The 2015 World Congress on
Advances in Structural Engineering and Mechanics (ASEM15)
Incheon, Korea, August 25-29, 2015

Parameter identification of hyperelastic and hyper-viscoelastic models

*Yi Feng Wu[1)], Ai Qun Li[2)] and Hao Wang[3)]

[1), 2), 3)] *School of Civil Engineering, Southeast University, Nanjing, 210096*
[2)] *Beijing University of Civil Engineering and Architecture, Beijing ,100044*

ABSTRACT

Based on the Ogden model and the Levenberg-Marquardt nonlinear optimization algorithm, a professional method that can realize the comprehensive fitting of the uniaxial tension, biaxial tension, planar tension and simple shear experimental data of rubbers was developed. The experiment data from Treloar(1944) was fitted very well, and the determined parameters by using this method were proved to be correct and practical in the numerical verification in ANSYS. Then, the constitutive model of the hyper-viscoelastic materials which combines the Ogden model with the generalized Maxwell model was explained in detail, and the parameter identification algorithm was proposed based on the integration of the relaxation modulus. Moreover, the restrictions of the initial values set for the undetermined parameters in the hyper-viscoelastic model were investigated, and the validity and the practicability of the estimated parameters was also verified in ANSYS.

1. INTRODUCTION

With the rapid development of modern industrialization, rubbers are one of the most remarkable materials having a wide range of applications in civil engineering, aerospace engineering, mechanical engineering, automotive engineering, etc. In order to meet various requirements of the industry, special fillers, like carbon black or silica, with different proportion are usually added during vulcanization for improving the strength and toughness properties, which in turn makes it difficult to accurately characterize the mechanic properties of rubbers(Amin et al. 2006). Rubbers usually present a number of interesting features like hyperelasticity and hyper-viscoelasticity. The stress-strain relationship of the hyperelasticity can be illuminated by a strain energy function W, the strain-invariant-based models of W mainly include the Mooney-Rivlin model, the Yeoh model and the Gent model, and the principal-stretch-based model mainly include the Ogden model(Yang et al. 2004). It is worth mentioning that the Ogden model breaks through the limitation of the strain energy function being the even power of the stretches, and it is capable of more accurately fitting the experimental data when rubbers undergo large deformation(Ogden 1973; Treloar 1975). The constitutive

[1)] PH.D Candidate
[2)] Professor
[3)] Associate Research Fellow

relations for the viscoelasticity can be characterized by combining the elastic components with the viscous components. The relaxation-modulus-based generalized Maxwell model is one of the most widely used models in the commercial finite element (FE) software, like ANSYS, ABAQUS, LS-DYNA, etc.

In order to uncover the viscoelasticity properties under finite deformation, Lianis(1963) proposed a relaxation-modulus constitutive function for the incompressible isotropic materials. In the framework of a multiplicative decomposition of the deformation gradient tensor, Huber et al.(2000) generalized the application range of the so-called three-parameter solids to the finite deformation. Yoshida et al.(2004) presented a constitutive model combined with a elastoplastic body with a strain-dependent isotropic hardening law and a hyperelastic body with a damage model, which coincided well with the experimental results. Amin et al.(2002; 2006a; 2006b) proposed an improved hyperelastic constitutive equation for rubbers in compression and shear regime, the parameters in the equation were identified by the data from the rate-independent instantaneous experiments and equilibrium experiments, and the nonlinear viscous coefficient was introduced to represent the rate-dependent properties of rubbers. With regard to the nonlinear material parameter estimation of the hyperelastic model, Gendy et al.(2000) developed an professional scheme named COMPARE to get the parameters by using uniaxial tension(ST), biaxial tension(ET) and planar tension (PT) data, which was based on the Ogden model and consisted of a sensitivity analysis as well as an optimization procedure. Wang et al.(2007) provided an hyper-viscoelasticity relation to simulate the high damping materials, and the parameters of the model were identified from the experiment data.

The investigations hereinbefore show that there have been few researches conducted on the parameter identification of the hyper-viscoelastic model. In this paper, a professional method based on the Ogden model and the Levenberg-Marquardt (L-M) optimization algorithm was developed to fit the ST, PT, ET, and simple shear(SS) experimental data of hyperelastic materials. Then, Taking the combination of the generalized Maxwell model with the Ogden model as the constitutive model of hyper-viscoelastic materials, and the integration of relaxation modulus which is based on the Boltzmann superposition principle was employed as the fundamental algorithm, the method that can identify the parameters in the hyper-viscoelastic models by using SS data with different strain loading velocities was developed. All the estimated parameters in the hyperelastic and the hyper-viscoelastic models were verified by ANSYS.

