for the linear elastic case, it was found that it does not matter as to whether one treats the problem as a plane-stresss (disc) or a plane-strain (cylinder) case in relation to the predicted values of σ_x and, σ_y all along the vertical diameter.

Other numerical studies by Raoof and Lin (1999) suggested that the non-linear nature of the assumed constitutive relations under multi-axial states of stresses are of little importance as far as the cylinder splitting problem is concerned and the controlling factor is, indeed, the assumed criteria used for the final failure of concrete. In the multi-axial crushing and/or cracking mode(s), in their non-linear isoparametric model, Raoof and Lin (1999) had used the concrete failure envelope after Hseih et al. (1979).

Based on theoretical parametric studies, using their non-linear isoparametric finite strip model, on a cylinder with diameter d = 150 mm (which is in accordance with the, then, British Standard BS 1881: Part 117: 1983) with four different widths of packing strips equal to 12, 13, 14 and 15 mm, and guided by the above observations, Raoof and Lin (1999) showed that for given values of cylinder diameter and width of the packing strip, the ratio of concrete uniaxial tensile strength to cylinder splitting tensile strength, f_t/f_t', is very nearly a sole function of variations in the ratio of f_t/f_c where f_c = cylinder compressive strength: Figure 4, for example, presents plots of $y = f_t/f_t'$ against $x = f_t/f_c$ for a packing strip width of 15 mm and $0.085 \leq f_t/f_c \leq 0.14$.

From plots such as the one presented in Figure 4, for each width of packing strip, the relationship between f_t/f_t' and f_t/f_c was found to be almost exactly linear, with the fitted straight line being defined by the following expression:

$$\frac{f_t}{f_t'} = a\frac{f_t}{f_c} + b \qquad (2)$$

where, the values of the constants a and b in Eq. (2), for each value of packing strip width, within the range from 12 to 15 mm, are given in Table 1.

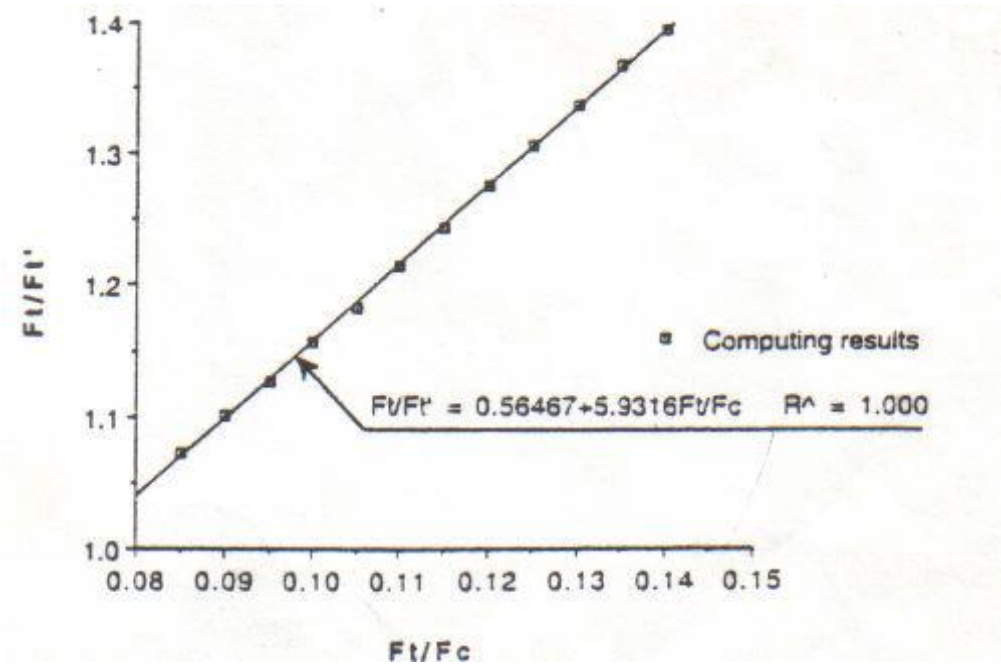

Fig. 4 Determination of the correction factor, k, for a width of strip packing = 15 mm- after Raoof and Lin (1999).

Finally, by simple algebraic manipulations, Eq. (2) was re-written as:

$$f_t = kf_t^{'} \tag{3}$$

where, the correction factor, k, is:

$$k = \frac{b}{1.0 - a\dfrac{f_t^{'}}{f_c}} \tag{4}$$

Raoof and Lin (1999), then, suggested that with f_c and Q determined by the cylinder splitting test, Eq. (1) should be used to calculate f_t'. For a given width of packing strip, Table 1 gives values of a and b. Eq. (4) may then be used to calculate the correction factor, k, which will, finally, enable one to estimate the true uniaxial tensile strength $f_t = kf_t^{'}$.

Raoof and Lin (1999) argued that the theoretical values of the correction factor, k, over the full range of packing strip widths studied (i.e. 12-15 mm) vary (for a given ratio of f_t/f_c) by only a maximum difference margin of 6%, and considering the usual wide scatter problem associated with concrete as a material, therefore, it was suggested that one may, in practice, reasonably use the so-called general values of constants a and b

Table 1 Constants a and b for different widths of packing strip, Raoof and Lin (1999).

Packing strip width (mm)	12	13	14	15
a	7.0566	6.6969	6.301	5.9316
b	0.4937	0.5148	0.5398	0.5647

relating to only a packing strip width of 15 mm which (incidently) is the one recommended by the British Standard BS 1881: Part 117 (1983). It is, also, perhaps, worth mentioning that although in Raoof and Lin's (1999) study the external load was reasonably assumed to be uniformly distributed over the full width of the packing strip(s), in accordance with the Hertzian contact stress theory, the normal stresses at either edge of the packing strip are zero. However, in view of the fact that tensile cracking along the vertical diameter of the cylinder occurs away from the points of application of the external load(s), the exact form of the assumed distribution of the external normal stresses over the packing strip is not believed to have a significant influence on the estimated cylinder splitting strength.

