The 2015 World Congress on
Advances in Structural Engineering and Mechanics (ASEM15)
Incheon, Korea, August 25-29, 2015

Quantitative of uncertainties in Earth Structures

*Stefanos Drakos[1] and Gyan Pande[2]

[1], [2] *International Center for Computational Engineering, Rhodes, Greece*
[1] *stefanos.drakos@gmail.com*
[2] *G.N.Pande@swansea.ac.uk*

ABSTRACT

The design of earth structures requires the calculation of soil deformation which is generally a difficult task because of the uncertainty and spatial variability of the properties of soil materials. This paper presents a procedure of conducting Stochastic Finite Element Analysis using Polynomial Chaos in order to quantitative the uncertainties. Among other methods the procedure leads to an efficient computational cost for real practical problems. This is achieved by polynomial chaos expansion of the displacement field. An example of a plane-strain strip load on a semi-infinite elastic foundation is presented and the results of settlement are compared to those obtained from the closed form solution method. A close matching of the two is observed.

1. INTRODUCTION

The design of earth structures requires the calculation of soil deformation which is generally difficult because of the uncertainty and spatial variability of the properties of soil materials. Various forms of uncertainties arise which depend on the nature of geological formation, the extent of site investigation, the type and the accuracy of design calculations etc. In recent years there has been considerable interest amongst engineers and researchers in the issues related to quantification of uncertainty as it affects safety, design as well as the cost of projects.

A number of approaches using statistical concepts have been proposed in geotechnical engineering in the past 25 years or so. These include the Stochastic Finite Element Method (SFEM)(Phoon et al 1990, Mellah et al. 2000 Eloseily et al 2008), and the Random Finite Element Method (RFEM) (Fenton & Vanmarcke 1990, Paice, et all 1996, Fenton & Griffiths 2002, Fenton & Griffiths 2005, Fenton & Griffiths 2008). The RFEM involves generating a random field of soil properties with controlled mean, standard deviation and spatial correlation length, which is then mapped onto a finite element mesh.

[1] President
[2] General Secretary

In the past SFEM has been developed using different expansions of stochastic variables. In this paper we present SFEM using the method of Generalized Polynomial Chaos (GPC). To descretise the stochastic process of material the Karhunen-Loeve Expansion was used and it is presented. A numerical example of foundation settlement given in the last part of the paper and the results compared with those arises from closed form solution.

2. PROBLEM DESCRIPTION AND MODEL FORMULATION

Let us consider a general boundary value problem of computation of probable deformation of a body of arbitrary shape having randomly varying material properties caused by the application of a randomly varying load as shown in Fig. 1.

According to the elasticity theory a boundary value problem can be described as follow:

$$\begin{cases} \sigma_{ij,j}(x,\omega) = f(x,\omega) \text{ in } D \times \Omega \\ \sigma_{ij}(x,\omega) = C_{ijkl}(x,\omega)\varepsilon_{kl}(x,\omega) \text{ in } D \times \Omega \\ u(x,\omega) = g_D \text{ in } B_D \\ \sigma_{ij}(x,\omega)n_j = g_N \text{ in } B_D \end{cases} \tag{1}$$

And in the weak form as:

$$a(u,v) = l(v) \tag{2}$$

Where

$$a(u,v) = \int_D \varepsilon^T(v)C(x,\omega)\varepsilon(u)\,dx \tag{3}$$

$$l(v) = \int_D f(x,\omega)\cdot v dx + \int_{B_N} g_N \cdot v dx - \int_{B_D} \varepsilon^T(v)C(x,\omega)\varepsilon(u)\,dx \cdot g_D \tag{4}$$

In the case of homogenous boundaries conditions the test function and the operators are determined as follows:

$v \in V^h \in V = L^2(\Omega, L^2(D))$
$a: V^h \times V^h \to \mathbb{R}$
$l: V^h \to \mathbb{R}$

In essence the solution of the problem is a function of the form $u \in \Omega \times D \to \mathbb{R}$ i.e. a random field and is not a deterministic function.

Although the procedure presented in the following sections is general and applicable to any boundary value problem, a specific problem of a plane-strain strip load on a semi-infinite elastic foundation with elastic modulus (E) varying randomly in space is considered for simplicity of illustration (Fig. 2).The foundation loading in general form is not specified and also can varies randomly. In order to model the problem assuming the sample space $(\Omega, \mathcal{F}, \mathbb{P})$ where $\mathcal{F}$ is the σ- algebra and is considered to contain all the information that is available, $\mathbb{P}$ is the probability measure and the spatial domain of the soil is $D \subset \mathbb{R}^2$. The Elasticity modulus $\{E(x,\omega): \in D \times \Omega\}$ and the foundation load $\{f(x,\omega): \in D \times \Omega\}$ considered as second order random fields and their functions are determined $E, f: D \times \Omega \rightarrow \mathbb{R} \in V = L^2(\Omega, L^2(D))$ and characterized by specific distribution where in our case as lognormal. The expected value of a quantity of the problem is given by the following norm:

$$\|\cdot\|_{L^2(\Omega,L^2(D))} = \int_\Omega \int_D |\cdot|^2(x,\omega)dxd\mathbb{P} = \mathbb{E}\left(\|\cdot\|_{L^2(D)}\right) < \infty \quad (5)$$

