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Effect of Pressure-Dependence of the Yield Criterion on Residual Stresses in Rotating Annular Discs

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ABSTRACT

The Drucker-Prager yield criterion is combined with an equilibrium equation to provide the elastic-plastic stress distribution within rotating annular discs and the residual stress distribution when the angular speed becomes zero. It is verified that unloading is purely elastic for the range of parameters used in the present study. Numerical techniques are only necessary to solve an ordinary differential equation of first order and a system of transcendental equations. The primary objective of this paper is to examine the effect of the parameter that controls the deviation of the Drucker-Prager yield criterion from the von Mises yield criterion on the stress distributions. It is shown that this parameter significantly influences the position of the elastic/plastic boundary and the distribution of the stresses.

1. INTRODUCTION

The Tresca yield criterion has long been associated with the solutions for the stresses and strains in a thin rotating, elastic/plastic disc. A review of such solutions is provided, for example, in Eraslan and Orcan (2002). Many of these solutions are analytic. Solutions for other yield criteria require numerical techniques of different levels of complexity. In particular, several solutions are available for the von Mises yield criterion used in the conjunction with the deformation theory of plasticity (You and and Zhang 1999, You *et all*. 2000, Eraslan and Argeso 2002, Eraslan and Akis 2003, and Hojjati and Hassani 2008). It is worthy of note that the deformation theory of plasticity is valid only when dealing with proportional loadings. On the other hand, it is known that the strain path is not proportional in thin discs, for example Pirumov *et all*. (2013). Comparisons between stress solutions based on these two criteria are presented in Rees (1999), and Alexandrova and Alexandrov (2004). Few further solutions that show the effect of yield criteria on the distribution of stresses and strains in rotating discs have appeared in the literature. In particular, plastic anisotropy has been taken into

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account in Karakuzu and Sayman (1994), Singh and Ray (2002), Callioglu (2004), and Alexandrova and Alexandrov (2004). Quite a general piece-wise linear yield surface has been adopted in Guo-wei and Mao-hong (1995), and Ma *et all*. (2001). All of the aforementioned solutions are for pressure-independent plasticity models. However, many metallic materials reveal pressure-dependence of plastic yielding (Spitzig *et all*. 1976, Kao *et all*. 1990, Wilson 2002, and Liu 2006). Plastic behavior of some metallic materials is adequately represented by the Drucker-Prager yield criterion in Drucker and Prager (1952) (see Wilson 2002, for a review). This yield criterion reduces to the von Mises yield criterion at specific values of input parameters. The effect of the deviation of the Drucker-Prager yield criterion from the von Mises yield criterion on the distribution of stresses in thin discs and plates subject to various loading conditions has been demonstrated in Pirumov *et all*. (2013), Alexandrov *et all*. (2011), Alexandrov *et all*. (2012), and Alexandrov *et all*. (2014).The present paper extends these analyses to rotating discs. It is worthy of note that, in contrast to classical metal plasticity, there is no commonly accepted model of pressure-dependent plasticity. Reviews and original models are provided in Spencer (1982), Druyanov (1993), and Harris (2014). However, the problem solved is statically determinate. Therefore, the distribution of stresses is found without using any equations that connect stress and plastic strain or strain rate. Numerical techniques are only necessary to solve an ordinary differential equation of first order and a system of transcendental equations.

2. STATEMENT OF THE PROBLEM

Consider a thin rotating disc with inner and outer radii a_0 and b_0, respectively (Fig. 1). The thickness of the disc is constant and its inner and outer radii are stress free. Introduce a cylindrical coordinate system (r, θ, z) whose $z-$ axis coincides with the axis of symmetry of the disc.

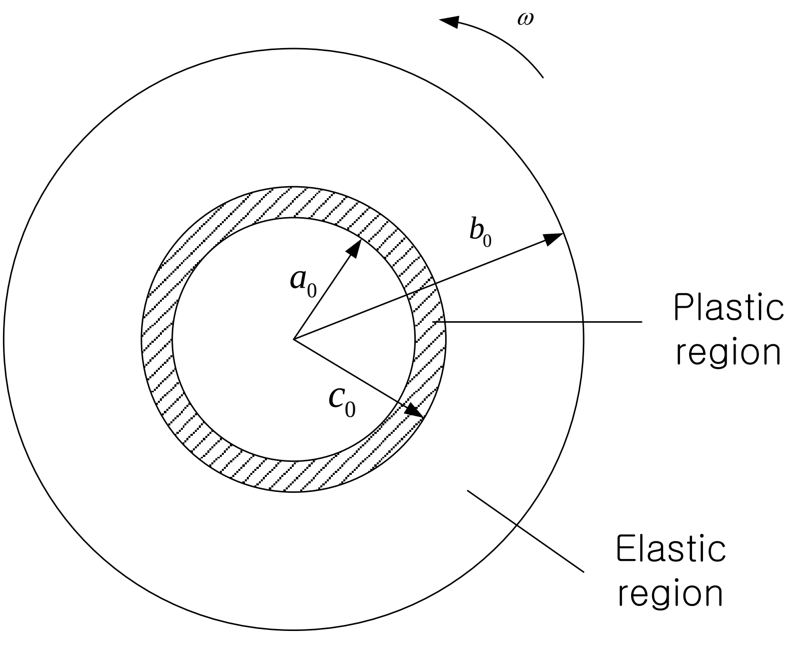

Figure 1. Disc configuration.

Let σ_r, σ_θ and σ_z be the normal stresses in this coordinate system. These stresses are the principal stresses. The plane state of stress is adopted. Therefore, $\sigma_z = 0$. The stress boundary conditions are

$$\sigma_r = 0 \tag{1}$$

for $r = a_0$ and $r = b_0$. The only non-trivial equilibrium equation is

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = -\rho\omega^2 r . \tag{2}$$

Here ρ is the density of the material and ω is the angular velocity of the disc about the *z*-axis. The angular velocity slowly increases from zero to its current value. The elastic portion of the strain tensor obeys Hooke's law. No relation between stress and plastic strain (or plastic strain rate) is required for stress analysis. The yield criterion in Drucker and Prager (1952) is,

$$\frac{\alpha}{3}\left(\sigma_r + \sigma_\theta + \sigma_z\right) + \frac{1}{\sqrt{2}}\sqrt{\left(\sigma_r - \sigma_\theta\right)^2 + \left(\sigma_\theta - \sigma_z\right)^2 + \left(\sigma_z - \sigma_r\right)^2} = \sigma_0 . \tag{3}$$

Here α and σ_0 are material constants. The yield criterion (3) reduces to the von Mises yield criterion at $\alpha = 0$. Thus α is a measure of pressure-dependence of plastic yielding. Under plane stress conditions, the yield criterion (3) becomes

$$\left(1 - \frac{\alpha^2}{9}\right)\sigma_r^2 + \left(1 - \frac{\alpha^2}{9}\right)\sigma_\theta^2 - \left(1 + \frac{2\alpha^2}{9}\right)\sigma_r\sigma_\theta + \frac{2\alpha}{3}\sigma_0\left(\sigma_r + \sigma_\theta\right) = \sigma_0^2 . \tag{4}$$

It is convenient to introduce the following dimensionless quantities

$$\Omega = \frac{\rho\omega^2 b_0^2}{\sigma_0}, \quad a = \frac{a_0}{b_0}, \quad \gamma = \frac{r}{b_0}. \tag{5}$$

3. PURELY ELASTIC SOLUTION

The general elastic solution to the problem formulated in the previous section is well known in Timoshenko and Goodier (1970) and can be written as

$$\frac{\sigma_r}{\sigma_0} = A + \frac{B}{\gamma^2} - \frac{\Omega\left(3+\nu\right)}{8}\gamma^2, \quad \frac{\sigma_\theta}{\sigma_0} = A - \frac{B}{\gamma^2} - \frac{\Omega\left(1+3\nu\right)}{8}\gamma^2. \tag{6}$$

Here ν is Poisson's ratio and *A* and *B* are constants of integration. At small ω the entire disc is elastic. In this case the stress distribution (6) must satisfy the boundary conditions (1). As a result,

$$A = A_e = \frac{\Omega\left(3+\nu\right)\left(1+a^2\right)}{8}, \quad B = B_e = -\frac{\Omega\left(3+\nu\right)a^2}{8}. \tag{7}$$

Substituting (7) into (6) supplies the stress distribution in the purely elastic disc in the form

$$\begin{aligned} \frac{\sigma_r}{\sigma_0} &= \frac{\Omega\left(3+\nu\right)}{8\gamma^2}\left(a^2 - \gamma^2\right)\left(\gamma^2 - 1\right), \\ \frac{\sigma_\theta}{\sigma_0} &= \frac{\Omega}{8\gamma^2}\left\{a^2\left(3+\nu\right)\left(1+\gamma^2\right) + \gamma^2\left[3+\nu-\left(1+3\nu\right)\gamma^2\right]\right\}. \end{aligned} \tag{8}$$

Assume that plastic yielding begins at $\gamma = a$. This assumption should be verified *a posteriori*. In this case, it follows from (4) and (8) that

$$\frac{\Omega_e^2}{16}\left(1-\frac{\alpha^2}{9}\right)\left[3+\nu+a^2\left(1-\nu\right)\right]^2+\frac{\alpha\Omega_e}{6}\left[3+\nu+a^2\left(1-\nu\right)\right]=1. \qquad (9)$$

Here Ω_e is the value of Ω corresponding to the initiation of plastic yielding. The solution of equation (9) is

$$\Omega_e=\frac{12}{\left(3+\alpha\right)\left[3+\nu+a^2\left(1-\nu\right)\right]}. \qquad (10)$$