2. PARAMETER IDENTIFICATION OF THE HYPERELASTIC MODEL

The constitutive models representing the hyperelastic properties of rubbers mainly include the statistical models, the strain-invariant-based models and the principal-stretch-based models, Among which the Ogden model is able to fit the experiment data when rubbers undergo large deformations($\lambda < 7$). In the following sections, the analytical theories of the rubbers subjected to ST, ET, PT and SS deformations and the numerical verification with the experiment data are firstly presented in detail.

2.1 Theories of the Ogden model

According to the theories of the continuum mechanics, there exists a strain energy function W for the hyperelasticity properties of rubbers. The stresses can be obtained by the partial derivative of the variable W with respect to the strain.

$$S = 2\frac{\partial W}{\partial C} \tag{1}$$

Where C is the right Cauchy-Green deformation gradient tensor, S is the second Piola-Kirchhoff stress.

After transforming the second Piola-Kirchhoff stress to the Cauchy stress (Lai et al. 2010), we will get

$$T = -pI + 2\frac{\partial W}{\partial I_1}B - 2\frac{\partial W}{\partial I_2}B^{-1} \tag{2}$$

$$B = FF^T \tag{3}$$

Where F is the deformation gradient tensor, B is the left Cauchy-Green deformation gradient tensor, I_1, I_2, I_3 are the three strain invariants of B, p is the undetermined hydrostatic pressure which can be decided by the underlying equilibrium and boundary conditions of the particular problem.

In view of the incompressible properties of the volume of rubbers, the strain energy of the Ogden model can be defined with the Eq.(4) without regard for the contribution of the volume strain.

$$W = \sum_n \frac{\mu_n}{\alpha_n}(\lambda_1^{\alpha_n} + \lambda_2^{\alpha_n} + \lambda_3^{\alpha_n} - 3) \tag{4}$$

In Eq. (4), n is the number of terms considered for the Ogden model, μ_n , α_n are the undetermined parameters of the model, λ_i is the principal stretch.

In the ST experiment, the deformation gradient tensor F and the left Cauchy-Green deformation gradient tensor B are described as

$$F = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & 1/\sqrt{\lambda_1} & 0 \\ 0 & 0 & 1/\sqrt{\lambda_1} \end{pmatrix} \qquad B = \begin{pmatrix} \lambda_1^2 & 0 & 0 \\ 0 & 1/\lambda_1 & 0 \\ 0 & 0 & 1/\lambda_1 \end{pmatrix} \tag{5}$$

and the three strain invariants of the tensor B are

$$I_1 = \frac{2}{\lambda_1} + \lambda_1^2 \qquad I_2 = \frac{1}{\lambda_1^2} + 2\lambda_1 \qquad I_3 = 1 \tag{6}$$

Taking the boundary condition $T_{22} = T_{33} = 0$ into consideration, the expression for Cauchy stress T_{11} can be expressed by using the Eq. (2) as

$$T_{11} = 2(\lambda_1^2 - \frac{1}{\lambda_1})(\frac{\partial W}{\partial I_1} + \frac{1}{\lambda_1}\frac{\partial W}{\partial I_2}) \tag{7}$$

Then, substituting the Eq. (4) and the Eq. (6) in the Eq. (7), we will obtain

$$T_{11} = \sum_n \mu_n (\lambda_1^{\alpha_n} - \lambda_1^{-\alpha_n/2}) \tag{8a}$$

and the corresponding nominal stress is

$$f_1 = T_{11} / \lambda_1 = \sum_n \mu_n (\lambda_1^{\alpha_n - 1} - \lambda_1^{-\alpha_n/2-1}) \tag{8b}$$

In the ET experiment, the boundary conditions $T_{22} = T_{33}, T_{11} = 0, \lambda_2 = \lambda_3$. After the similar derivative processes of the unixial tension, the Cauchy stress T_{22}, as well as T_{33}, can be expressed as

$$T_{22} = T_{33} = \sum_n \mu_n (\lambda_2^{\alpha_n} - \lambda_2^{-2\alpha_n}) \tag{9a}$$

and the corresponding nominal stress is

$$f_2 = T_{22} / \lambda_2 = \sum_n \mu_n (\lambda_2^{\alpha_n - 1} - \lambda_2^{-2\alpha_n - 1}) \tag{9b}$$

Similarly, Cauchy stress T_{11} of the PT experiment is

$$T_{11} = \sum_n \mu_n (\lambda_1^{\alpha_n} - \lambda_1^{-\alpha_n}) \tag{10a}$$

where the boundary condition is $T_{33} = 0, \lambda_2 = 1$, and the nominal stress is

$$f_3 = T_{11} / \lambda_1 = \sum_n \mu_n (\lambda_1^{\alpha_n - 1} - \lambda_1^{-\alpha_n - 1}) \tag{10b}$$

As for the SS experiment, the direction of the principal stretch does not in consistent with the direction of the applied deformation, rather it involves a rotation of axes (Amin et al., 2006). The deformation gradient tensor F and the left Cauchy-Green deformation tensor B are expressed as

$$F = \begin{pmatrix} 1 & \gamma & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 1+\gamma^2 & \gamma & 0 \\ \gamma & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{11}$$

and the three invariants of the tensor B are

$$I_1 = I_2 = 3+\gamma^2 \qquad I_3 = 1 \tag{12}$$

the Cauchy stress T_{12} is

$$T_{12} = 2\gamma(\frac{\partial W}{\partial I_1} + \frac{\partial W}{\partial I_2}) \tag{13}$$

where γ is the shear strain.