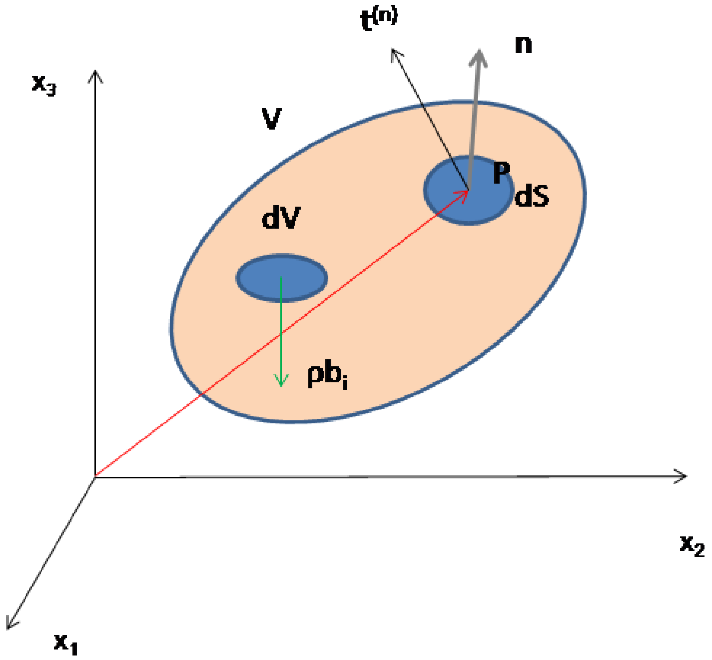

Figure 1. Body of arbitrary shape

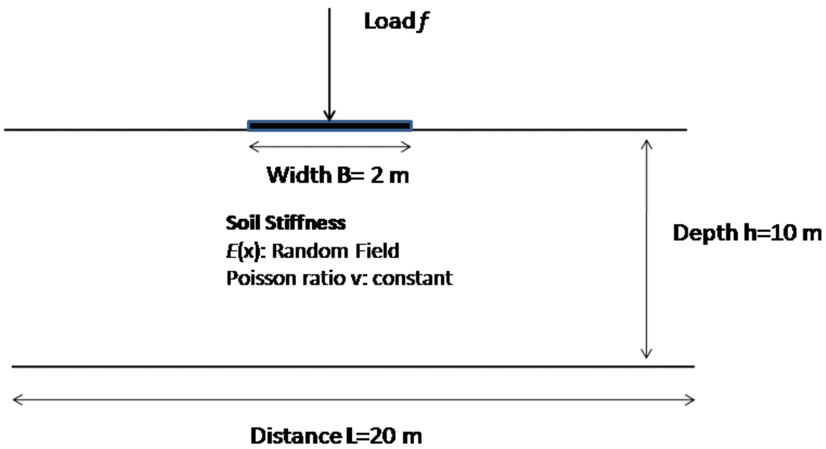

Figure 2. Plane-strain strip load on a semi-infinite elastic foundation

3. THE STOCHASTIC FINITE ELEMENT METHOD (SFEM)

The SFEM (Ghanem & Spanos 1991) have a wide range of applications and are used to solve problems in various branches of science. In the following paragraphs we introduce the procedure to solve problems in geotechnical engineering using the Stochastic Finite Element Method based on Generalized Polynomial Chaos.

3.1 *Karhunen-Loeve Expansion*

One of the major points of the SFEM is the separation of deterministic part from the stochastic part of the formulation. Thus the method has two types of discretization, the ordinary FEM discretization of geometry and the stochastic discretization of random fields. In the current paper in order to reach in these results the Karhunen-Loeve expansion has been used which is the most efficient method for the discretization of a random field, requiring the smallest number of random variables to represent the field within a given level of accuracy. Based on that the stochastic process of Young modulus over the spatial domain with a known mean value $\widetilde{\mathrm{E}}(\mathrm{x})$ and covariance matrix $Cov(\mathrm{x}_1, \mathrm{x}_2)$ assuming lognormal distribution is given by:

$$E\big(\boldsymbol{x}, \xi(\omega)\big) = \exp(\tilde{E}(\boldsymbol{x}) + \sum_{\kappa}^{\infty} \sqrt{\lambda_\kappa} \varphi_\kappa(\boldsymbol{x}) \xi_\kappa(\boldsymbol{\omega})) \tag{6}$$

In practice, calculations were carried out over a finite number of summations (for example 1-5) so the approximate stochastic representation is given by the trancuated part of expansion:

$$E\big(\boldsymbol{x}, \xi(\omega)\big) = \exp(\tilde{E}(\boldsymbol{x}) + \sum_{\kappa=1}^{K} \sqrt{\lambda_\kappa} \varphi_\kappa \xi_\kappa(\omega)) \tag{7}$$

Where:

λ_κ : are the eingenvalues of the covariance function
$\varphi_\kappa(\boldsymbol{x})$: are the eingenfunctions of the covariance function $Cov(\mathrm{x}_1, \mathrm{x}_2)$
$x \in$ D and $\boldsymbol{\omega} \in \Omega$
$\xi = [\xi_1, \xi_1, \dots, \xi_M]: \Omega \to \Gamma \subset \mathbb{R}^M$ and

$\Gamma = \Gamma_1 \times \Gamma_1 \times \dots \times \Gamma_M$

The pairs of eingenvalues and eingenfunctions arised by the equation:

$$\int_D C(x_1, x_2) \varphi_\kappa(x_2) = \lambda_\kappa \varphi_\kappa(x_1) \tag{8}$$

For two a dimensional Domain $D = [-a_1, a_1] \times [-a_2, a_2]$ the eigenvalues are $\lambda_m = \lambda_1 \lambda_2$ and eigenfunctions are equal to $\varphi_m(x) = \varphi_1(x_1)\varphi_2(x_2)$ where the values $\{\lambda_1, \lambda_2\}$ and $\{\varphi_1, \varphi_2\}$ calculated by the following equation:

$$\int_D C(x_1, x_2)\varphi_m(x_2) = \lambda_m \varphi_m(x_1), \quad m = 1,2 \tag{9}$$

Considering the covariance matrix:

$$C(x, y) = \sigma^2 \exp(-\frac{|x_j - x_i|}{\lambda_x} - \frac{|y_j - y_i|}{\lambda_y}) \tag{10}$$

In Figures 3 to 6, examples of random fields realization for different correlation lengths of covariance above matrix are presented.