Moreover, it is interesting to note that, according to the studies of Raoof and Lin, the ratio of the compressive/tensile stresses within the central (i.e. critical) portion of the cylinder is (as one approaches the predicted failure load) about 3-3.5 which is in line with the corresponding prediction based on linear elastic theory (as applied to a disc), with the latter approach predicting this ratio to be equal to 3. According to well-established experimental observations (e.g. Gedling et al. 1986), the tensile strength of plane concrete subjected to a biaxial tension-compression state of stresses with such associated ratios of external compression/tension will be significantly reduced (cf. the corresponding uniaxial tensile strength): it is, therefore, not surprising that the traditional formula for the cylinder splitting test (i.e. Eq. (1)) must (for obtaining a reasonable estimate of the true concrete tensile strength) be multiplied by a correction factor, k, as suggested by Raoof and Lin (1999) with the theoretical values of, k, (for $20 \leq f_c \leq 50$ N/mm^2) being as high as, say, 1.4, Figure 4.

3. RAOOF AND SENUSSI SEMI-EMPIRICAL MODEL

3.1 Background

It is not possible to verify directly the conclusions of the theoretical study of Raoof and Lin (1999) using experimental data, because only two out of the three strength parameters which appear in Eq. (2) (i.e. f_c, and f_t') can be experimentally measured. If the modulus of rupture (MOR) is substituted into Eq. (2) for the true uniaxial tensile strength, f_t , the resulting equation becomes:

$$\frac{MOR}{f_t^{'}} = a\frac{MOR}{f_c} + b \qquad (5)$$

As a plausible suggestion (at least on a theoretical basis), Eq. (5) indicates that if values of the ratio MOR/f_t' are plotted on the y–axis against the corresponding values of the ratio MOR/f_c (on the x-axis), the resulting relationship should be a straight line with a gradient of 'a' and an intercept (on the y-axis) equal to 'b'. Based on this premise, then, the relationship (in practice) between the three strength parameters, MOR, f_t', and f_c, as defined by Eq. (5), will be reasonably verified in what follows. This will be done using the extensive experimental database created by the present authors which includes the wide ranging test data generated as part of the PhD work of the second Author (Senussi 2004) as well as the first Author and his undergraduate students (Raoof et al. 2005-2010), combined with certain previously reported test data by others (Akazawa 1953, Walker and Bloem 1960, and Grieb and Werner 1962).

3.2 Relationship between MOR/f_c and MOR/f'_t

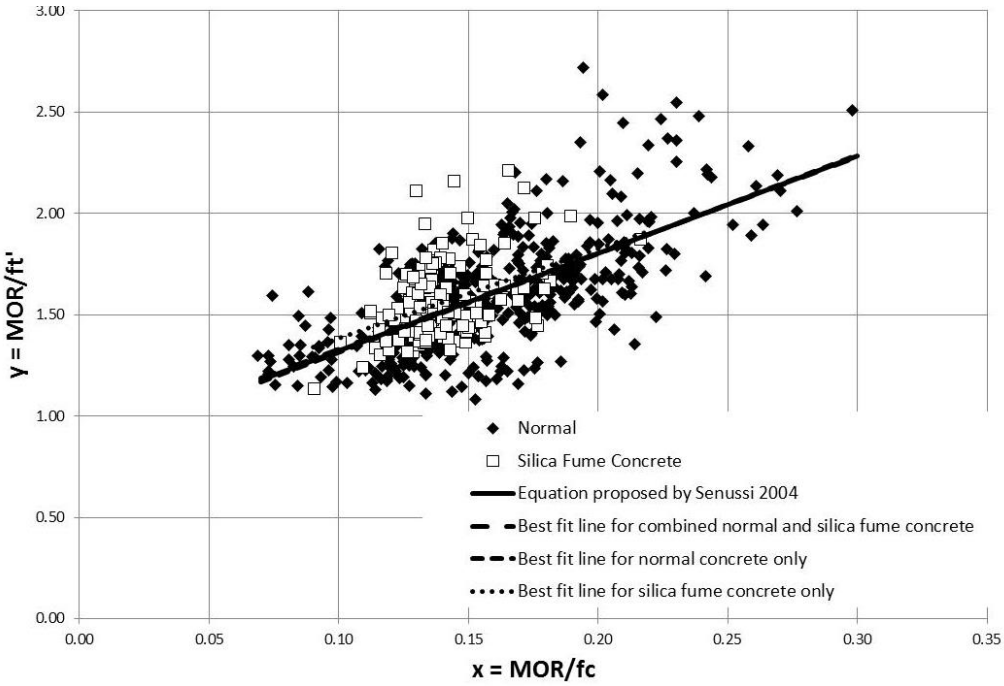

Fig. 5 Relationship between MOR/f_c and MOR/f_t'

Figure 5 presents plots of the measured values of MOR/f_c versus MOR/f_t' associated with each of the set of data for normal (510) and silica fume (153) concrete batches in the above mentioned test database, where for each given batch, test data was obtained for the cylinder and cube compressive strengths (f_c and f_{cu}, respectively) as well as the cylinder splitting and modulus of rupture tensile strengths (f'_t and MOR, respectively).

As demonstrated in considerable detail, elsewhere (Senussi 2004), strictly speaking, the plots of MOR/f'_t versus MOR/f_c are somewhat dependent on the rate of loading and the age at testing, with the type and size of the aggregates in the concrete mix not

having a practically significant influence on this relationship. For practical design purposes, despite the mild dependency of this relationship on the specific type of concrete mix design and testing procedures adopted in practice, a unified form of this relationship may be proposed which corresponds to the best fitted line through all the available test data (based on a total of 663 different concrete batches) irrespective of the specific type of concrete mix design, age at testing and the testing procedures adopted in practice (which can be according to different national or international standards).