4. ELASTIC/PLASTIC SOLUTION

If $\Omega \geq \Omega_e$, a plastic region appears in the disc. The angular velocity at which the entire disc becomes plastic is denoted by Ω_p. In the range $\Omega_p > \Omega > \Omega_e$ the disc consists of an inner plastic region, $a \leq \gamma \leq \gamma_c$ (or $a_0 \leq r \leq c_0$), surrounded by an outer elastic region, $\gamma_c \leq \gamma \leq 1$ (or $c_0 \leq r \leq b_0$). Here c_0 is the radius of the elastic/plastic boundary and $\gamma_c = c_0/b_0$ is its dimensionless representation (Fig. 1). The general solution (6) is valid in the elastic region. However, *A* and *B* are not determined by (7). The solution (6) must satisfy the boundary condition (1) at $\gamma = 1$. Therefore,

$$A+B=\frac{\Omega\left(3+\nu\right)}{8}. \qquad (11)$$

Consider the plastic region. The yield criterion (4) is satisfied by the following substitution (Alexandrov *et all*. 2011),

$$\frac{\sigma_r}{\sigma_0}=3\beta_0-\frac{\beta_1}{2}\left(1+3\sqrt{3}\beta_1\right)\sin\psi+\frac{\sqrt{3}}{2}\beta_1\left(1-\sqrt{3}\beta_1\right)\cos\psi,$$
$$\frac{\sigma_\theta}{\sigma_0}=3\beta_0+\frac{\beta_1}{2}\left(1-3\sqrt{3}\beta_1\right)\sin\psi-\frac{\sqrt{3}}{2}\beta_1\left(1+\sqrt{3}\beta_1\right)\cos\psi, \qquad (12)$$
$$\beta_0=\frac{2\alpha}{4\alpha^2-9},\quad \beta_1=\frac{\sqrt{3}}{\sqrt{9-4\alpha^2}}.$$

Here ψ is a new function of γ. Substituting (12) into (2) and using (5) yield

$$\gamma\left[\sin\left(\psi+\frac{\pi}{6}\right)-\frac{3\sqrt{3}}{\sqrt{9-4\alpha^2}}\sin\left(\psi-\frac{\pi}{3}\right)\right]\frac{\partial\psi}{\partial\gamma}+2\sin\left(\psi-\frac{\pi}{3}\right)=\frac{\gamma^2\Omega\sqrt{9-4\alpha^2}}{\sqrt{3}}. \qquad (13)$$

This equation should be solved numerically. Let ψ_a be the value of ψ at $\gamma = a$. Then, the boundary condition to equation (13) is

$$\psi=\psi_a \qquad (14)$$

for $\gamma = a$. It follows from (12) that

$$\frac{\sigma_\theta-\sigma_r}{\sigma_0}=\frac{2\sqrt{3}}{\sqrt{9-4\alpha^2}}\sin\left(\psi-\frac{\pi}{3}\right),\quad \frac{\sigma_\theta+\sigma_r}{\sigma_0}=\frac{12\alpha}{4\alpha^2-9}+\frac{18}{\left(4\alpha^2-9\right)}\sin\left(\psi+\frac{\pi}{6}\right). \qquad (15)$$

It is evident that $\sigma_\theta > \sigma_r$. Therefore, it is seen from (15) that

$$\frac{\pi}{3} < \psi < \frac{4\pi}{3}. \tag{16}$$

The boundary condition (1) at $\gamma = a$ and (12) combine to give

$$\psi_a = \frac{4\pi}{3} - \arcsin q, \quad q = \frac{\sqrt{3}\sqrt{9-4\alpha^2}}{2(3+\alpha)}. \tag{17}$$

Here the inequality (16) has been taken into account. Let ψ_c be the value of ψ at $\gamma = \gamma_c$ .The radial and circumferential stresses must be continuous across the elastic/plastic boundary. Therefore, it follows from (6), (11) and (15) that

$$\begin{gathered}\frac{\Omega(1-\nu)\gamma_c^4 - 8B}{4\gamma_c^2} = \frac{2\sqrt{3}}{\sqrt{9-4\alpha^2}}\sin\left(\psi_c - \frac{\pi}{3}\right),\\ 2B - \frac{\Omega}{4}\left[3+\nu-2(1+\nu)\gamma_c^2\right] = \frac{6}{\left(9-4\alpha^2\right)}\left[2\alpha + 3\sin\left(\psi_c + \frac{\pi}{6}\right)\right].\end{gathered} \tag{18}$$

Eliminating *B* between these two equations yields

$$\begin{aligned}(1-\nu)\Omega\gamma_c^4 - \frac{24}{\left(9-4\alpha^2\right)}\left[2\alpha + 3\sin\left(\psi_c + \frac{\pi}{6}\right)\right] - \Omega\left[3+\nu-2(1+\nu)\gamma_c^2\right] =\\ = \frac{8\sqrt{3}\gamma_c^2}{\sqrt{9-4\alpha^2}}\sin\left(\psi_c - \frac{\pi}{3}\right).\end{aligned} \tag{19}$$

Then, the value of *B* can be found from any of equations (18). As a result,

$$B = \frac{\Omega}{8}\left[3+\nu-2(1+\nu)\gamma_c^2\right] + \frac{3}{\left(9-4\alpha^2\right)}\left[2\alpha + 3\sin\left(\psi_c + \frac{\pi}{6}\right)\right]. \tag{20}$$

The solution to equation (13) supplies the dependence of ψ_c on γ_c . This dependence and (19) constitute the system of equation to determine ψ_c and γ_c at any given value of Ω in the range $\Omega_p > \Omega > \Omega_e$. This system should be solved numerically. The value of Ω_p can be found without considering the solution in the elastic region. Since $\sigma_r = 0$ at $\gamma = 1$, it is evident that $\psi = \psi_a$ at $\gamma = 1$ in the fully plastic disc. This condition and the solution to equation (13) satisfying the boundary condition (17) determine the value of Ω_p. The variation of Ω_p with *a* at $\nu = 0.3$ and several values of α is depicted in Fig. 2. The effect of $\alpha-$ value on γ_c and ψ_c in the range $\Omega_p > \Omega > \Omega_e$ is illustrated in Figs 3 to 8 ($a = 0.1$ in Figs. 4 and 6, $a = 0.3$ in Figs. 4 and 7, and $a = 0.5$ in Figs. 5 and 8). The distribution of stresses can now be found from (6), (11) and (20) in the elastic region, $\gamma_c \le \gamma \le 1$, and from (12) and the solution to equation (13) in the plastic region, $a \le \gamma \le \gamma_c$. In particular, the dependence of the radial and circumferential stresses within a $a = 0.3$ disc at $\nu = 0.3$ is shown in Figs. 9 to 14. In order to reveal the effect of $\alpha-$ value on these stress distributions, the parameters needed for this calculation have been chosen as follows. Firstly, it is assumed that $\gamma_c = 0.4$ at $\alpha = 0$.

Then, the corresponding values of ψ_c and Ω are found from (19) and the solution to equation (13). Secondly, this value of Ω is substituted into (19) and the system of equations consisting of this equation and the solution to equation (1) is solved for γ_c and ψ_c at $\alpha = 0.1$, $\alpha = 0.2$, and $\alpha = 0.3$. As a result, the curves shown in Figs. 9 and 12 are obtained. The curves depicted in Figs. 10, 11, 13, and 14 are obtained in a similar manner assuming that $\gamma_c = 0.5$ at $\alpha = 0$ (Figs. 10 and 13) and $\gamma_c = 0.6$ at $\alpha = 0$ (Figs. 11 and 14). The values of the parameters found are shown in Table 1. The validity of the solution has been confirmed by the inequality

$$\left(1-\frac{\alpha^2}{9}\right)\left(\frac{\sigma_r}{\sigma_0}\right)^2+\left(1-\frac{\alpha^2}{9}\right)\left(\frac{\sigma_\theta}{\sigma_0}\right)^2-\left(1+\frac{2\alpha^2}{9}\right)\frac{\sigma_r}{\sigma_0}\frac{\sigma_\theta}{\sigma_0}+\frac{2\alpha}{3}\left(\frac{\sigma_r}{\sigma_0}+\frac{\sigma_\theta}{\sigma_0}\right)-1\le 0$$

in the elastic region. This inequality follows from (4).

Table 1. Parameters used to calculate the distribution of stresses at different values of α

γ_c, $\alpha=0$	Ψ_c, $\alpha=0$	Ω	γ_c, $\alpha=0.1$	Ψ_c, $\alpha=0.1$	γ_c, $\alpha=0.2$	Ψ_c, $\alpha=0.2$	γ_c, $\alpha=0.3$	Ψ_c, $\alpha=0.3$
0.4	3.33	1.62	0.42	3.4	0.45	3.47	0.48	3.59
0.5	3.41	1.88	0.54	3.47	0.6	3.52	0.66	3.55
0.6	3.43	2.05	0.67	3.46	0.76	3.46	0.9	3.39

5. RESIDUAL STRESSES

Let Ω_f be the maximum value of the angular velocity. Then, assuming that unloading is purely elastic the stress increments, $\Delta\sigma_r$ and $\Delta\sigma_\theta$, when the angular velocity decreases from Ω_f to zero are determined by (6) and (7) where Ω should be replaces with $-\Omega_f$, A with ΔA, and B with ΔB. In particular,

$$\Delta A = -\frac{\Omega_f(3+\nu)(1+a^2)}{8}, \quad \Delta B = \frac{\Omega_f(3+\nu)a^2}{8},$$
$$\frac{\Delta\sigma_r}{\sigma_0} = \Delta A + \frac{\Delta B}{\gamma^2} + \frac{\Omega_f(3+\nu)}{8}\gamma^2, \quad \frac{\Delta\sigma_\theta}{\sigma_0} = \Delta A - \frac{\Delta B}{\gamma^2} + \frac{\Omega_f(1+3\nu)}{8}\gamma^2. \qquad (21)$$