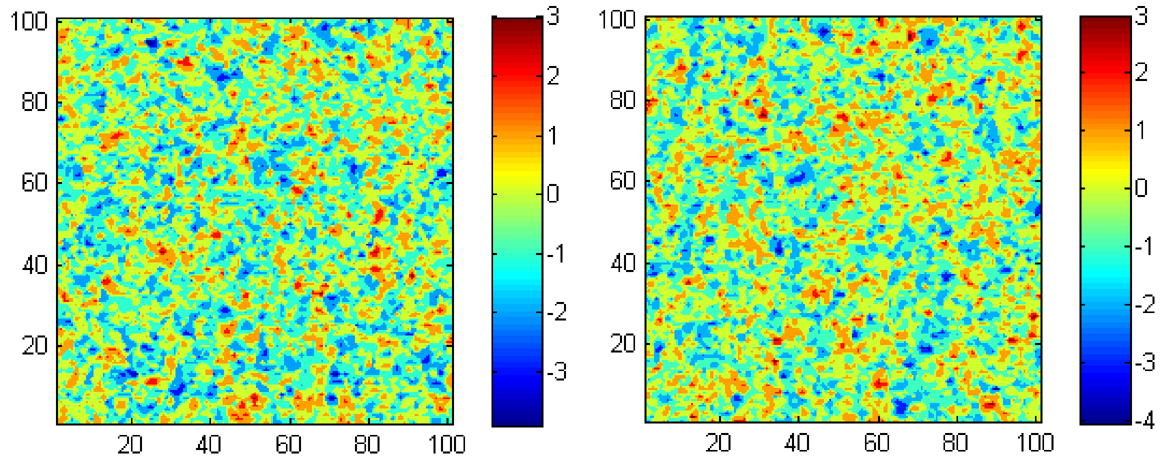

Figure 3. Random field with dimension $D = [0,100] \times [0,100]$ and correlation length $\lambda_x = \lambda_y = \frac{1}{10}$

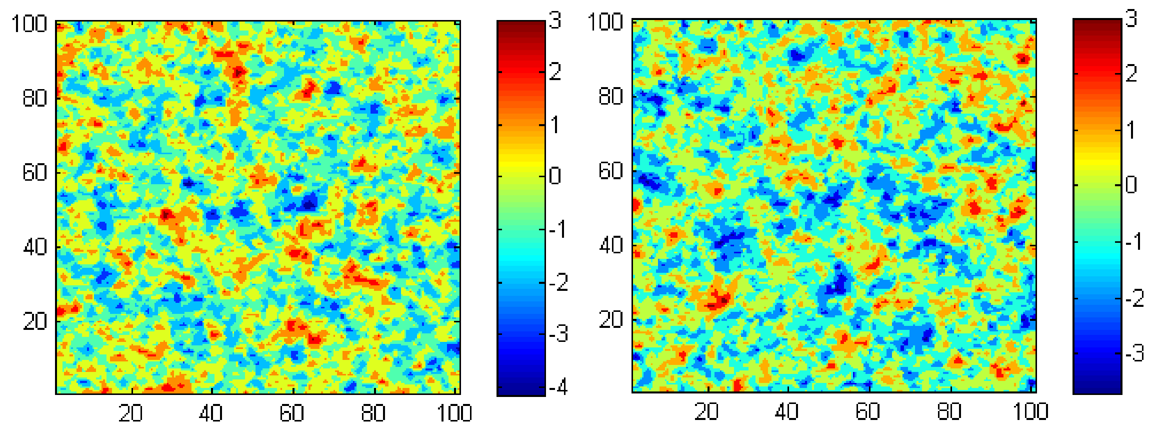

Figure 4. Random field with dimension $D = [0,100] \times [0,100]$ and correlation length $\lambda_x = \lambda_y = \frac{2}{10}$

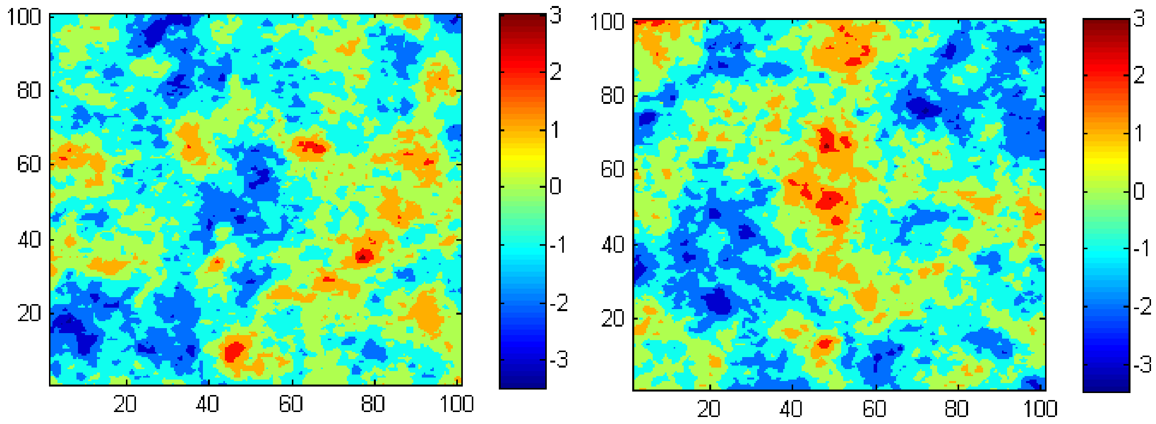

Figure 5. Random field with dimension $D = [0,100] \times [0,100]$ and correlation length $\lambda_x = \lambda_y = 1$