Then, after substituting the Eq. (4) and the Eq. (12) in the Eq. (13), we will get the shear stress and the nominal stress as shown in the Eq.(14).

$$f_4 = T_{12} = \frac{\sum_n \mu_n (\lambda_1^{\alpha_n} - \lambda_1^{-\alpha_n})}{\lambda_1 + 1/\lambda_1} \tag{14}$$

where the principal stretch λ_1 can be obtained by calculating the eigenvalues of the tensor B.

$$\lambda_1 = \sqrt{1+\frac{\gamma^2}{2}+\gamma\sqrt{1+\frac{\gamma^2}{4}}} \tag{15}$$

2.2 The object function for parameter identification

Previous researches have shown that the constitutive parameters of a hyperelastic model identified form a particular deformation mode may be invalid for other modes. For example, Charlton et al.(1994) claimed that the parameters identified from the uniaxial test data fail in predicting biaxial or planar tension responses. So, it is of much significance to take the four types of experiment data listed above into consideration. This goal can be achieved by the use of the least square procedure to make the deviations of the experiment data and the fitted data the least. The object function can be defined as

$$min: \sum_{q=1}^{4} \omega_q (\sum_{i=1}^{n_q} ((f_q)_i - (\sigma_q)_i)^2) \tag{16}$$

Where ω_q is the weight of different experiment type, if there exists no experiment data for a specific type, we just need to set its ω_q to be zero, if we want to emphasis the experiment data of a specific type, we can set its ω_q larger than others'. n_q represents the number of the experiment data for each type of experiment, $(f_q)_i$ is the theoretical value which is calculated by the Eq.8(b), Eq.9 (b), Eq.10(b) and Eq.14, $(\sigma_q)_i$ is the experiment data.

2.3 The identified parameters and the numerical verification

The object function Eq. (16) can be solved by the L-M nonlinear optimization algorithm(Levenberg, 1944). In this paper, the ST, ET and PT data from Treloar(1944) are taken as the experiment data, the preceding algorithm is realized by MATLAB software(2010), and the Ogden model is chosen as the hyperelastic model owing to its favorable performances under large deformations. When the number of terms considered for the Ogden model are respectively set as 3 and 4, the results of the undetermined parameters and the deviation S are listed in Table 1, the estimated parameters presented by Treloar(1944) are listed as well, the unit of μ_n in Table 1 is Mpa.

Table. 1 Results of the identified parameters in Ogden model

Treloar	$\mu_1 = 0.6174$ $\alpha_1 = 1.3$	$\mu_2 = 0.001176$ $\alpha_2 = 5$	$\mu_3 = -0.0098$ $\alpha_3 = -2$		S=0.1056
n=3	$\mu_1 = 0.492565$ $\alpha_1 = 1.52803$	$\mu_2 = 3.99430E-4$ $\alpha_2 = 5.516205$	$\mu_3 = -8.76610E-3$ $\alpha_3 = -2.08808$		S=0.0336
n=4	$\mu_1 = 6.29178$ $\alpha_1 = 0.0725356$	$\mu_2 = 0.111267$ $\alpha_2 = 2.5301145$	$\mu_3 = 4.46956E-6$ $\alpha_3 = 7.58336$	$\mu_4 = -7.33255E-4$ $\alpha_4 = -2.85904$	S=0.0118

As the Table 1 shows, these three groups of parameters are all capable of fitting the experiment data very well, and the last group is the best. Since there is no need to work out the globally optimal solution for the Eq.(16), the estimated parameters are not unique as long as they make the deviation lower than a certain limit. However, in order to make the tangent stiffness matrix to be positive definite, it is necessary to ensure that the product of μ_n and α_n is positive.

For the purpose of verifying the estimated parameters, the last group of the identified parameters is taken as the example to predict the response of rubbers in different types of experiments in ANSYS. The results of the numerical simulation, the fitted data obtained by the use of the proceeding algorithm, along with the experiment data from Treloar(1944) are all plotted in Fig. 1.