It is, perhaps, worth mentioning that the test data from the present authors' experiments (Senussi 2004 and Raoof et al. 2005-2010) covered different types and sizes of aggregates (10 and 20 mm), a wide range of rates of loading and different ages at testing (7, 14 or 28 days), with the specimens produced in accordance with British Standards and the results covering a wide range of cylinder compressive strengths from 12 MPa to 94 MPa, while the presently used extensive results from the literature not only cover an even wider range of types and sizes of aggregates (from 10 to 64 mm), but also relate to ages at testing from 7 to 365 days, and even data for air-entrained concrete, with the specimens having been produced in accordance with, for example, the old ASTM and Japanese Standards. It is, therefore, perhaps not surprising that (bearing in mind the very wide range of various variables covered by the presently used test database) there is a significant scatter of data about the best fitted line (defining the relationship between the modulus of rupture and the cylinder splitting strength). Based on the fitted line in Figure 5, a correlation factor, k_1, may, then, be proposed with k_1 being a function of the cylinder compressive strength, f_c, and the cylinder splitting strength, f_t', where the modulus of rupture:

$$MOR = k_1 f_t' \tag{6}$$

with the factor k_1 given by:

$$k_1 = \frac{b'}{1.0 - a' \dfrac{f_t'}{f_c}} \tag{7}$$

In the above, the values of the constants a' and b' are determined from the line of best fit through the combined (i.e. normal plus silica fume) data in Figure 5 – i.e. a' = 4.8147 and b' = 0.8408 (Senussi 2004).

3.3 Determination of f_t from MOR

As already discussed, Raoof and Lin (1999) have theoretically shown how the uniaxial tensile strength is related to the cylinder splitting strength. In this section, the outcome of the above will be employed to demonstrate as to how the uniaxial tensile strength is related to the modulus of rupture (MOR).

Combining Eq. (3) and (6):

$$\frac{MOR}{f_t} = \frac{k_1}{k} \tag{8}$$

where, f_t=uniaxial tensile strength of concrete and MOR= modulus of rupture with the factors k and k_1 given by Eq. (4) and (7).

For the present purposes, it will be assumed that, irrespective of the exact value of packing strip width, a = 5.9316 with b = 0.5647, and from the equation of the best fitted straight line by Senussi (2004), Figure 5, a' = 4.8147 with b' = 0.8408.

It should be noted that the values of a and b are based on the purely theoretical work of Raoof and Lin (1999), while the presently quoted values of a' and b' are empirical in nature. Using Equation (8), the ratio MOR/f_t was calculated for each of the 510 batches of test data from Senussi (2004), Raoof et al (2005-2010), Akazawa (1953), Walker and Bloem (1960), and Grieb and Werner (1962), by using the measured values of f_t' and f_c as well as the theoretical values of a and b and the empirical values of a' and b'. In Figure 6, the so-obtained values of MOR/f_c are plotted against the corresponding measured values of the cylinder compressive strength, f_c.

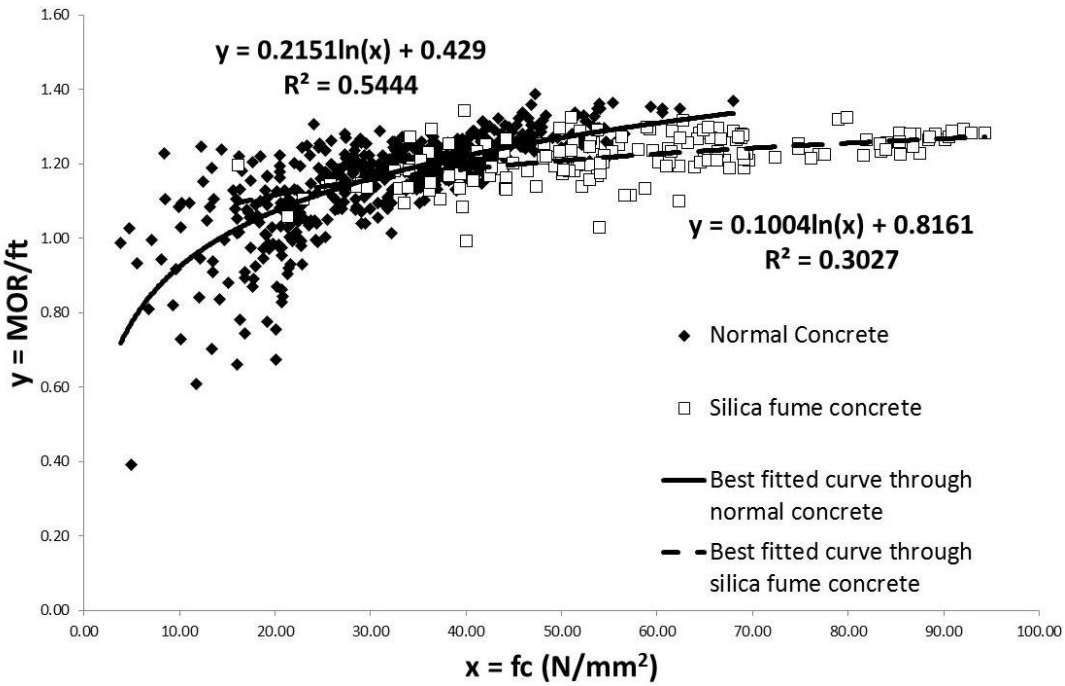

Fig. 6: The best fitted curves through the MOR/f_t versus f_c plots for all of the test data.

The two fitted curves through the data in Figure 6 (one through the normal and the other through the silica fume concrete data) demonstrate that there is a clear trend in the relationship between the ratio MOR/f_t and the cylinder compressive strength, f_c. In

both cases, the ratio MOR/f_t is found to reach a somewhat constant plateau for sufficiently high values of the cylinder compressive strength, f_c, and decreases as the cylinder compressive strength decreases, with the scatter in the data increasing significantly with decreasing values of f_c. The plots in Figure 6 suggest that logarithmic curves may be fitted through the results for both normal as well as silica fume concrete.

An examination of the results in Figure 6 suggests that, for sufficiently low compressive strengths, there is a high degree of scatter in the associated values of MOR/f_t: this has been found (as discussed in section 3.4) to be true especially in connection with those cases where the values of cylinder compressive strengths are less than, say, 22 MPa. Perhaps, it is also worth pointing out that the range of cylinder compressive strengths, f_c, used in the theoretical parametric studies of Raoof and Lin (1999) was 20 MPa ≤ f_c ≤ 50 MPa, and their proposed correction factors k have only been theoretically verified within this range of f_c: the validity (or otherwise) of Eq. (4) (for determining k) outside the range 20 MPa ≤ f_c ≤ 50 MPa has not been previously investigated by either theory or experiments.