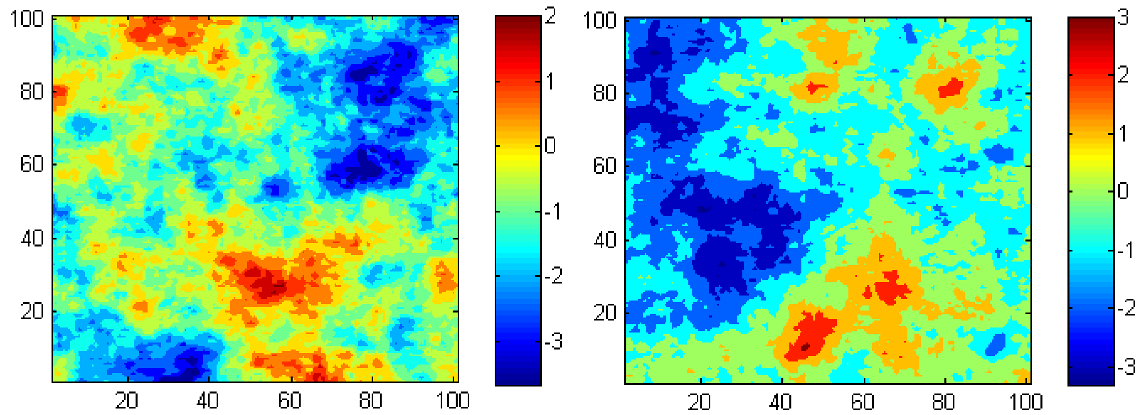

Figure 6. Random field with dimension $D = [0,100] \times [0,100]$ and correlation length $\lambda_x = \lambda_y = 2$

The rate of eigenvalue decay is inversely proportional to the correlation length. Thus for high correlation length (strong correlation) there is fast decay of the eigenvalues. For small correlation length (weak correlation) we have low decay. For zero correlation length there is no correlation and there is not decay of the eigenvalues. As the correlation length increases the decay rate increasing. If the correlation length is very small i.e correlation length =0.01 then the decay rate is not noticeable.

Implemented the Mercem theorem the first four eigenfunction are presented in Fig. 7 where in Fig. 8 and 9 the comparison of initial correlation matrix and the calculated based on the Mercer theorem is shown for different values of variation of diffusion coefficient.

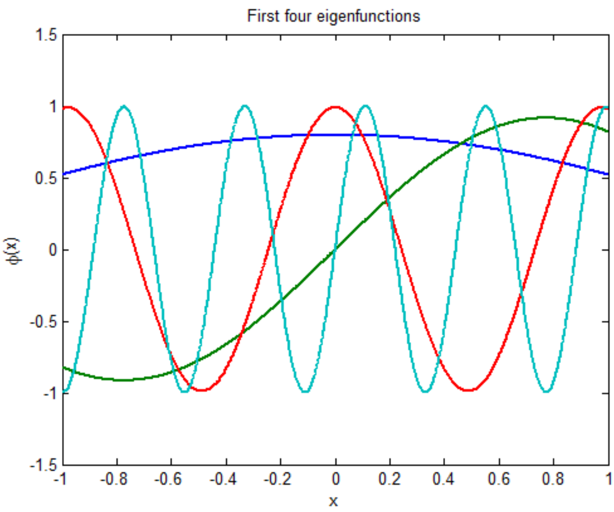

Figure 7. First four eigenfunctions in the domain D=[-1,1]

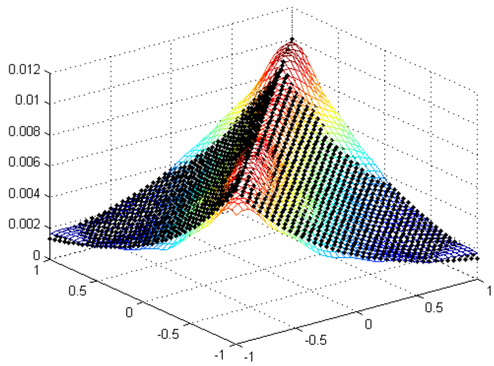

Figure 8. Comparison of initial covariance matrix and its numerical approach with correlation length=1 and sigma=0.1

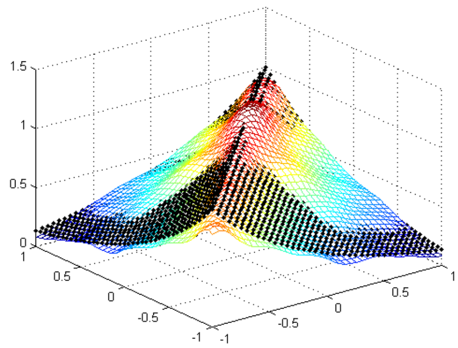

Figure 9. Comparison of initial covariance matrix and its numerical approach with correlation length=1 and sigma=1

3.2 Galerkin approximation

The Karhunen-Loeve expansion method enables to replace the calculating procedure for the expected value using instead of the abstract space Ω of random fields ξ their figures and finally to solve a deterministic problem in space $D \times \Gamma \subset \mathbb{R}^M$ instead of space $D \times \Omega$. By performing such replacements in fact a deterministic problem is solved, in contrast to the case of Monte Carlo where a large number of problems carried out. According that the test function of the weak form determined by $v \in L_p^2(\Gamma, H_0^1(D))$ while the solution of the problems in the general form of the boundaries conditions is a function $\tilde{u} \in W = L_p^2(\Gamma, H_g^1(D))$ which is satisfied the equation:

$$\tilde{\alpha}(\tilde{u}, v) = l(v)\ \forall\ v \in L_p^2(\Gamma, H_0^1(D)) \tag{12}$$

And
$\tilde{\alpha}: W \times V \to \mathbb{R}$
$l: V \to \mathbb{R}$

$$\tilde{\alpha}(\tilde{u}, v) = \int_{\Gamma} \rho(\boldsymbol{y}) \int_{D} \varepsilon^T(v) C(\boldsymbol{x}, \boldsymbol{y}) \varepsilon(u) dx dy \tag{13}$$

In the general case where and the load presents randomness:

$$\tilde{l}(v) = \int_{\Gamma} \rho(\boldsymbol{y})[\int_D f(\boldsymbol{x}, \boldsymbol{\omega}) \cdot v dx + \int_{B_N} g_N \cdot v dx - \int_{B_D} \varepsilon^T(v) C(\boldsymbol{x}, \boldsymbol{\omega}) \varepsilon(u)\, dx \cdot g_D] dy \tag{14}$$