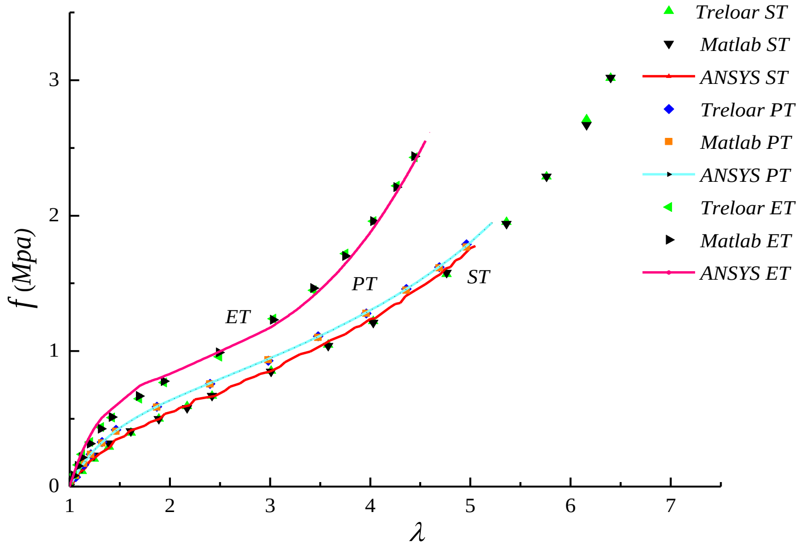
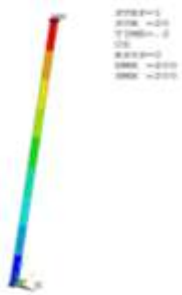


Fig. 1 Verification of the determined parameters in Ogden model

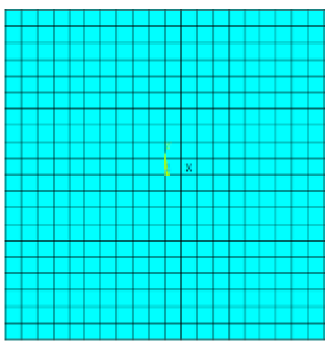
As illustrated in the Fig.1, The results of the algorithm developed in MATLAB fit the experiment data very well. Besides, in the numerical simulation tests, the results of ST and PT coincide well with the experiment data (when the principal stretch is larger than 5, the FE analysis of ST can't converge due to some uncertain reasons). However, the results of ET differs a bit from the experiment data, the reason for the errors lies in that the boundary conditions of the ET experiment can't be accurately satisfied when the principal stretch ranges from 1.5 to 1.9 in the simulation test.

The FE initial models of ST, PT, ET experiment and their deformed shapes with the stretch being 5, 6.14, 5.04 are presented respectively in Fig. 2(a)-Fig. 2(f).

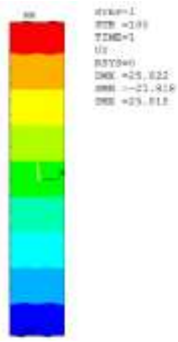
(a) initial model of ST experiment



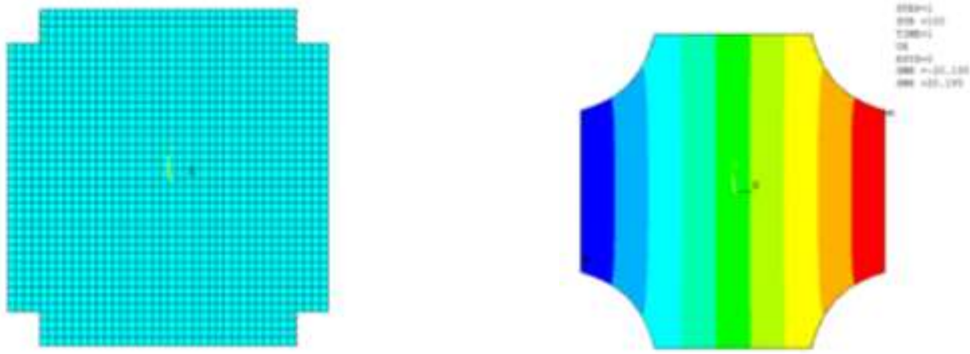
(b) deformed model of ST experiment(λ=5)



(c) initial model of PT experiment



(d) deformed model of PT experiment(λ=6.164)



(e) initial model of ET experiment (f) deformed model of ET experiment(λ=5.04)

Fig. 2 The FE models for numerical verification

3. PARAMETER IDENTIFICATION OF THE HYPER-VISCOELASTIC MODEL

In the previous section, the parameter identification of the Ogden model for hyperelastic materials was explained in detail, this model is suitable for the numerical simulation of natural rubbers with low damping. As for the high damping rubbers, they can be simulated by the hyper-viscoelastic model which combines the Ogden model with the generalized Maxwell model. In the following, the integration of the relaxation modulus was employed as the fundamental algorithm, and the parameter identification of the hyper-viscoelastic model for high damping rubbers was expounded.