Bearing the above comments in mind, the plots in Figure 6 are reproduced in Figure 7, with the proviso that in Figure 7 the data points with corresponding cylinder compressive strengths less than 22 MPa have been excluded.

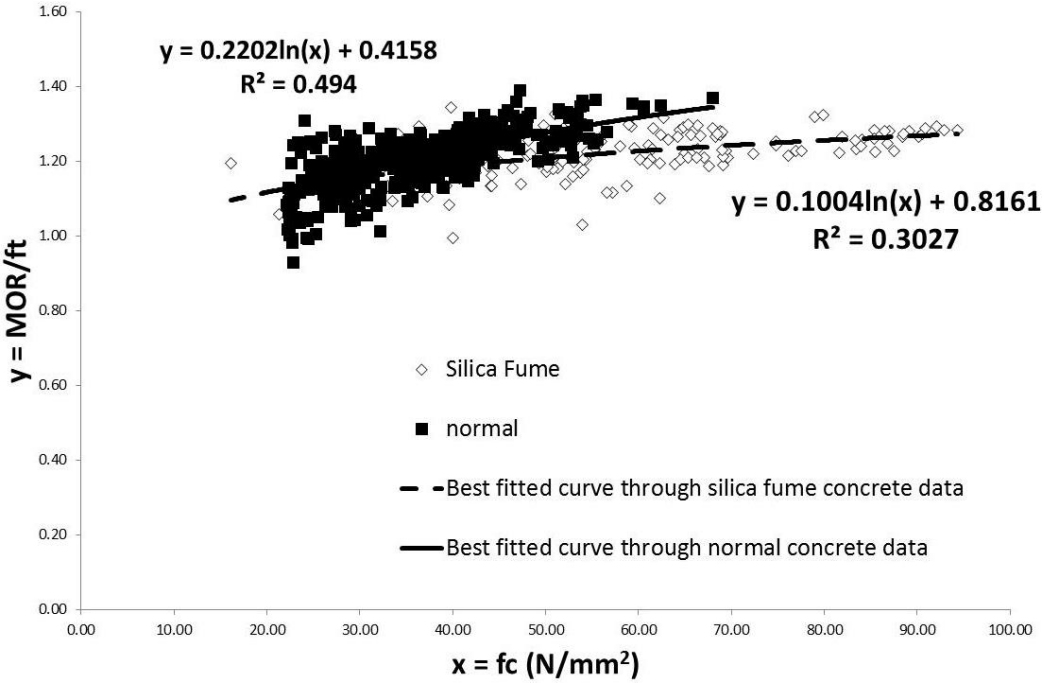

Fig. 7 The best fitted curves through the data after removal of those points with f_c ≤ 22N/mm^2

As far as the fitted curves through the data in Figure 7 are concerned, for the silica fume data, the fitted curve is found to remain the same as the corresponding one in Figure 6, with the same equation defining this curve. As for the normal concrete data, the original equation defining the fitted curve (in Figure 6) is now found to have changed slightly (but noticeably) after removal of the data for $f_c \leq 22$ N/mm^2

3.4 Comparison of the two alternative predicted values of f_t

Based on the presently reported developments, for each of the two distinctly different types of concrete (i.e. normal and silica fume) there are two (alternative) methods which may be used to calculate the true uniaxial tensile strength, depending on which of the two different indirect methods of cylinder splitting and modulus of rupture are to be used. The first method (which will be referred to as method 1) involves measuring both the cylinder splitting and the cylinder compressive strengths and by using Eq. (4) with a = 5.9316 and b = 0.5647 calculating k and hence f_t. This method was first proposed by Raoof and Lin (1999) and is purely theoretical in origin: the outcome based on this will be referred to as f_{tc}. The second method (method 2), on the other hand, involves measuring both the modulus of rupture and the cylinder compressive strength and, depending as to whether the concrete mix is of a normal or silica fume type, Eq. (9) or Eq. (10), respectively, are used to calculate f_t,: the outcome for this will, then, be referred to as f_{ta}, where

For normal concrete:

$$f_t = \frac{MOR}{0.2202 \ln f_c + 0.4158} \tag{9}$$

For silica fume concrete:

$$f_t = \frac{MOR}{0.1004 \ln f_c + 0.8161} \tag{10}$$

3.5 Practical limits of methods 1 and 2

Figures 8 and 9 present the correlations between the estimated values of f_{tc} and f_{ta} for the data associated with all the individual test batches in the previously mentioned database, with Figure 8 showing data for all the original (unscreened) batches and the screened plots in Figure 9 relating to only those concrete batches with $f_c > 22$ MPa. An examination of the data in Figure 8 suggests that the points generally lie around the 45 degree line, although there is much scatter, with the greatest degree of scatter associated with the data points for normal concrete from the literature (Senussi 2004). Some of the data are, however, too far out: upon detailed investigation, it was found that the vast majority of the grossly erroneous points corresponded to concrete mixes with cylinder compressive strengths below a value of about 21-23 MPa. Bearing this in mind, Figure 9, then, presents similar plots to those in Figure 8 with the provision that in Figure 9 all the potentially undesirable data with $f_c \leq 22$ MPa are deleted from the database – hence, the suggested range of concrete cylinder compressive strength

within which the presently proposed correction factors (for both normal and silica fume concrete) are valid is $22 < f_c \leq 94$ MPa.