Where:

$\rho : \Gamma \to \mathbb{R}$ is the η joint density of independent random variables **ξ**

In order to solve the problem according to the finite element method in the current paper we consider a triangle *K* with nodes $N_i(x^{(i)}, y^{(i)}), i = 1,2,3$. To each node $\mathrm{N_i}$ there is a hat function φ_i associated, which takes the value 1 at node N_i and 0 at the other two nodes. Each hat function is a linear function on *K* so it has the form:

$$\varphi_\mathrm{i} = a_i + b_i x + c_i y \tag{15}$$

The test v function belongs to the space:

$$\mathrm{V^h} = span\{\varphi_1, \varphi_2, \dots, \varphi_N\} \subset \mathrm{H_0^1(D)} \tag{16}$$

Any type of higher order shape functions can be used although it will increase the computational cost.

In order to solve the problem 1 we have to create the new space $\mathrm{L_p^2(\Gamma, H_0^1(D))}$. For that reason the subspace $\mathrm{S^k} \subset \mathrm{L_p^2(\Gamma)}$ is considered as (Lord et all 2014).

$$S^k = span\{\psi_1, \psi_2, \dots, \psi_\kappa\} \tag{17}$$

Using the dyadic product of the space $\mathrm{V^h}$, S^k the space $\mathrm{L^2_p}(\Gamma, \mathrm{H^1_0}(\mathrm{D}))$ created. Thus

$$V^{hk} = V^h \otimes V^k = span\{\varphi_i \psi_j, \mathrm{i} = 1 \dots N, \mathrm{j} = 1, \dots Q\} \tag{18}$$

The space V^{hk} has dimension *QN* and regards the test function *v*. In the case where exists N_B finite element supported by boundaries condition then the subspace of solution belongs is:

$$\mathrm{W^{hk}} = \mathrm{V^{hk}} \oplus span\{\varphi_{N+1}, \varphi_{N+2}, \dots, \varphi_{N+NB}\} \tag{19}$$

3.3 Generalized Polynomial of chaos and stochastic Galerkin solution

Assuming that the S_i^k represents a space of univariate orthonormal polynomial of variable $y_i \subset \Gamma_i \subset \mathbb{R}$ with order k or lower and:

$$S_i^k = \left\{P^i_{a_i}(y_i), a_i = 1,2, \dots k\right\}, i = 1, \dots M \tag{20}$$

The tensor product of the *M* S_i^k subspace results the space of the *Generalized Polynomial Chaos*:

$$S^k = S_1 \otimes S_2 \dots \otimes S_M \tag{21}$$

Xiu & Karniadakis (2003) show the application of the method for different kind of orthonormal polynomials and in the current paper the Hermite polynomial was used with the following characteristics:

$$P_0 = 1, < P_i >= 0, i > 0$$
$$< P_m P_n >= \int_\Gamma P_m(\boldsymbol{y}) P_n(\boldsymbol{y}) \rho(\boldsymbol{y}) d\boldsymbol{y} = \gamma_n \delta_{mn} \tag{22}$$

Where:

$\gamma_n = < P_n^2 >$: are the normalization factors.

δ_{mn} is the *Kronecker* delta

$\rho(\boldsymbol{y}) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x}{2}}$: is the density function

(23)

And:

$$P_n = (-1)^n e^{\frac{x}{2}} \frac{d^n}{dx^n} e^{-\frac{x}{2}}$$

(24)

The function $u \in W^{hk}$ can be written as the summation of S^k polynomials base as:

$$u(\boldsymbol{x}, \boldsymbol{y}) = \sum_{k=1}^{P} u_k(\boldsymbol{x}) \psi_\kappa(\boldsymbol{y})$$

(25)

According that and using the inner product of the weak form equation on each polynomial of the S^k base and get:

$$< a\left(\sum_{k=1}^{P} u_k(\boldsymbol{x}) \psi_\kappa(\boldsymbol{y}), v\right), \psi_p >=< l(v), \psi_p >$$

(26)

The LHS of the equation can be written based on the solution's polynomial chaos expansion as:

$$< a\left(\sum_{k=1}^{P} u_k(\boldsymbol{x}) \psi_\kappa(\boldsymbol{y}), v\right), \psi_p >= \sum_{i=1}^{ndof} \sum_{k=1}^{P} u_{ik} \int_{\Gamma} \rho(\boldsymbol{y}) \psi_\kappa(\boldsymbol{y}) \psi_p(\boldsymbol{y}) \int_{D} B^T C(\boldsymbol{x}, \boldsymbol{y}) B dx dy$$

(27)

Where B is strain displacement matrix.

Using the Karhunen-Loeve expansion the stochastic elasticity tensor is given by:

$$C_{ijkl}(\boldsymbol{x}, \boldsymbol{y}) = E(\boldsymbol{x}) C_{ijkl}^*(\boldsymbol{x})$$

(28)

$C^*_{ijkl}(x)$: is expressed in terms of (deterministic) Poisson's ratio as

$$C^*_{ijkl}(\boldsymbol{x}) = \frac{v}{(1+v)}\delta_{ij}\delta_{kl} + \frac{1}{2(1+v)}(\delta_{ik}\delta_{jl} + \delta_{ll}\delta_{jk}) \tag{29}$$

In the case of plane strain conditions:

$$C^*(\boldsymbol{x}) = \frac{1}{1+v}\begin{bmatrix} 1-v & 0 & 0 \\ v & 1-v & 0 \\ 0 & 0 & \frac{1-2v}{v} \end{bmatrix} \tag{30}$$