3.1 Theories of the hyper-viscoelastic model

The integration of the relaxation modulus for viscoelasticity is expressed in Eq. (17)

$$\sigma(t) = \int_{-\infty}^{t} Y(t-\xi) d\varepsilon(\xi) \tag{17}$$

where $Y(t-\xi)$ is the relaxation modulus function for characterizing the viscoelastic properties, $d\varepsilon(\xi)$ is the micro-strain of materials. Generally, when $\xi < 0$, the outer action is assumed to be 0, so the Eq. (17) will be changed to

$$\sigma(t) = \int_{0}^{t} Y(t-\xi) d\varepsilon(\xi) \tag{18}$$

If the strain loading speed is constant, namely $\varepsilon(\xi) = \lambda\xi$, then

$$\sigma(t) = \int_{0}^{t} Y(t-\xi) \lambda d\xi \tag{19}$$

where λ is the strain loading velocity.

The Eq. (19)can be calculated by dividing the time t into n equal portions, then we will get

$$\sigma(t) = \lambda \sum_{i=1}^{n} \int_{t_i}^{t_{i+1}} Y(t-\xi) d\xi \tag{20}$$

When $\xi \in (t_i, t_{i+1})$, if the Young modulus of materials is assumed to be a constant E_i, and the Prony series is introduced as the relaxation coefficient, then the Eq. (20) will become

$$\sigma(t) = \lambda \sum_{i=1}^{n} \int_{t_i}^{t_{i+1}} E_i (\mu_0 + \sum_{k=1}^{m} \mu_k e^{-(t-\xi)/t_k}) d\xi \tag{21}$$

where μ_k is the weight of modulus corresponding to the relaxation time t_k, m is the number of terms considered for the Prony series, μ_0 is the weight of the equilibrium modulus. It is obvious for the equation $\sum_{k=0}^{m} \mu_k = 1$ to be true, and the physical model of the Eq. (21) is presented in Fig. 3.

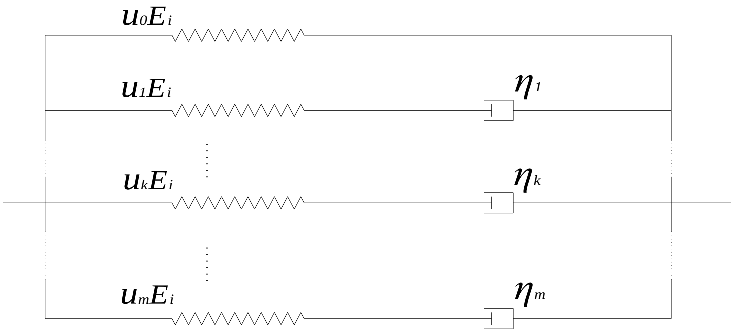


Fig. 3 Physical model of the hyper-viscoelastic model

In Fig. (3), E_i, μ_0, μ_k shares the same meaning with that in Eq. (21), η_k represents the viscosity coefficient and $\eta_k = \mu_k E_i t_k$, t_k is the relaxation time included in Eq. (21).

This physical model includes two main features, the first one is that it substitutes the stretch-dependent modulus spring for the constant modulus spring of the generalized Maxwell model, which is based on an assumption that the modulus corresponded to the viscosity coefficient η_k is obtained by multiplying the instantaneous modulus E_i and the weigh u_k. The second one is that the instantaneous modulus E_i can be decided by the hyperelasticity properties of materials, and the weight u_k is the undetermined parameters which is time independent.

If we stipulate that $t_k = 10^{k-1} t_1$, which is adopted by the FE software LS-DYNA(2007). Then, $\sigma(t)$ can be expressed as

$$\sigma(t) = \lambda \sum_{i=1}^{n} \int_{t_i}^{t_{i+1}} E_i (\mu_0 + \sum_{k=1}^{m} \mu_k e^{-(t-\xi)/(10^{k-1} t_1)}) d\xi \tag{22}$$

As mentioned before, E_i represents the instantaneous modulus of materials within the time range from t_i to t_{i+1}. It can be solved by the following procedures. Firstly, the fast stretch-stress experiment data is obtained by conducting experiment at sufficiently high speed to exclude the viscous effect. Then they will be fitted by the Ogden model to decide the undetermined parameters, which is explained in previous section. After acquiring the stretch-stress curve based on Ogden model, E_i will be solved by the derivation of the stress σ with respect to the principal stretch λ, namely, $\left(\frac{\partial\sigma}{\partial\lambda}\right)_{t=t_i}$, or taking the secant modulus as a substitution for the tangent modulus, namely, $(\sigma_{i+1}-\sigma_i)/(\lambda_{i+1}-\lambda_i)$.