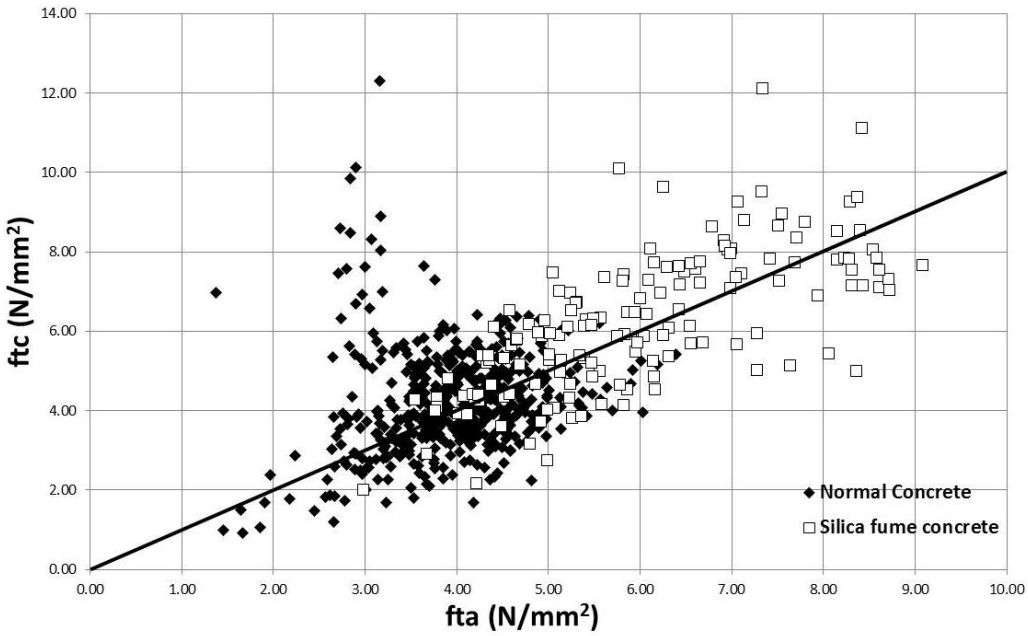

Fig. 8 Correlations between the estimated values of uniaxial tensile strength based on methods 1 and 2, f_{tc} and f_{ta}, respectively, relating to all the test data from the present investigation as well as all the test data from the literature

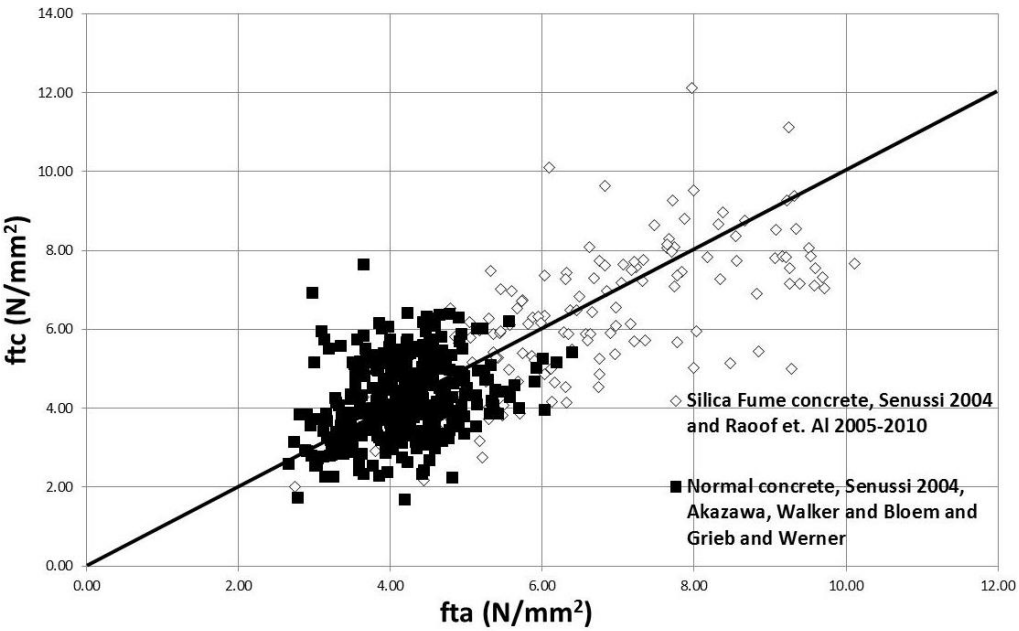

Fig. 9 Correlations between the estimated values of uniaxial tensile strength based on methods 1 and 2, f_{tc} and f_{ta}, respectively – $f_c > 22 N/mm^2$

In reference to the plots of f_{tc} versus f_{ta} as presented in Figures 8 & 9, where the data forms a cluster, Figures 10-12 present histograms for the f_{ta}/f_{tc} ratios associated with the silica fume data, the normal concrete data with f_c > 22 MPa, and the combined normal and silica fume concrete data with f_c > 22 MPa from the database, respectively. Most importantly, Table 2 presents, assuming a normal distribution, the Means and Standard Deviations for the histograms in Figures 10-12, where the Mean values for the histograms are 1.062, 1.025, and 1.035, respectively, which is quite remarkable. Furthermore, using a confidence interval of 95%, for each histogram in Figures 10 – 12 one may delete any of the data which lies outside of the range 'Mean ± 2 Standard Deviation' (i.e. the outliers), and recalculate the new Means and Standard Deviations of the so-obtained screened data: it is most interesting that, as shown in Table 2, in the absence of outliers, the Mean values for the amended histograms have then turned out to be 1.030, 1.007 and 1.013 for silica fume concrete, normal concrete and combined silica fume and normal concrete, respectively, which is extremely encouraging. It is also noteworthy that the Standard Deviations associated with the Mean values as given in Table 2 are well within the practically acceptable range for concrete as a material. This, then, reinforces the fact that, using the presently proposed simple approaches, the predictions of concrete uniaxial tensile strength based on the two alternative indirect tests (i.e. the cylinder splitting and modulus of rupture) are (for all practical purposes) rather close.

Finally, in view of the reasonable closeness of the trend of the data in the plots of MOR/f_t vs f_c in Figure 7, in Figure 13 rather than two separate fitted curves, a unified (single) curve has been fitted through all of the normal and silica fume concrete data with the Equation defining the curve being:

$$f_t = \frac{MOR}{0.1243 \ln f_c + 0.7488} \tag{11}$$

A careful examination of the results presented in Table 3, strongly suggests that when one uses the unified Eq. (11) in preference to Eq. (9) and (10), the so-obtained Means and Standard Deviations of the resulting histograms, in both the presence and the absence of outliers are almost identical to the corresponding ones in Table 2 – hence, the suggestion by the present authors for the adoption of the unified curve as defined by Eq. (11) for use in design.