The residual stresses are found as

$$\frac{\sigma_r^{res}}{\sigma_0} = \frac{\sigma_r}{\sigma_0} + \frac{\Delta\sigma_r}{\sigma_0}, \quad \frac{\sigma_\theta^{res}}{\sigma_0} = \frac{\sigma_\theta}{\sigma_0} + \frac{\Delta\sigma_\theta}{\sigma_0}. \qquad (22)$$

Here σ_r are σ_θ are understood to be known from the solution of Section 4 at $\Omega = \Omega_f$. Then, (21) and (22) supply the distribution of the residual stresses. The validity of this solution is controlled by the following condition that can be derived from (4)

$$\left(1-\frac{\alpha^2}{9}\right)\left(\frac{\sigma_r^{res}}{\sigma_0}\right)^2+\left(1-\frac{\alpha^2}{9}\right)\left(\frac{\sigma_\theta^{res}}{\sigma_0}\right)^2- \\ -\left(1+\frac{2\alpha^2}{9}\right)\frac{\sigma_r^{res}}{\sigma_0}\frac{\sigma_\theta^{res}}{\sigma_0}+\frac{2\alpha}{3}\left(\frac{\sigma_r^{res}}{\sigma_0}+\frac{\sigma_\theta^{res}}{\sigma_0}\right)-1\le 0. \tag{23}$$

The variation of the residual stresses with γ corresponding to the distribution of the stresses at $\Omega = \Omega_f$ depicted in Figs. 9 to 14 is shown in Figs. 15 to 20. The inequality (23) has been verified numerically.

6. CONCLUSION

A new semi-analytic solution for thin rotating annular discs has been found. Numerical techniques are only necessary to solve differential equation (13) and the system of transcendental equations consisting of equation (19) and the solution to equation (13). The primary objective of the present paper is to reveal the effect of α on the solution. Note that $\alpha = 0$ corresponds to the von Mises yield criterion. Therefore, the value of α is a measure of the deviation of the Drucker-Prager yield criterion from the von Mises yield criterion. The effect of α on the value of the angular velocity at which the entire disc becomes plastic is illustrated in Fig. 2. It is seen from this figure that the larger α, the less Ω_p. It is seen from Figs. 3, 5, and 7 that the value of α significantly influences the radius of the elastic/plastic boundary. The variation of ψ_c with Ω and α for a $a = 0.3$ disc is illustrated in Figs. 4, 6, and 8. Even though ψ_c is an auxiliary parameter, its value is important since reasonable initial values of ψ_c and γ_c should be guessed to solve the system of equations consisting of equation (19) and the solution to equation (13). The effect of α on the distribution of the stresses within a rotating $a = 0.3$ disc is seen from Figs. 9 to 14. This effect is pronounced in the vicinity of the elastic/plastic boundary, especially for the circumferential stress (Fig. 12 to 14). In general, the radial stress is less affected by the value of α than the circumferential stress. This is not surprisingly because the magnitude of the radial stress is fixed at the inner and outer radii of the disc and this magnitude is independent of α. However, the effect of pressure dependence of the yield criterion on the radial stress is larger than the effect of plastic anisotropy reported in Singh and Ray (2002). The effect of α on the distribution of the residual stresses within the same $a = 0.3$ disc is illustrated in Figs. 15 to 20. It is seen from Fig. 15 to 17 that the distribution of the residual radial stresses is more affected by the value of α than the corresponding distribution of the radial stresses within the rotating disc at $\Omega = \Omega_f$. On the other hand, the effect of α – value on the distribution of the residual circumferential stresses (Fig. 18 to 20) is less pronounced than on the corresponding distribution of the circumferential stresses within the rotating disc at $\Omega = \Omega_f$.

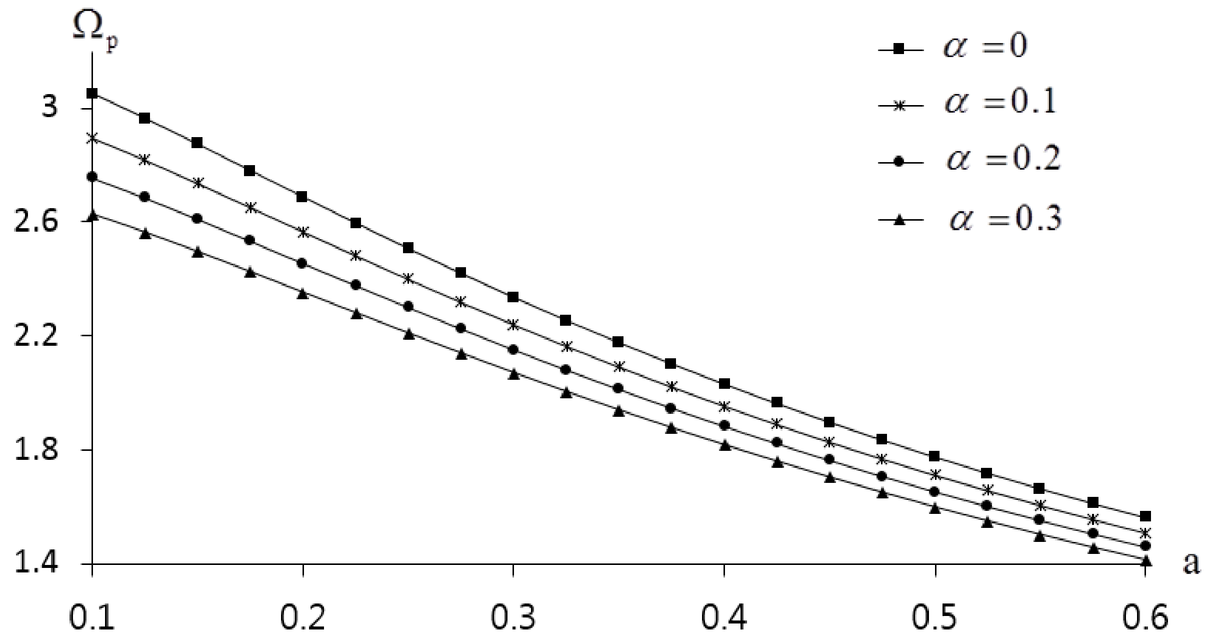

Figure 2. Variation of Ω_p with *a* at $\nu = 0.3$ and several $\alpha -$ values.

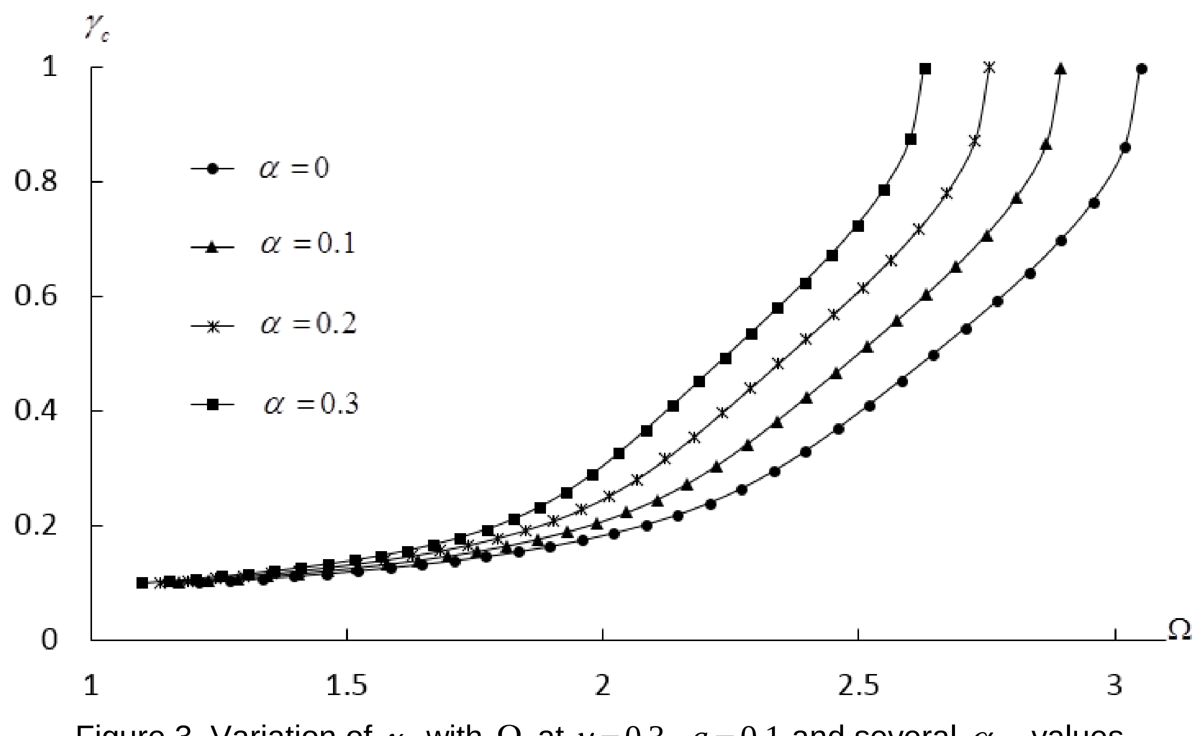

Figure 3. Variation of γ_c with Ω at $\nu = 0.3$, $a = 0.1$ and several $\alpha -$ values.