From the above:

$$\begin{aligned}
&< a\left(\sum_{k=1}^{P} u_k(\boldsymbol{x})\psi_\kappa(\boldsymbol{y}), v\right), \psi_p >= \\
&\sum_{i=1}^{ndof}\sum_{k=1}^{P} u_{ik}\int_\Gamma \rho(\boldsymbol{y})\psi_\kappa(\boldsymbol{y})\psi_p(\boldsymbol{y})\int_D B^T E(\boldsymbol{x})C^*(\boldsymbol{x})B dx dy = \\
&\sum_{i=1}^{ndof}\sum_{k=1}^{P} u_{ik}\int_\Gamma \rho(\boldsymbol{y})\psi_\kappa(\boldsymbol{y})\psi_p(\boldsymbol{y})\int_D B^T \exp(\tilde{E}(\boldsymbol{x}) + \sum_{\kappa=1}^{K}\sqrt{\lambda_\kappa}\varphi_\kappa\xi_\kappa(y))C^*(\boldsymbol{x})B dx dy = \\
&\sum_{i=1}^{ndof}\sum_{k=1}^{P} u_{ik}\int_\Gamma \rho(\boldsymbol{y})\psi_\kappa(\boldsymbol{y})\psi_p(\boldsymbol{y})\exp(\sum_{\kappa=1}^{K}\sqrt{\lambda_\kappa}\varphi_\kappa\xi_\kappa(y))\int_D B^T \exp(\tilde{E}(\boldsymbol{x}))C^*(\boldsymbol{x})B dx dy
\end{aligned} \tag{31}$$

If we set

$$\begin{aligned}
Q_m &= \int_\Gamma \rho(\boldsymbol{y})\psi_\kappa(\boldsymbol{y})\psi_p(\boldsymbol{y})\exp(\sum_{\kappa=1}^{K}\sqrt{\lambda_\kappa}\varphi_\kappa\xi_\kappa(\boldsymbol{y}))\, dy \\
K_m &= \int_D B^T \exp(\tilde{E}(\boldsymbol{x}))C^*(\boldsymbol{x})B dx dy
\end{aligned} \tag{32}$$

The LHS of the weak form equation can be written as:

$$a(u, v) = Q_m \otimes K_m \tag{33}$$

And the RHS of the weak form assuming constant load for simplicity:

$$< l(v), \psi_p > = \int_{\Gamma} \rho(\boldsymbol{y})\psi_p[\int_D \varphi^T f(\boldsymbol{x})dx + \int_{B_N} \varphi^T g_N ds - \int_{B_D} B^T(v)C(\boldsymbol{x},\boldsymbol{y})B(u)\, dx \cdot g_D]dy \tag{34}$$

If we set

$$q_0 = \int_{\Gamma} \rho(\boldsymbol{y})\psi_p(\boldsymbol{y})\psi_1(\boldsymbol{y})\, dy$$
$$K_{Bm} = \int_{BD} B^T \exp(\tilde{E}(\boldsymbol{x}))C^*(\boldsymbol{x})Bdxdy$$
$$f_0 = \int_D \varphi^T f(\boldsymbol{x})dx$$
$$t_{gN} = \int_{BN} \varphi^T \cdot g_N ds \tag{35}$$

And

$$< l(v), \psi_p > = q_0 \otimes \left(f_{0+} t_{gN}\right) - Q_m \otimes K_{Bm} \cdot g_d \tag{36}$$

Finally the system has the form:

$$\boldsymbol{K} \cdot u = \boldsymbol{F}, \qquad \boldsymbol{K} \in \mathbb{R}^{\mathrm{ndof\cdot P \times ndof\cdot P}}, \text{and } \boldsymbol{F} \in \mathbb{R}^{\mathrm{ndof\cdot P}} \tag{37}$$

The statistical moments of the displacement field arise by the properties of the Polynomial of Chaos expansion:

The expected value

$$\mathbb{E}[u(\boldsymbol{x},\boldsymbol{y})] = \mathbb{E}[\sum_{k=0}^{P} u_k(\boldsymbol{x})\psi_\kappa(\boldsymbol{y})] =$$
$$u_0(\boldsymbol{x}) \underbrace{\mathbb{E}[\psi_0(\boldsymbol{y})]}_{1} + \underbrace{\sum_{k=1}^{P} u_k(\boldsymbol{x})\mathbb{E}[\psi_\kappa(\boldsymbol{y})]}_{0} = u_0(\boldsymbol{x}) \tag{38}$$

And the variance:

$$\sigma^2 = \mathbb{E}(u(\boldsymbol{x},\boldsymbol{y}) - \mathbb{E}[u(\boldsymbol{x},\boldsymbol{y})])^2 =$$
$$\mathbb{E}(\sum_{k=0}^{P} u_k(\boldsymbol{x})\mathbb{E}[\psi_\kappa(\boldsymbol{y})] - u_0(\boldsymbol{x})) \Rightarrow$$

$$\sigma^2 = \sum_{k=0}^{P} u_k^2(\boldsymbol{x})\mathbb{E}[\psi_\kappa^2 \quad] \tag{39}$$

4 NUMERICAL EXAMPLE

A shallow foundation problem for various values of variation's coefficient v_e is solved taken to account the randomness of the ground. To estimate the statistical moments of the soil deformation the numerical algorithm of SFEM using the Generalized Polynomial Chaos as described in the previous paragraphs is applied and the results are compared to those obtained by the closed form solution.
To avoid the negative values of the elastic modulus assumed to have lognormal distribution and given by the following equation:

$$E = \exp(\mu_{\ln E} + \sigma_{\ln E} Z(\omega))$$

Where:

$\sigma_{\ln E}^2 = \ln(1 + v_e^2)$

$\mu_{\ln E} = \ln(\mu_E) - \frac{1}{2}\sigma_{\ln E}^2$

$v_e = \frac{\sigma_E}{\mu_E}$

And μ_E, σ_E are the mean value and standard deviation of elasticity modulus E.