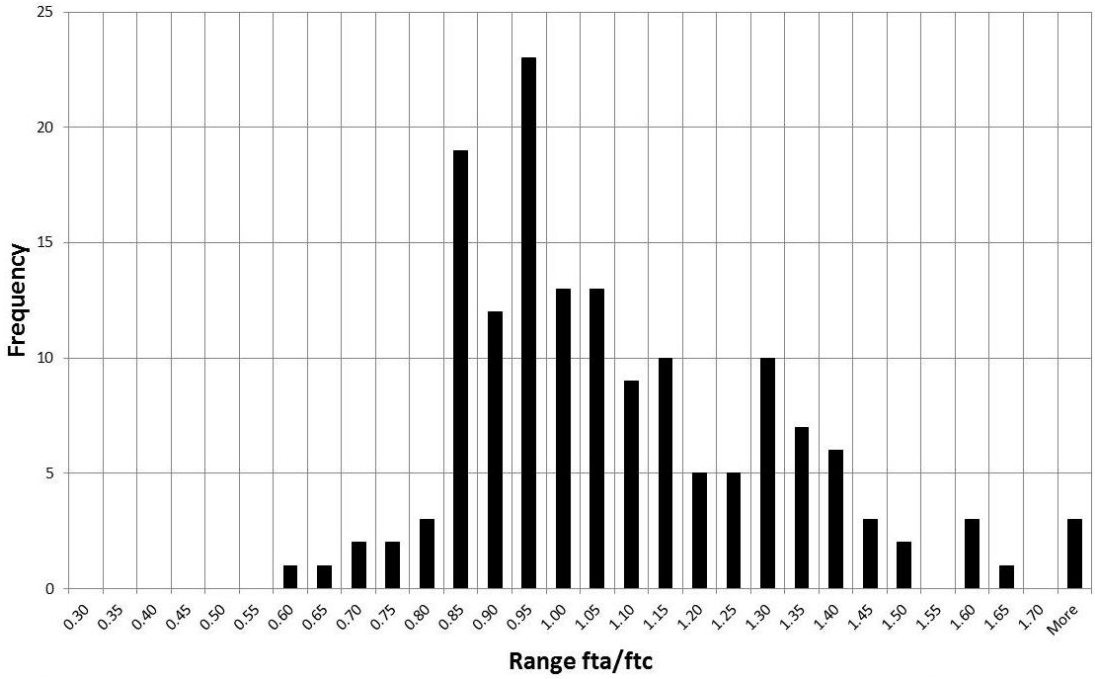

Fig. 10 Histogram for the ratios of f_{ta}/f_{tc} for silica fume concrete with Eq. (10) used for f_{ta} – total number of data = 153.

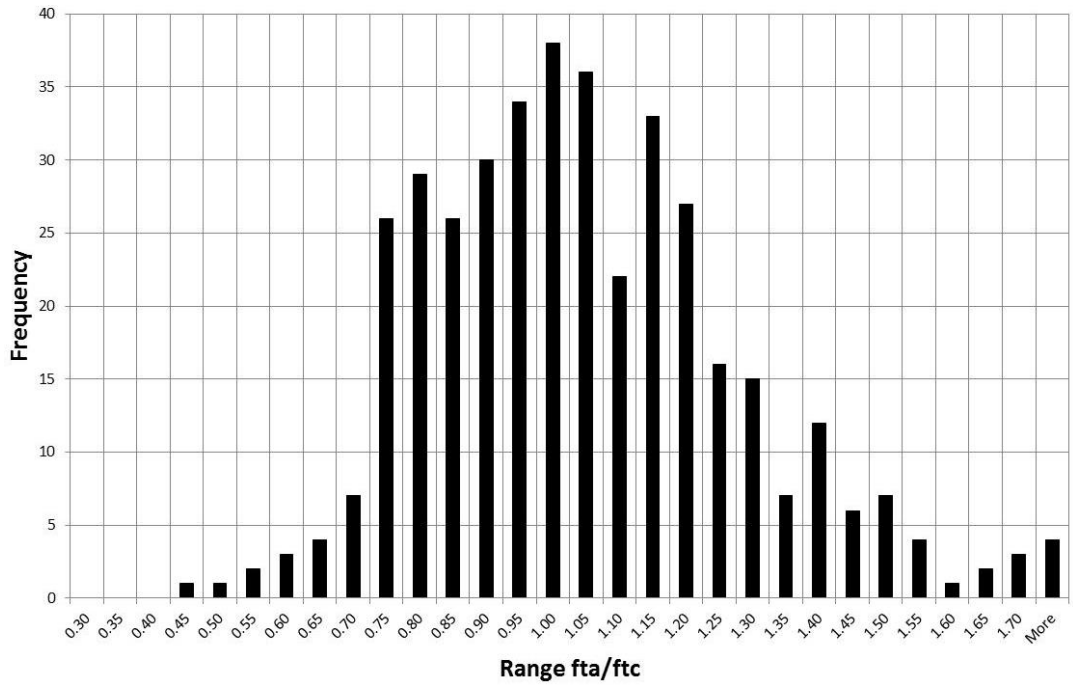

Fig. 11 Histogram for the ratios of f_{ta}/f_{tc} for normal concrete, with Eq. (9) used for f_{ta} – total number of data = 396.