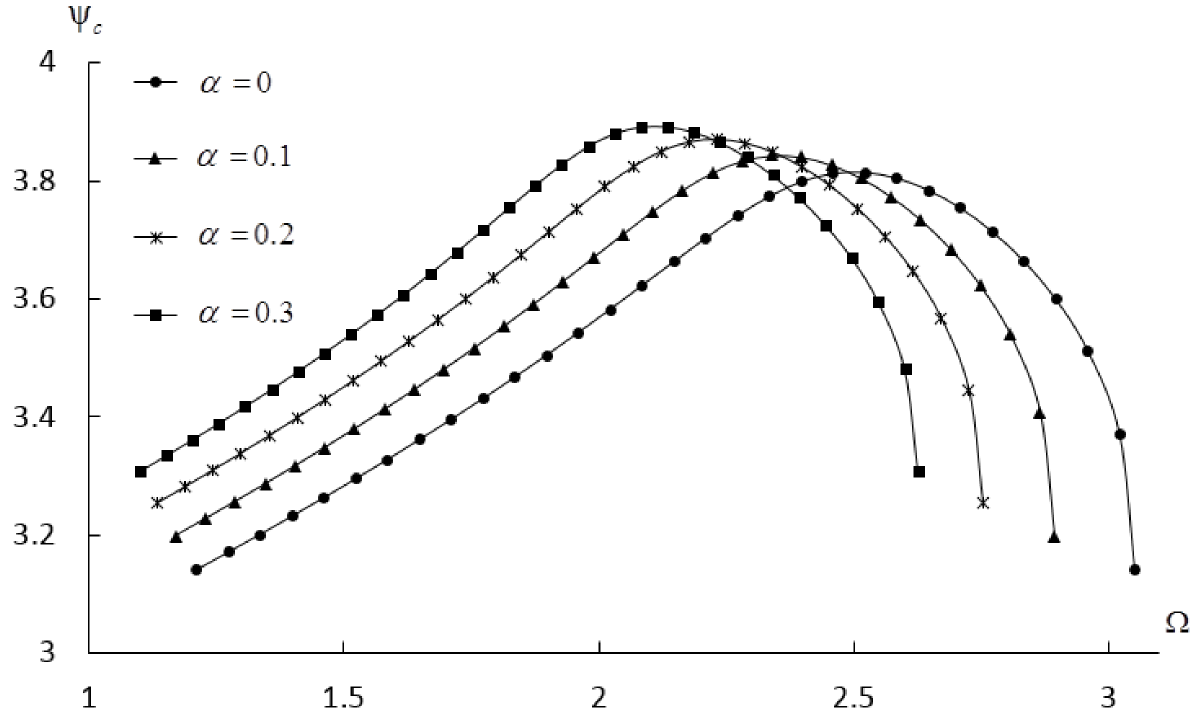

Figure 4. Variation of ψ_c with Ω at $\nu = 0.3$, $a = 0.1$ and several $\alpha -$ values.

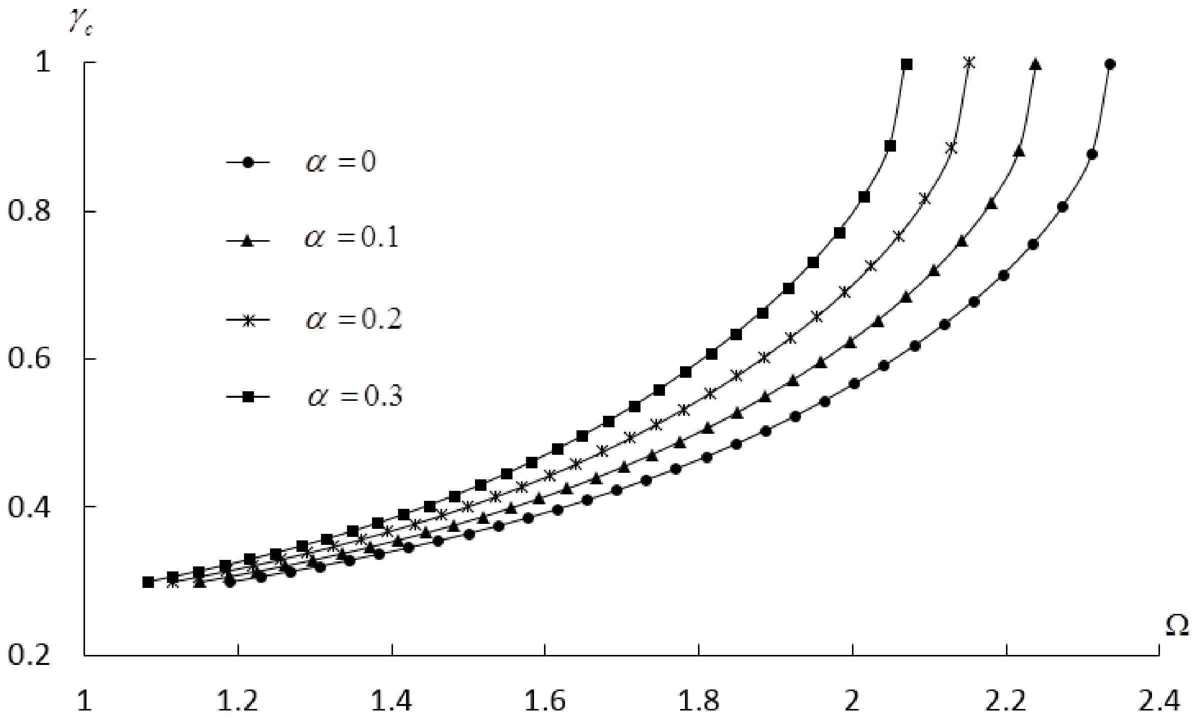

Figure 5. Variation of γ_c with Ω at $\nu = 0.3$, $a = 0.3$ and several $\alpha -$ values.

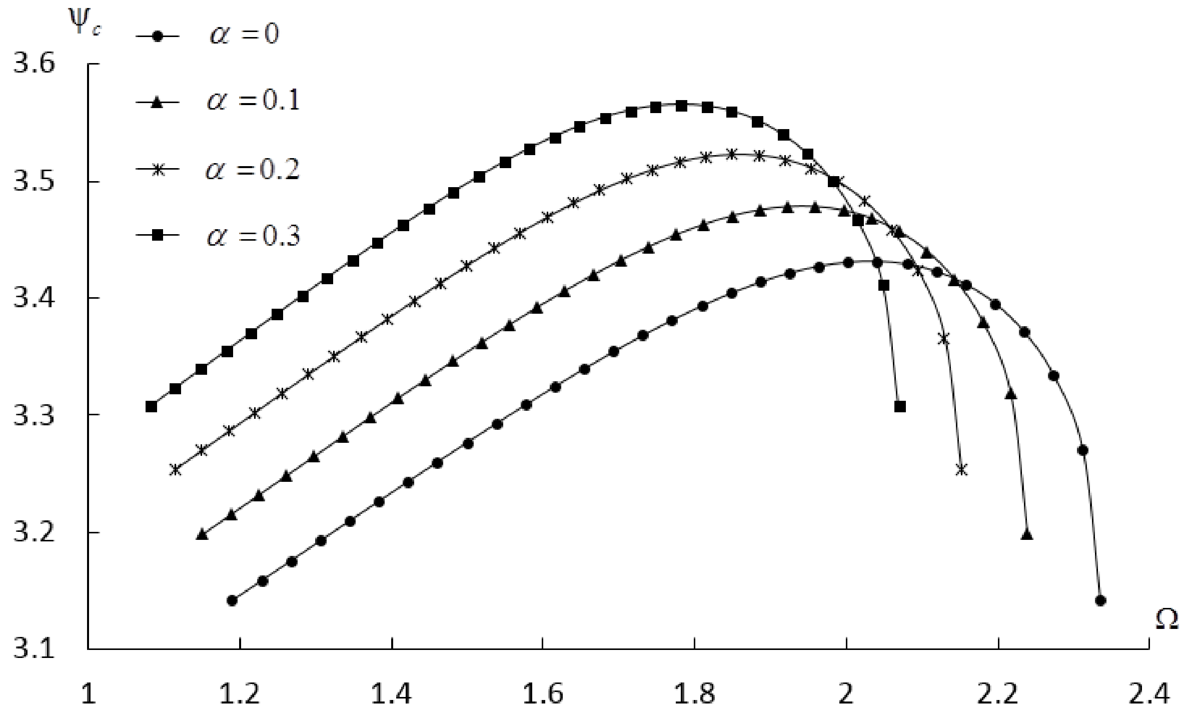

Figure 6. Variation of ψ_c with Ω at $\nu = 0.3$, $a = 0.3$ and several $\alpha -$ values.

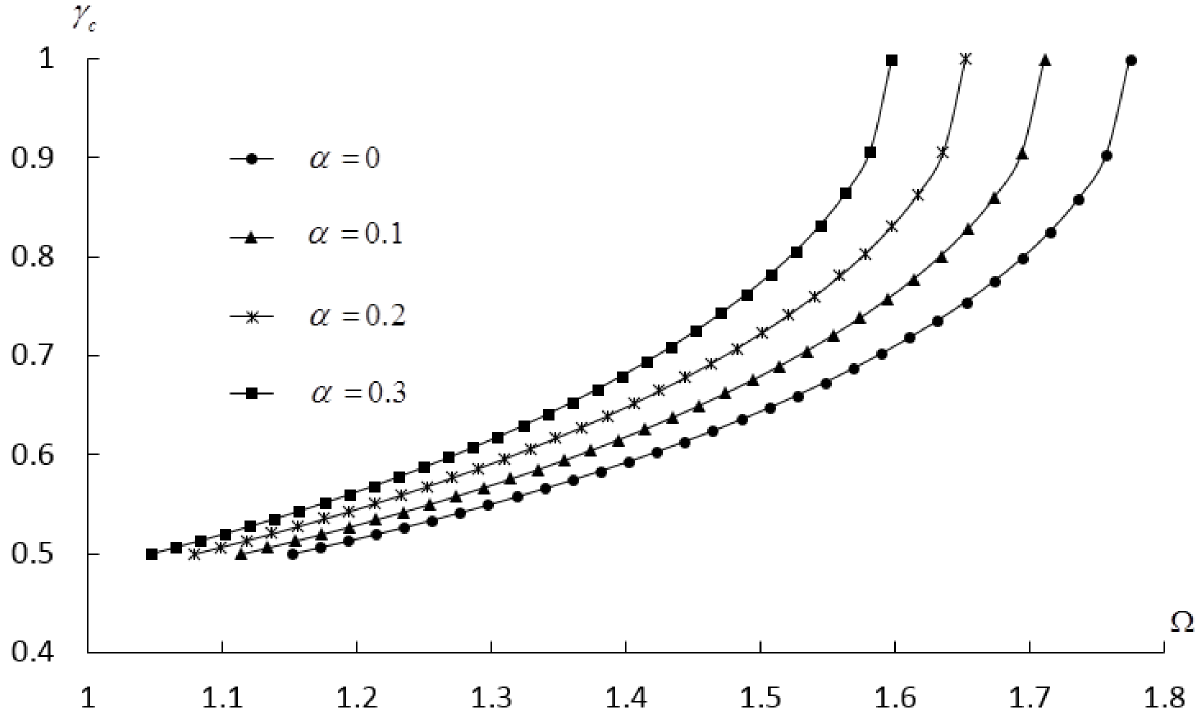

Figure 7. Variation of γ_c with Ω at $\nu = 0.3$, $a = 0.5$ and several $\alpha -$ values.