It is known that the settlement beneath a foundation with uniform but random elastic modulus is given by the equation (Fenton & Grifith 2008):

$$s = \frac{s_{det}\mu_E}{E}$$

Where s_{det} is the deterministic value of settlement with $E = \mu_E$ everywhere.
Assuming lognormal distribution for the settlements the mean values is equal to

$$\mu_{\ln(s)} = \ln(s_{det}) + \frac{1}{2}\sigma_{\ln(E)}^2$$

The geometry of the finite elements used for the simulation of the problem presented in Fig. 10. The input data of the problem is the random field modulus with a constant average value equal to 100 Mpa and a fixed Poisson ratio equal to 0.25. Calculations have been made for ten different coefficients $v_e = \frac{\sigma_E}{\mu_E}$ of the elastic modulus with a minimum value of 0.1 and then with step 0.1 to a maximum value equal to 1. The randomness of Elasticity modulus in Fig. 11 is shown. For SFEM one dimensional Hermite GPC with order 5 (Xiu & Karniadakis 2003) were used. In the Fig. 12 the results of SFEM method comparatively with the closed form solution are shown and they present great accuracy. It is observed that for of v_e = 0.5 the error is equal to

0.8%. In the Fig 13 and 14 the expected value and the standard deviation of the settlements are presented for various value of variation's coefficient.

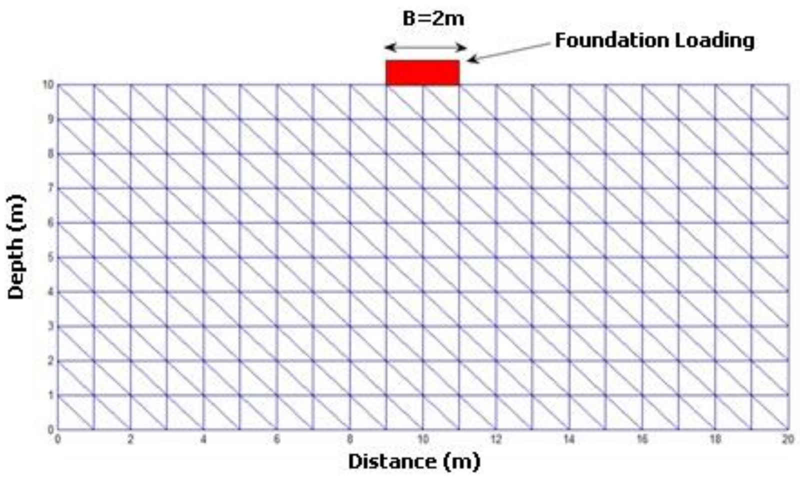

Figure 10. Finite element mesh

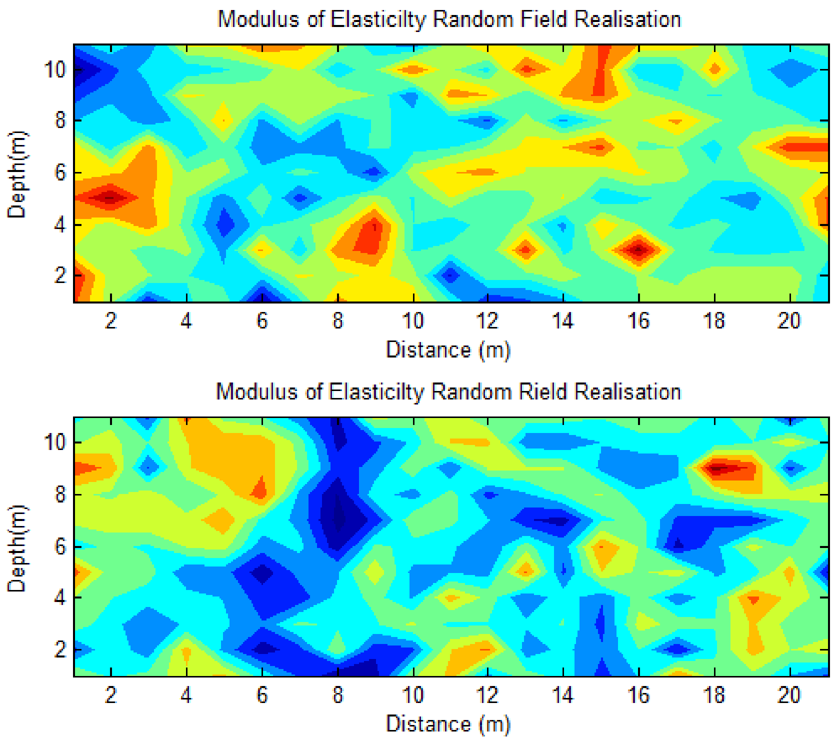

Figure 11. Modulus of Elasticity Random field realization

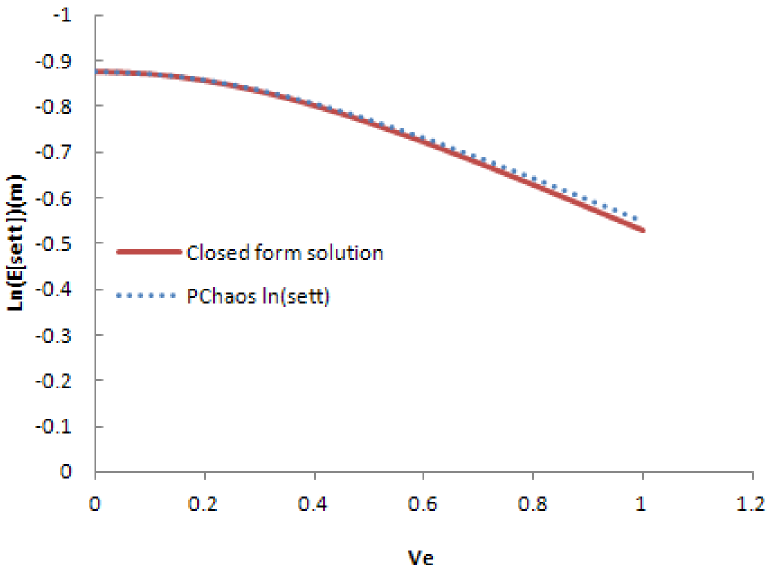

Figure 12. Logarithmic results of the expected settlements.