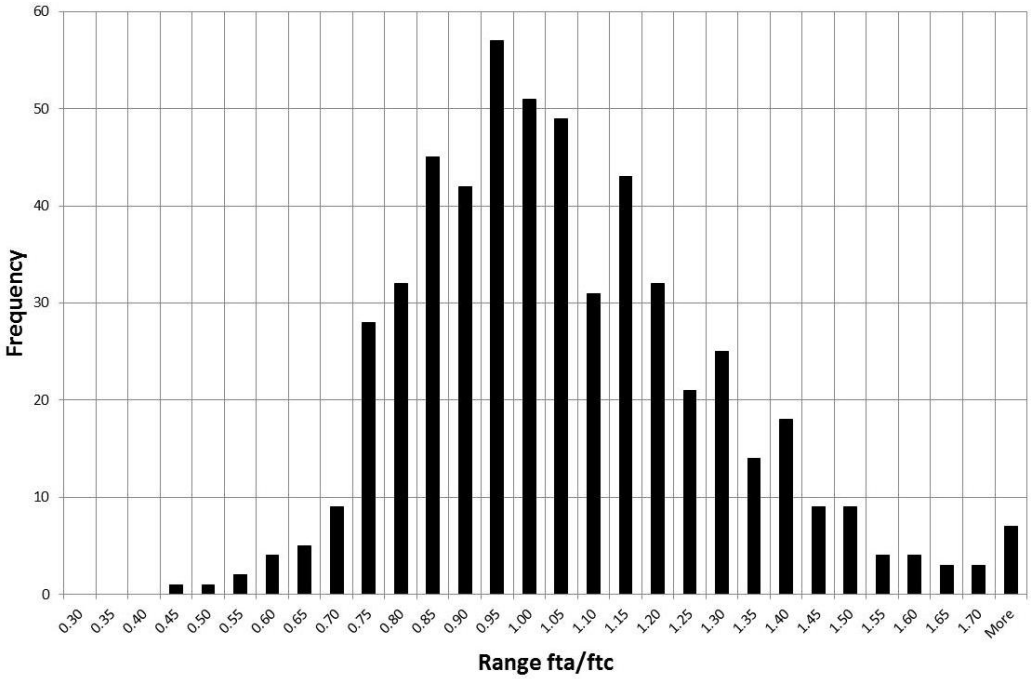

Fig. 12 Histogram for the ratios of f_{ta}/f_{tc} for both normal and silica fume concrete, using Eq. (9) and Eq. (10) for f_{ta}, respectively – total number of data = 549.

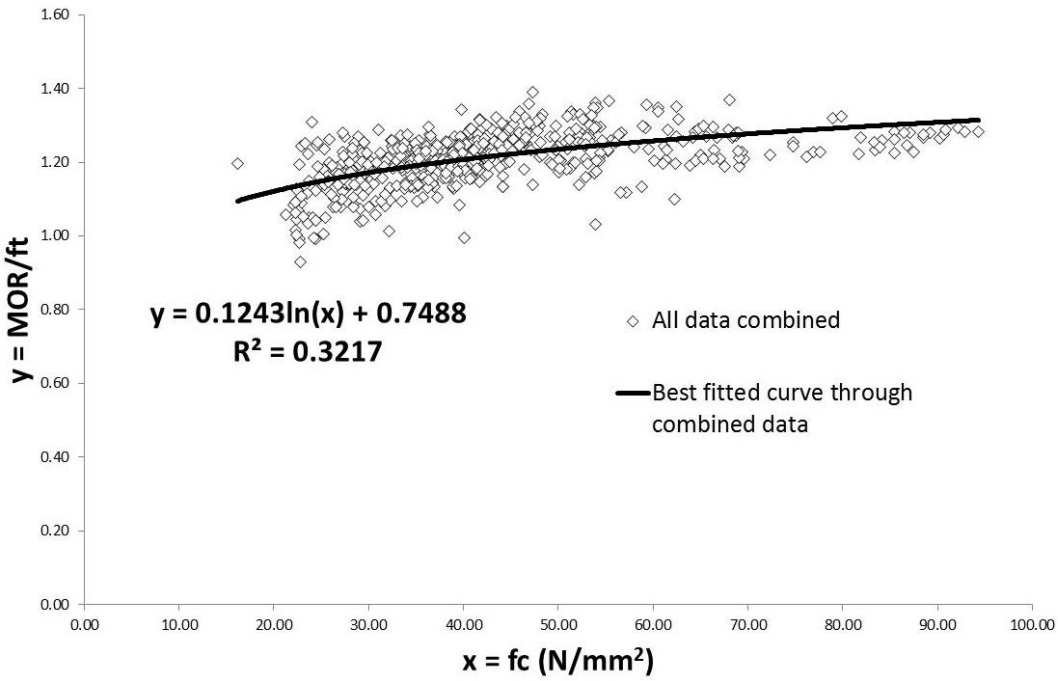

Fig. 13 Unified fitted curve through all normal, with f_c > 22 MPa, and silica fume concrete data

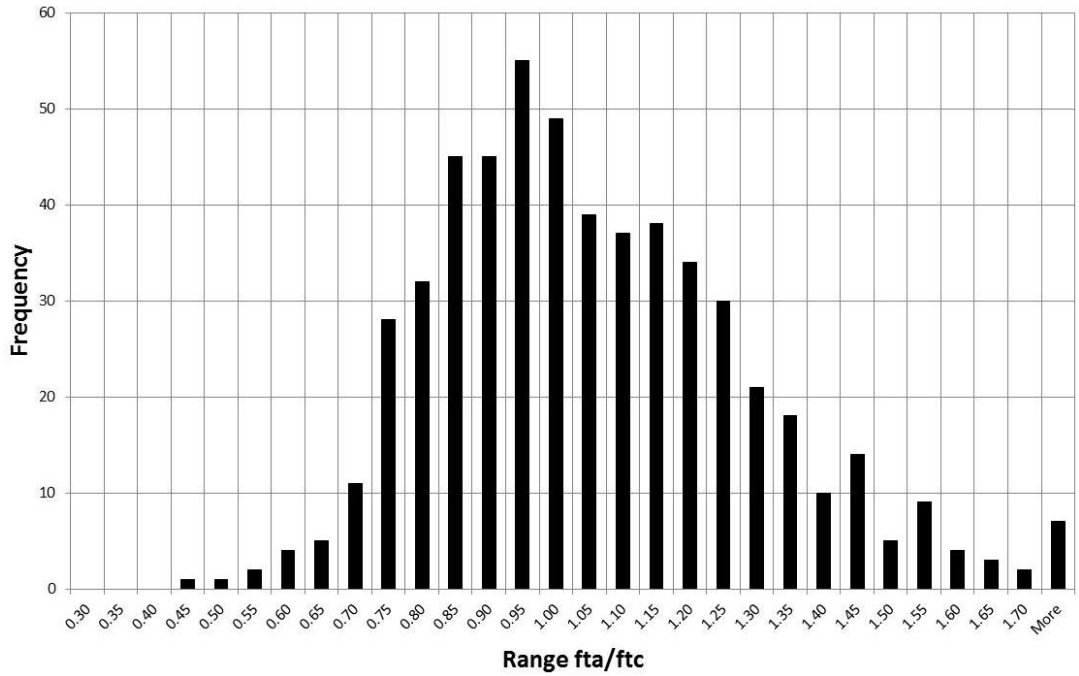

Fig. 14 Histogram for the ratios of f_{ta}/f_{tc} associated with the unified equation for f_{ta} – total number of data (normal plus silica fume concrete) = 549.