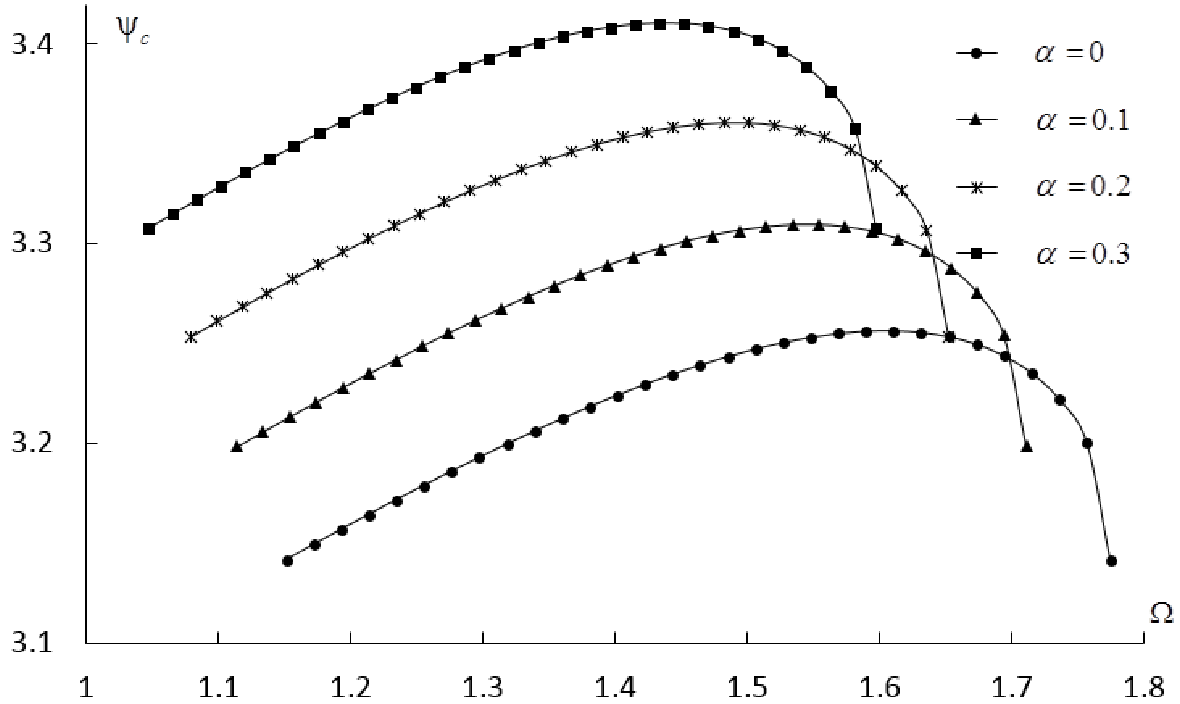

Figure 8. Variation of ψ_c with Ω at $\nu = 0.3$, $a = 0.5$ and several $\alpha -$ values.

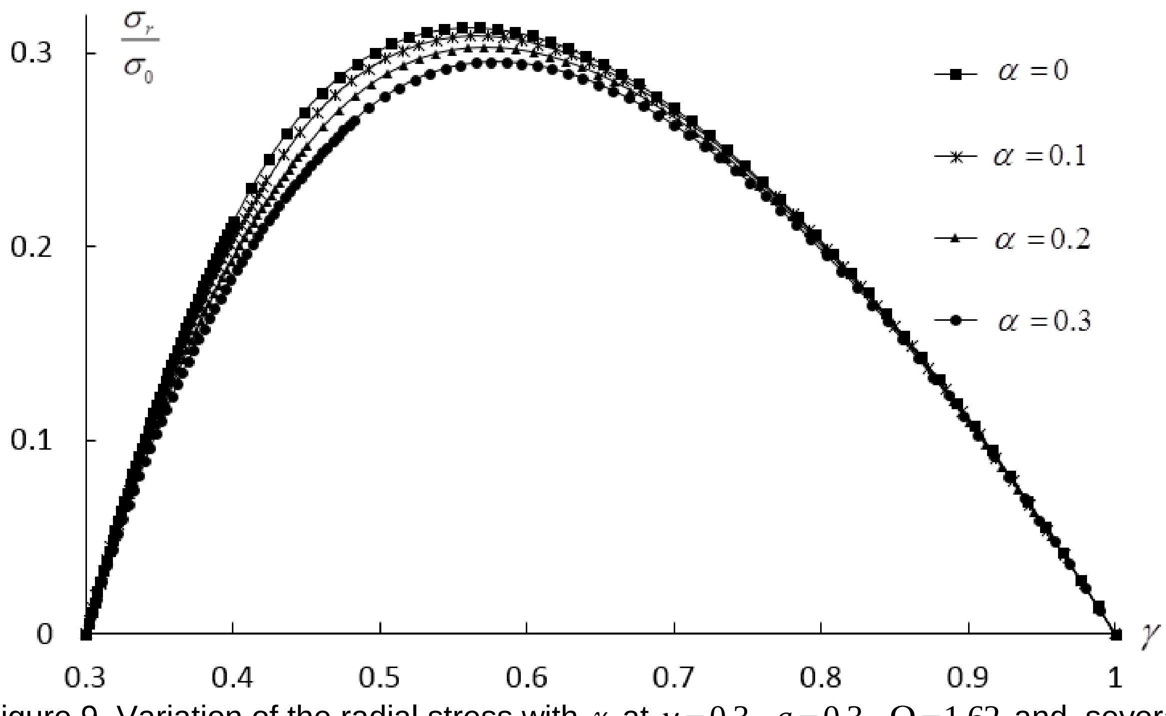

Figure 9. Variation of the radial stress with γ at $\nu = 0.3$, $a = 0.3$, $\Omega = 1.62$ and several $\alpha -$ values.

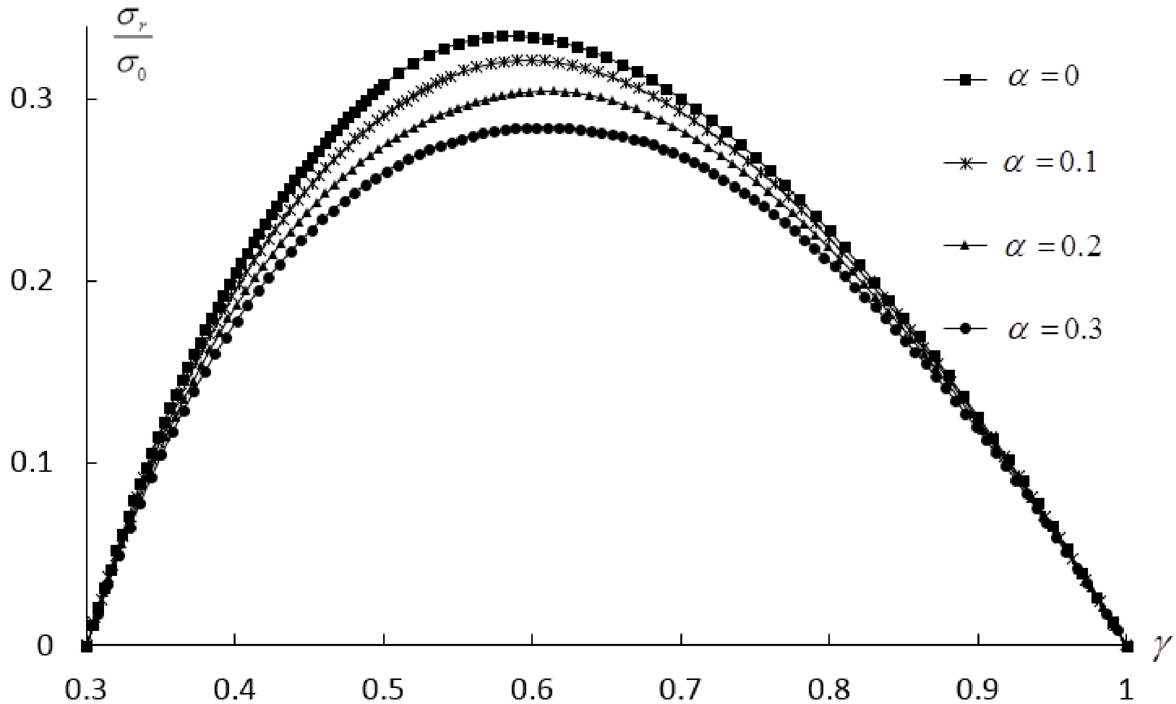

Figure 10. Variation of the radial stress with γ at $\nu = 0.3$, $a = 0.3$, $\Omega = 1.88$ and several $\alpha -$ values.