In this section, the SS test is taken as an example, and the second way to get the instantaneous modulus is adopted, then the Eq. (22) will be converted into

$$T_{12} = \lambda\sum_{i=1}^{n}\frac{\tau_{i+1}-\tau_i}{\gamma_{i+1}-\gamma_i}\int_{t_i}^{t_{i+1}}(1-\sum_{k=1}^{m}\mu_k+\sum_{k=1}^{m}\mu_k e^{-(t-\xi)/(10^{k-1}t_1)})d\xi \tag{23}$$

where τ_i, γ_i is the corresponding shear stress and shear strain with the time t_i, and the shear stress can be solved by Eq. (14) and (15).

The Eq. (23) is the computational formula for the SS experiment of hyper-viscoelastic materials, and it is the foundation for the next section to realize the parameter identification of hyper-viscoelastic materials. It should be mentioned that the calculation error may be up to 10% when the shear strain is larger than 300%, which results from neglecting the influence of large deformation.

3.2 The parameter identification algorithm for the hyper-viscoelastic model

According to the Eq. (23), the number of the undetermined parameters is $m+1$, including u_k and t_1. Similarly to the Eq. (16), the object function for the parameter identification of hyper-viscoelastic model can be defined as follows,

$$min: \sum_{j=1}^{p}\sum_{i=1}^{n_j}(f_i-T_i)^2 \tag{24}$$

where p represents the number of the tests with different loading speed, n_j is the number of the experiment data for the jth group of experiment, and in this paper, n_j is set as a constant *r*, for simplicity. T_i is the experiment data, f_i is the theoretical value which is calculated by the Eq. (23).

The process of the parameter identification for the hyper-viscoelastic model is summarized into two steps. Firstly, the instantaneous shear modulus G_i at each time node t_i is calculated by the method proposed in previous section. Then, on the basis of the L-M nonlinear algorithm, the p groups of experiment data are fitted to realize the parameter identification.

Owing to the complexity of the Eq. (24), It is necessary here to explain how to get the sensitivity matrix in the L-M algorithm. As we can see from the Eq. (24), the object function is a summation of $p \cdot r$ functions, and the number of the undetermined parameters is $m+1$, so, the sensitivity matrix A can be defined as the Eq. (25).

$$A=\begin{bmatrix} \frac{\partial f_1}{\partial t_1} & \frac{\partial f_1}{\partial u_1} & \cdots & \frac{\partial f_1}{\partial u_k} & \cdots & \frac{\partial f_1}{\partial u_m} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{\partial f_i}{\partial t_1} & \frac{\partial f_i}{\partial u_1} & \cdots & \frac{\partial f_i}{\partial u_k} & \cdots & \frac{\partial f_i}{\partial u_m} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{\partial f_{pr}}{\partial t_1} & \frac{\partial f_{pr}}{\partial u_1} & \cdots & \frac{\partial f_{pr}}{\partial u_k} & \cdots & \frac{\partial f_{pr}}{\partial u_m} \end{bmatrix}_{pr\times(m+1)} \tag{25}$$

where $\frac{\partial f_i}{\partial u_k}$ can be solved by Eq. (23).

For the lack of the appropriate experiment data for SS experiment, we assume a type of high damping rubbers whose hyperealstic model is characterized by the last group of parameter in Table 1, and the viscoelastic model is characterized by the following Prony series,

$$P(\xi)=0.4e^{-(t-\xi)/10}+0.1e^{-(t-\xi)/100} \tag{26}$$

Then, we will generate four groups of experiment data by ANSYS software, the loading speed for each experiment is $0.5/s, 0.1/s, 0.01/s$ and $0.001/s$, the largest shear strain is set to be 200%, and the number r is set to be 9. All the figures generated are listed in Table 2,

Table. 2 The generated figures to be fitted

$\lambda=0.5/s$		$\lambda=0.1/s$		$\lambda=0.01/s$		$\lambda=0.001/s$	
γ	T_{12}	γ	T_{12}	γ	T_{12}	γ	T_{12}
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.03501	0.01293	0.03501	0.01286	0.03501	0.01212	0.03501	0.00889
0.14191	0.05213	0.14191	0.05097	0.14191	0.04221	0.14191	0.02959
0.41762	0.15035	0.41762	0.14123	0.41762	0.10239	0.41762	0.07994
0.71654	0.25024	0.71654	0.22585	0.71654	0.15934	0.71653	0.13213
1.01436	0.34158	1.01437	0.29705	1.01437	0.21031	1.01434	0.18124
1.31116	0.42469	1.31127	0.35685	1.31123	0.25613	1.31111	0.22747
1.60726	0.50106	1.60763	0.40808	1.60748	0.29803	1.60717	0.27157

1.98398	0.59172	1.98506	0.46532	1.98460	0.34783	1.98387	0.32617

It is necessary here to explain how to set the initial values for all of the undetermined parameters in Eq.(23). As for the weight u_k, it should be within the range from 0 to 1, and the summation of u_k, excluding u_0, should also be within the same range. While for the relaxation time t_1, it seems to be a bit more complicate. The suggested initial value for t_1 is the main relaxation time of materials, and the main relaxation time is the point corresponding to the relaxed stress $\sigma_{0.368}$ in the creep tests, $\sigma_{0.368}$ is defined as

$$\sigma_{0.368} = 0.368(\sigma_0 - \sigma_\infty) + \sigma_\infty \tag{27}$$

where σ_0 is the instantaneous stress and σ_∞ is the equilibrium stress.