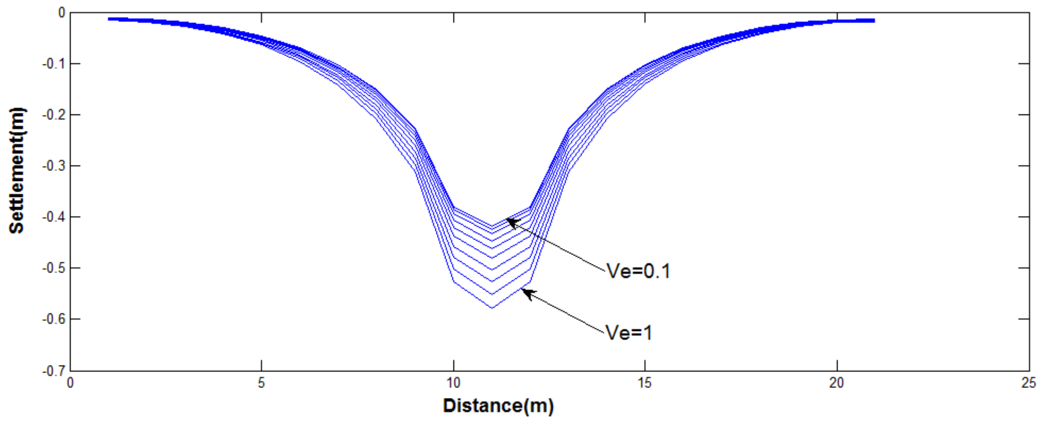

Figure 13. Expected settlements for various values of variation's coefficient.

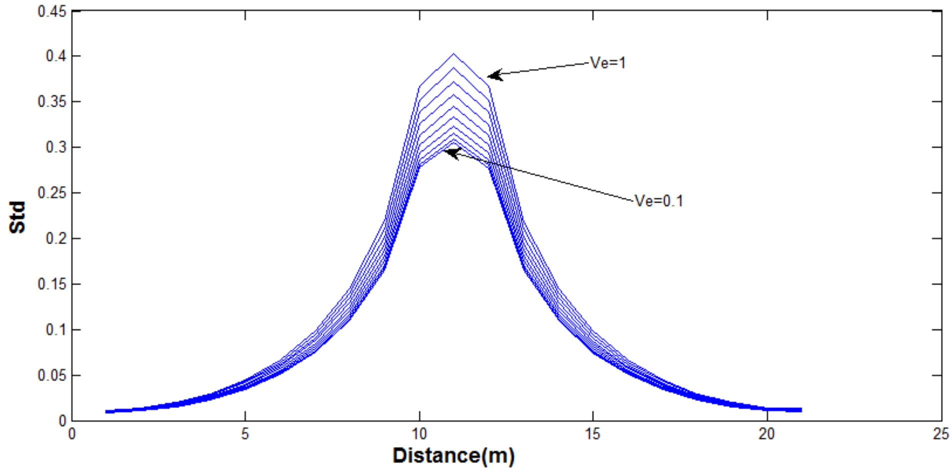

Figure 14. Standard deviation of settlements for various values of variation's coefficient.

5. CONCLUSIONS

To quantitative the uncertainties arises due to spatial variability of mechanical parameters of soil/rock in geotechnical structures design, a procedure of conducting a Stochastic Finite Element Analysis has been presented. Two different approaches in order to quantifying uncertainty are discussed. An algorithm of Stochastic Finite Element using Polynomial Chaos has been developed. An analysis of settlement of a plane strain strip load on an elastic foundation has been given as an example of the proposed approach. It is shown that the results of SFEM using polynomial chaos compare well with those obtained from closed form solution. The main advantage in using the proposed methodology is that a large number of realizations which have to be made for (Random Finite Element Method) avoided, thus making the procedure viable for realistic practical problems.

REFERENCES

Phoon, K., Quek, S., Chow, Y. and Lee, S (1990), “Reliability analysis of pile settlements”. Journal of Geotechnical Engineering, ASCE, 116(11), 1717–35.

Mellah, R., Auvinet, G. and Masrouri, F.(2000), “Stochastic finite element method applied to non-linear analysis of embankments”. Probabilistic Engineering Mechanics, 15 (3), 251–259.

Eloseily, K., Ayyub, B. and Patev, R. (2002), “Reliability assessment of pile groups in sands”. Journal of Structural Engineering, ASCE, 128(10), 1346–53.

Lord G., Powel C., Shardlow T. (2014), “An Introduction to Computational Stochastic PDEs”, Cambridge Texts in Applied Mathematics.

Fenton, G. A., and Vanmarcke, E. H. (1990), “Simulation of random fields via local average subdivision.” J. Eng. Mech., 116(8), 1733–1749.

Paice, G. M., Griffiths, D. V., and Fenton, G. A.(1996), “Finite element modeling of settlements on spatially random soil.” *J. Geotech. Eng.*, 122(9), 777–779.

Fenton, G. A., and Griffiths, D. V. (2002), “Probabilistic foundation settlement on spatially random soil.” *J. Geotech. Geoenviron. Eng.*,128(5), 381–390.

Fenton, G. A., and Griffiths, D. V. (2005), “Three-dimensional probabilistic foundation settlement.” *J. Geotech. Geoenviron. Eng.*, 131(2), 232–239.

Fenton, G. A., and Griffiths, D. V. (2008), “Risk assessment in geotechnical engineering”, Wiley, Hoboken, NJ.

Ghanem, R. G., and Spanos, P. D. (1991), *Stochastic finite elements: A spectral approach*, Springer-Verlag, New York.

D Xiu, G Em Karniadakis. (2003), "Modeling uncertainty in steady state diffusion problems via generalized polynomial chaos" *Computer Methods in Applied Mechanics and Engineering* 191 (43), 4927-4948