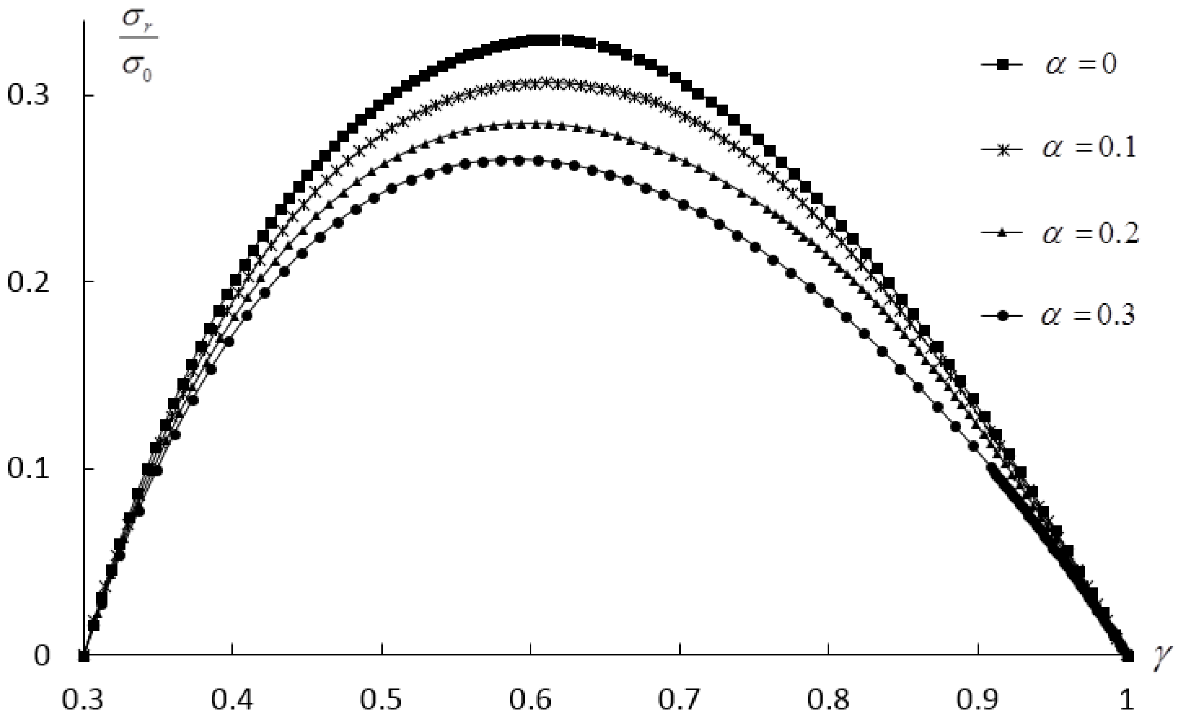

Figure 11. Variation of the radial stress with γ at $\nu = 0.3$, $a = 0.3$, $\Omega = 2.05$ and several $\alpha -$ values.

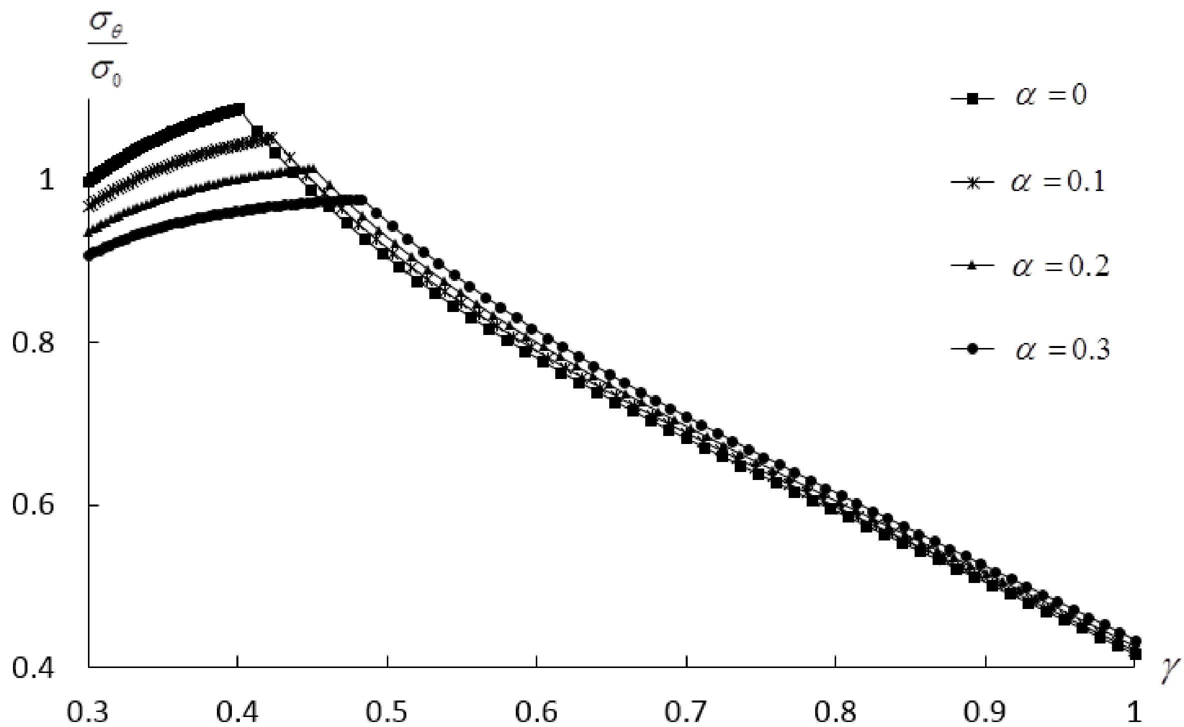

Figure 12. Variation of the circumferential stress with γ at $\nu = 0.3$, $a = 0.3$, $\Omega = 1.62$ and several $\alpha -$ values.

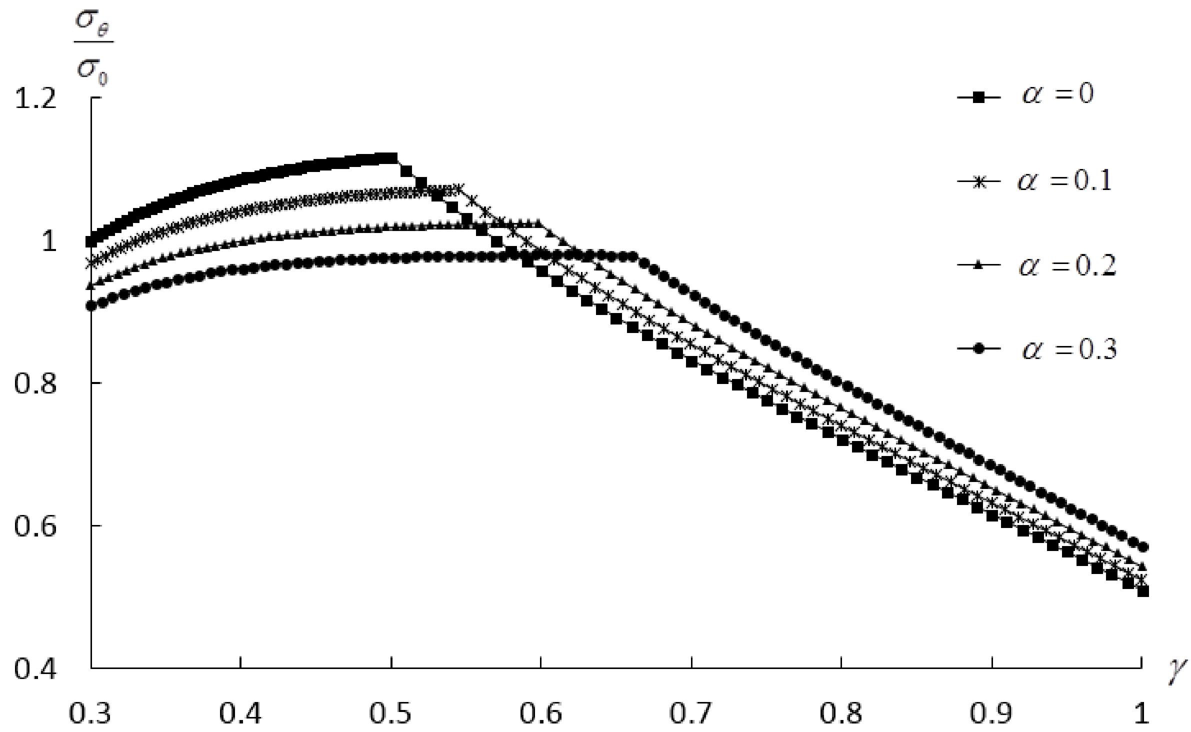

Figure 13. Variation of the circumferential stress with γ at $\nu = 0.3$, $a = 0.3$, $\Omega = 1.88$ and several $\alpha -$ values.