The creep experiment of the materials, which is characterized by the Eq. (26) and the last group of parameters in Table 1, is conducted in ANSYS, and the main relaxation time is decided as 12.54s.

With regard to the number of the terms considered for the Prony series, the figures in Table 2 will be fitted more accurately if the number becomes larger, however, this may leads to unreasonable estimated parameters. So the number to be decided is suggested to make the largest relaxation time in Eq. (23) the same order of magnitude with the largest experiment time. For example, the largest experiment time in Table 2 is 2000s, so, the number of the terms considered for Prony series is 3 since the largest relaxation time is 1254s. With all the restrictions presented above, the initial values for the undetermined parameters are set as $[t_1, \mu_1, \mu_2, \mu_3] = [12.54, 0.3, 0.3, 0.3]$.

3.3 The identified parameters and the numerical verification

On the basis of the proceeding L-M nonlinear algorithm for hyper-viscoelastic materials, a group of parameters are obtained by using the data in Table 2, that is $[t_1, \mu_1, \mu_2, \mu_3] = [9.85231, 0.436469, 0.057897, 0.0051455]$. The deviation of the experiment data and the fitted data is 3.43E-4, which means the error is sufficiently small.

Similarly, in order to verify the accuracy of the estimated parameters, these parameters are employed in ANSYS to predict the response of the high damping materials. The results calculated in ANSYS and the data in Table 2 is plotted in Fig. 4.

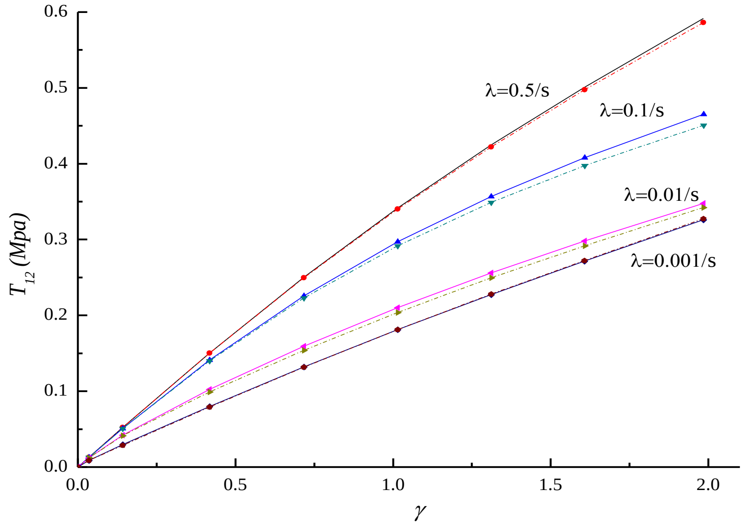
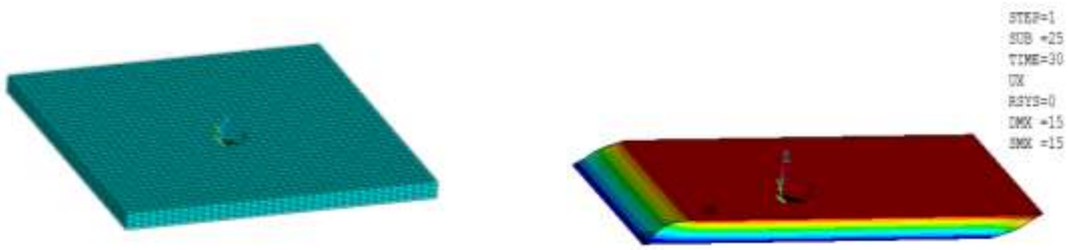


Fig. 4 Verification of the determined parameters in hyper-viscoelastic model

In Fig. 4, the solid lines represent the figures in Tab. (2), and the dotted line represent the data calculated in ANSYS. Obviously, the errors are very small since the largest one is only about 3.1%, which means the identification algorithm proposed in this paper is capable of estimating the parameters in hyper-viscoelastic models when the shear strain is lower than 200%, and the reason for the errors has been explained in the previous section. The FE initial model for SS experiment and its deformed shape with the shear strain being 200% are presented in Fig. 5.