Table 2: Values of mean and standard deviation for the histograms in Figures 10-12.

Type of data (f_{ta}/f_{tc})	with outliers			without outliers		
	Total Population	Mean	Standard deviation	Mean	Standard deviation	No. of outliers
silica fume concrete, Eq. (10)	153	1.062	0.241	1.030	0.193	7
normal concrete (with f_c > 22 Mpa), Eq. (9)	396	1.025	0.250	1.007	0.207	20
silica fume and normal concrete (with f_c > 22 Mpa), Eq. (10) and (9) respectively	549	1.035	0.248	1.013	0.203	27

Table 3: Recalculated values of mean and standard deviation using a unified equation to calculate f_{ta} from both the normal and silica fume concrete mixes

Type of data (f_{ta}/f_{tc})	with outliers			without outliers		
	Total Population	Mean	Standard deviation	Mean	Standard deviation	No. of outliers
silica fume concrete, Eq. (11)	153	1.037	0.235	1.010	0.192	6
normal concrete (with f_c > 22 Mpa), Eq. (11)	396	1.034	0.254	1.016	0.211	17
silica fume and normal concrete (with f_c > 22 Mpa), Eq. (11)	549	1.035	0.248	1.014	0.205	23

4. SUMMARY AND CONCLUSIONS

Back in 1999, a survey of available work, on the measurement of the uniaxial tensile strength of concrete, demonstrated the considerable disunity on the issue of how to obtain an accurate estimate of the uniaxial tensile strength. In particular, it was found that the three popular (traditional) methods for the measurement of the uniaxial tensile strength (i.e. the direct tensile test, the modulus of rupture test and the cylinder splitting test) are all riddled with complex stress conditions (shortcomings) which have been shown to lead to significant differences between their associated measured values of tensile strength. Based on the findings of others (as also confirmed by the work of the present authors), it was clear that, overall, for the same batch of concrete, modulus of rupture measurements, MOR, are invariably higher than the corresponding ones based on the cylinder splitting tests, f_t', with the measurements based on the direct tensile test being even lower than those obtained by the cylinder splitting method. The general view among researchers appeared to support the notion that the cylinder splitting test gives a closer estimate than the direct tensile test. It can also safely be said that the majority of researchers are aware of the shortcomings in these tests, and research on how to improve the estimate had taken the form of either developing new test methods or improving the methods already in use by, for example, developing empirical or theoretical correction factors in order to improve the accuracy of the tensile strength based on a particular test. It is, however, noteworthy that none of the later innovations, which have been developed to improve the estimates of the concrete uniaxial tensile strength (e.g. the double punch test, the cube or prism splitting test, or the various improvements on the direct tensile test), have actually been proven to be significant improvements in comparison to any of the three existing (traditional) test methods. In addition, the review also showed that (relatively speaking) there have been few detailed studies into developing correction factors to account for the inaccuracies in the traditional test methods. It was, therefore, concluded that, given the present state of uncertainty regarding the determination of the uniaxial tensile strength of concrete, there is a need for further investigation in this area. Further, it was decided that the methodology adopted should be semi-empirical in nature employing an extensive experimental study of the size and detail that, to the present authors' knowledge, had previously never been carried out.

Bearing the above in mind, using the results based on an extensive series of wide ranging tests carried out by the present authors (which included preparation and testing of nearly 4800 specimens), as well as extensive test data reported by others, a relationship has been established between the modulus of rupture, MOR, and the cylinder splitting, f_t', strengths. For practical design purposes, despite the mild dependency of this relationship on the specific concrete mix design and testing procedures adopted in practice, a unified form of this relationship is proposed irrespective of the specific type of concrete mix design, age at testing and the testing procedure adopted in practice (which can be according to different national or international standards). Most importantly, the very extensive test data used to develop this unified relationship, covers a very wide range of types and sizes of aggregates (including lightweight aggregate), rates of loading, ages at testing and cylinder crushing

strengths, f_c (up to 94 MPa) with the higher strengths achieved by using silica fume as an additive in the concrete mix.

By using the above mentioned empirical relationship in conjunction with the theoretical correction factor of Raoof and Lin, with the latter relating the uniaxial tensile strength of concrete, f_t, to the traditional cylinder splitting strength, f_t', very simple relationship(s) between the traditional modulus of rupture, MOR, and the uniaxial tensile strength, f_t, have been proposed for both normal and high strength (silica fume) concrete, with $MOR=k_1f_t'$, where k_1 is a simple correction factor. It is now possible to estimate the uniaxial tensile strength of concrete, f_t, by either using the cylinder splitting test or the modulus of rupture test, with the agreements between the predictions of f_t by these two different traditional indirect tests being practically reasonable. Most importantly, it is now shown that, over the range $22 < f_c < 94$ MPa, the modulus of rupture, MOR, is always greater than the corresponding uniaxial tensile strength, with the ratio MOR/f_c being dependent on the value of corresponding cylinder compressive strength, f_c. In the case of the cylinder splitting test, on the other hand, according to Raoof and Lin's model, over the range of $22 < fc < 94$ MPa, the uniaxial tensile strength, f_t, is always greater than the corresponding cylinder splitting strength, f_t', with the ratio f_t/f_t' (= k, the correction factor) being dependent on the ratio f_t'/f_c: for sufficiently high values of f_t'/f_c, the correction factor, k, can be as high as, say, 1.4 - i.e. the uniaxial tensile strength can be 40% higher than the corresponding cylinder splitting strength.

It is also noteworthy that both approaches for estimating the concrete uniaxial tensile strength (using either the modulus of rupture or the cylinder splitting tests) are amenable to simple hand calculations, using a pocket calculator, which is of obvious value to busy practicing engineers: using the presently proposed methods in everyday design, involves calculations which are comparable (in terms of their simplicity) to those associated with the traditionally used formulas for estimating the modulus of rupture and/or cylinder splitting strengths.

Finally, the presently reported work is believed to have come some way to resolve the longstanding problem of determining the true tensile strength of concrete which has eluded researchers since the early years of the 20th century.

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