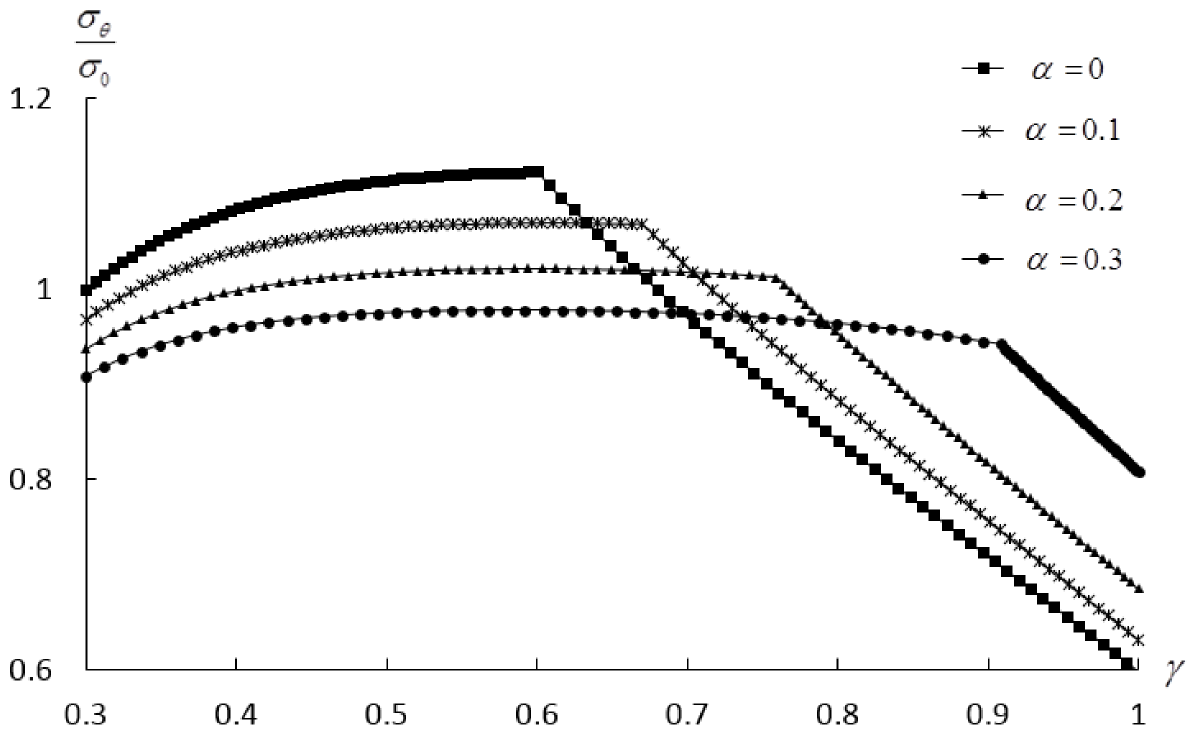

Figure 14. Variation of the circumferential stress with γ at $\nu = 0.3$, $a = 0.3$, $\Omega = 2.05$ and several $\alpha -$ values.

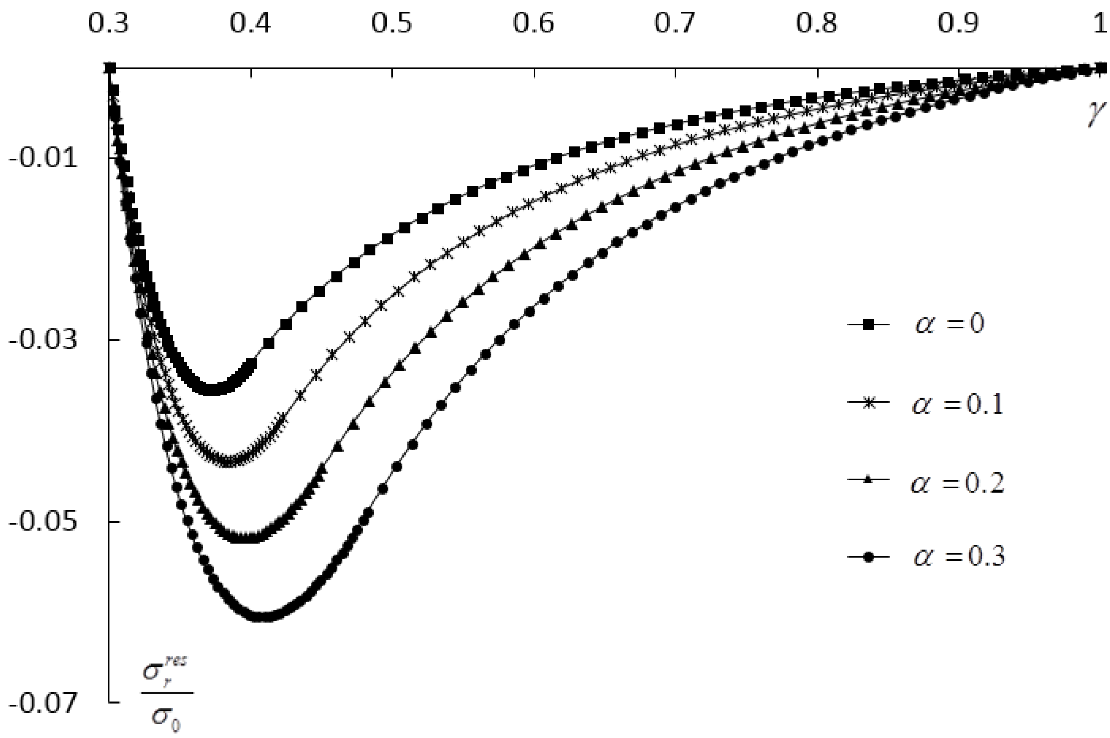

Figure 15. Variation of the residual radial stress with γ at $\nu = 0.3$, $a = 0.3$, $\Omega_f = 1.62$ and several $\alpha -$ values.

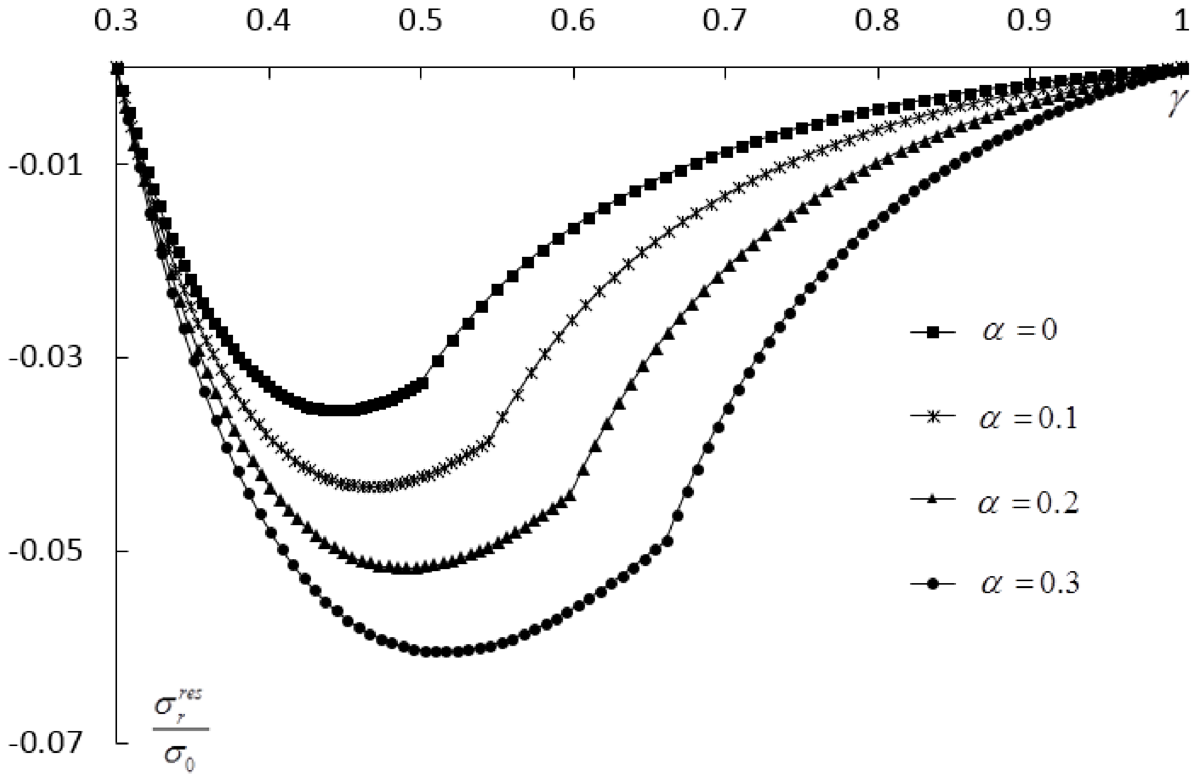

Figure 16. Variation of the residual radial stress with γ at $\nu = 0.3$, $a = 0.3$, $\Omega_f = 1.88$ and several $\alpha -$ values.

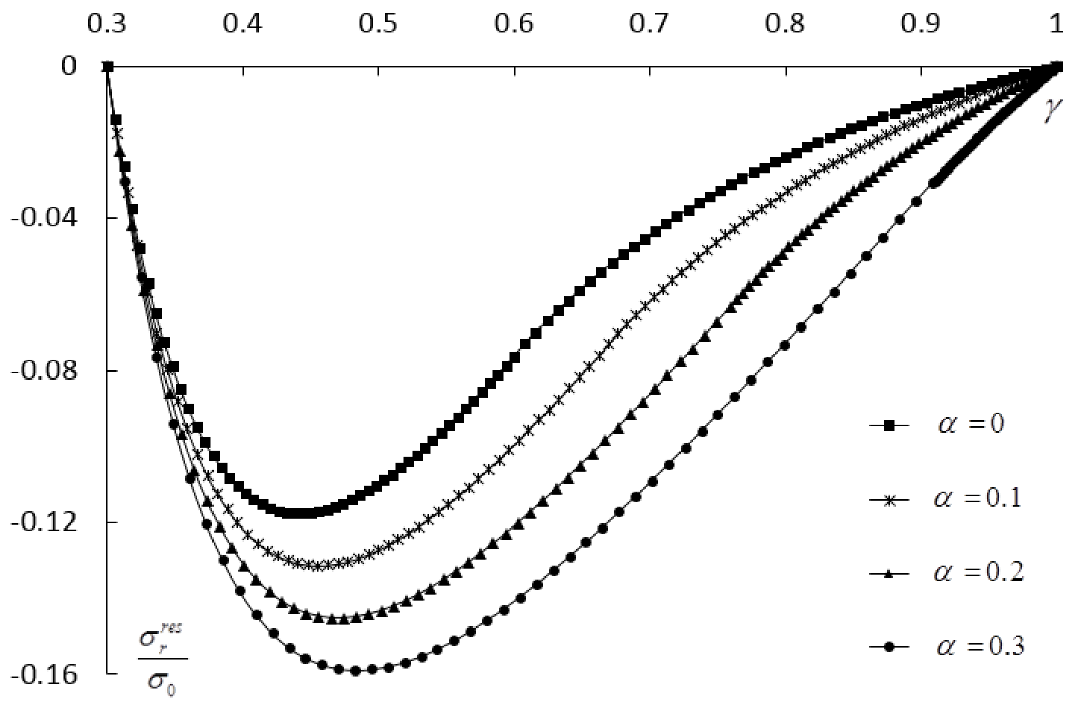

Figure 17. Variation of the residual radial stress with γ at $\nu = 0.3$, $a = 0.3$, $\Omega_f = 2.05$ and several $\alpha -$ values.