(a) initial model (b) deformed model ($\gamma = 200\%$)

Fig. 5 The FE models for numerical verification

There exist some interesting results when using different initial values for the undetermined parameters. In some circumstances, some identified results of the weights u_k are negative, it is unreasonable in the sense of the physical model presented in Fig. 3 . However, if we ignore the physical meanings of these weights, it seems acceptable. Moreover, in most cases the identified negative weights are proved to be practical in ANSYS and the fitting effect is comparatively better.

At last, it deserves to be emphasized that the parameter identification algorithm for the SS experiment is on the assumption that the Ogden model is able to characterize the simple shear properties of materials. In the same way, this algorithm can be also used to identify parameters in the hyper-viscoelastic model with the ST experiment data, ET experiment data as well as PT data, the only thing needs to be done is substituting E_i determined by corresponding instantaneous experiments for the term $\frac{\tau_{i+1}-\tau_i}{\gamma_{i+1}-\gamma_i}$ in Eq. (23).

4. CONCLUSIONS

In this study, we focus on the parameter identification of the hyperelastic and hyper-viscoelastic materials, and the following conclusions have been made:

(1) On the basis of the Ogden model and the L-M nonlinear optimization algorithm, the comprehensive fitting of the multi-type experiment data, including ST, PT, ET, SS, of rubbers has been realized.

(2) According to the fitting results of the experiment data form Treloar and the verification results of the numerical simulation in ANSYS, the method for identifying parameters in Ogden model proposed in this paper is practical and efficient.

(3) The parameter identification method for the hyper-viscoelastic materials which is characterized by combing the Ogden model and the generalized Maxwell model was developed.

(4) In this paper, the SS test was taken as the example, and the proposed algorithm for identifying parameters in the hyper-viscoelastic model was proved practical and efficient when the shear strain is lower than 200%.

REFERENCES

Amin, A.F.M.S., Alam, M.S., and Okui, Y.(2002), "An improved hyperelasticity relation in modeling viscoelasticity response of natural and high damping rubbers in compression: experiments, parameter identification and numerical verification", Mech. Mater., 34, 75-95.

Amin, A.F.M.S., Wiraguna, S.I., Bhuiyan, A.R., and Okui, Y. (2006), "Hyperelasticity model for finite element analysis of natural and high damping rubbers in compression and shear", J. Eng. Mech., 132, 54-64.

Amin, A.F.M.S., Lion, A., Sekita, S., and Okui, Y. (2006), "Nonlinear dependence of viscosity in modeling the rate-dependent response of natural and high damping rubbers in compression and shear: experimental identification and numerical verification", Int. J. Plasticity., 22, 1610-1657.

Charlton, D.J., Yang, J., and Teh, K.K. (1994), "A review of methods to characterize rubber elastic behavior for use in finite analysis", Rubber. Chem. Technol., 67, 481-503.

Gendy, A.S., Saleeb, A.F. (2000), "Nonlinear material parameter estimation for characterizing hyperelastic large strain models", Comput. Mech., 25, 66-77.

Huber, N., Tsakmakis, C. (2000), "Finite deformation viscoelasticity laws", Mech. Mater., 32, 1-18.

Lai, W.M., Rubin, D., Krempl, E. (2010), "Introduction to Continuum Mechanics(Fourth Edition)", Oxford, Elsevier.

Lianis, G. (1963), "Constitutive equations for viscoelastic solids under finite deformation", Purdue Univ Report, A and ES, 63-11.

Livermore Software Technology Corporation (2007)," Ls-dyna keywords user's manual, version 971", Livermore Software Technology Corporation, California.

Ogden, R.W. (1973), "Large deformation isotropic elasticity - on the correlation of theory and experiment for incompressible rubber-like solids", Rubber. Chem. Technol., 46, 398-416.

The MathWorks (2010), "Matlab＆Simulink release notes for r2010b", Natick, Mass, USA.

Treloar, L.R.G. (1944), "Stress-Strain data for vulcanized rubber under various types of deformations", Trans. Faraday Soc, 1944, 40, 59-70.

Treloar, L.R.G (1975), "The physics of rubber elasticity", Oxford, Clarendon Press.

Wang, R., Li, S.Q., Song, S.Y. (2007), "A viscohyperelastic constitutive model for rubber used in vibration isolation", Noise and Vibration Worldwide, 38, 12-19.

Yang, T.Q., Luo, W.B., Xu, P., et al. (2004), "Theories and applications of viscoelasticity properties", Beijing, Science Press.

Yoshida, J., Abe, M., Fujino, Y.(2004), "Constitutive model of high-damping rubber materials", J Eng Mech, 2004, 130, 129-141.

Zhang, G.C., Wang, W.J., Han, H.L., et al.(2005), "Computational method for nonlinear optimization", Beijing, High Education Press.