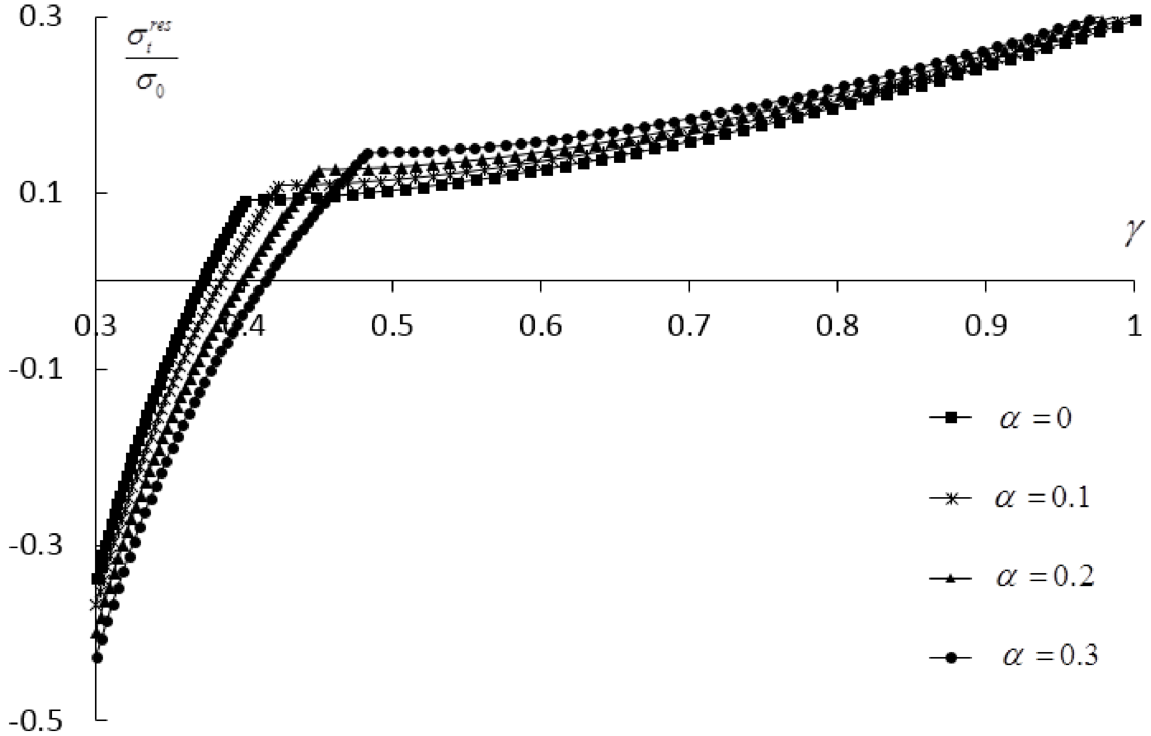

Figure 18. Variation of the residual circumferential stress with γ at $\nu = 0.3$, $a = 0.3$, $\Omega_f = 1.62$ and several $\alpha -$ values.

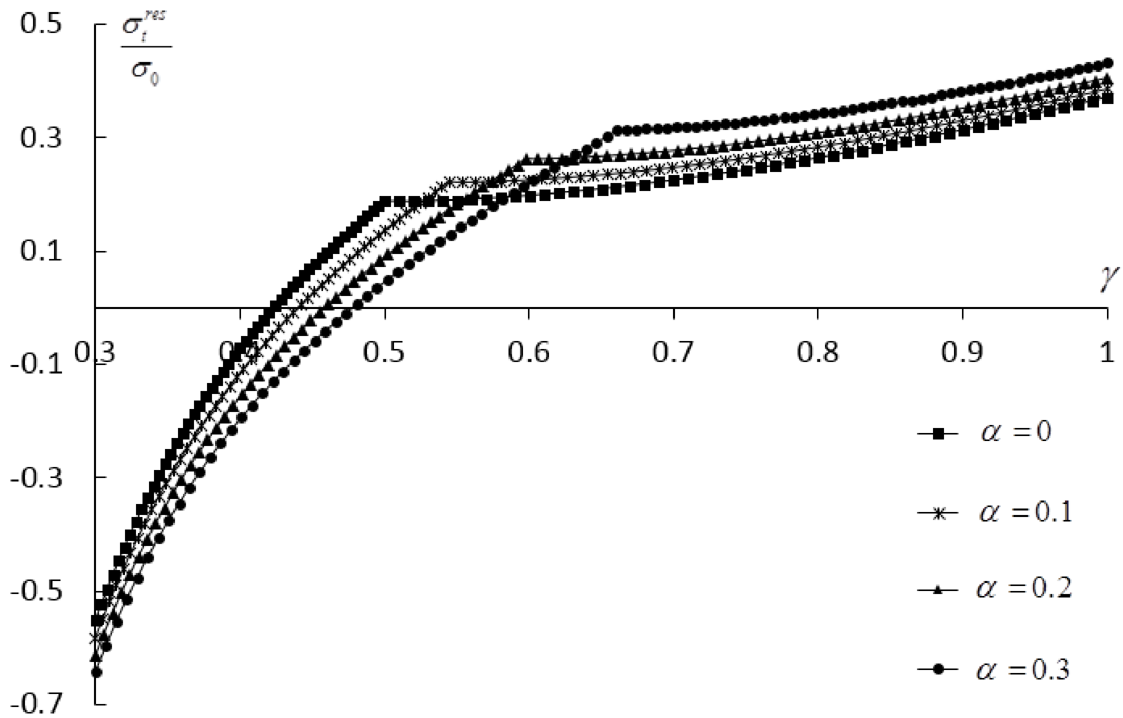

Figure 19. Variation of the residual circumferential stress with γ at $\nu = 0.3$, $a = 0.3$, $\Omega_f = 1.88$ and several $\alpha -$ values.

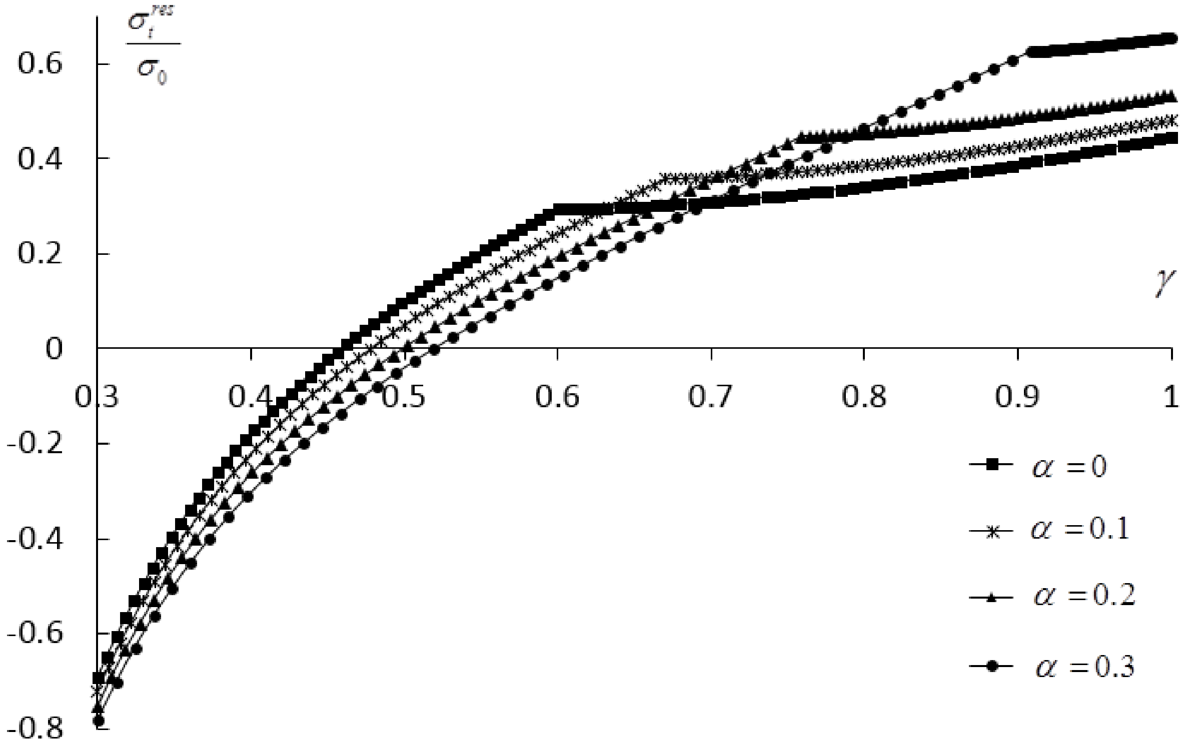

Figure 20. Variation of the residual circumferential stress with γ at $\nu = 0.3$, $a = 0.3$, $\Omega_f = 2.05$ and several $\alpha -$ values.

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