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Application of an explicit dynamic method for discontinuous analysis by using rigid bodies–spring model

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ABSTRACT

This paper proposes an explicit dynamic model to solve sequentially large deformation behavior; this model is expanded to include the rigid bodies–spring model (RBSM) that is suitable for analysis of progressive failures such as complex landslides. First, we define RBSM based on the hybrid principle of virtual work, and then, we describe a new RBSM approach for the contact mechanism and friction characteristics. Since the approach is similar to the distinct element method, it is possible to explain the behavior of the failure mechanism after the formation. Finally, to confirm the applicability of the stability analysis of landslide, we evaluate the dynamic behavior in multiple block models.

1. INTRODUCTION

In the analysis of fracture problems, the use of a discrete element model by RBSM is effective. (Kawai 1977) RBSM was developed as a numerical model for generalizing the limit analysis in plasticity, in which a structure to be analyzed is idealized as an assemblage of rigid bodies connected by normal and tangential springs. On the other hand, by using the principle of hybrid-type virtual work, the authors have applied an explicit method to each element in their previous study. (Yagi 2015) In this paper, we illustrate the formulization of RBSM that has been expanded to include the distinct element method (DEM). Further, we describe a new RBSM approach for the contact mechanism and friction characteristics. Since the approach is similar to the distinct element method, it is possible to explain the behavior of the failure mechanism after the formation. In the present work, it is intended to confirm the applicability of the stability analysis of landslides by evaluating the dynamic behavior in multiple block models.

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2. BRIEF FORMULATIONS

We introduce a subsidiary condition into the framework of the variational equation with Lagrange multipliers λ, such that the hybrid-type virtual work equation can be described as follows, within the subdomain M and intersection boundary N:

$$\sum_{e=1}^{M}\left(\int_{\Omega^{(e)}} \boldsymbol{\sigma} : \mathrm{grad}(\delta \boldsymbol{u})dV - \int_{\Omega^{(e)}} \boldsymbol{f}\cdot\delta\boldsymbol{u}dV - \int_{\Omega^{(e)}} \boldsymbol{f}_{\boldsymbol{\alpha}}\cdot\delta\boldsymbol{u}dV\right) - \sum_{s=1}^{N}\left(\delta\int_{\Gamma_{<s>}} \boldsymbol{\lambda}\cdot(\tilde{\boldsymbol{u}}^{(a)}-\tilde{\boldsymbol{u}}^{(b)})\,dS\right) - \int_{\Gamma_{\sigma}} \bar{\boldsymbol{t}}\cdot\delta\boldsymbol{u}\,dS = 0 \tag{1}$$

This equation implies that the Lagrange multiplier λ is the surface force on the boundary $\Gamma_{<ab>}$ in the subdomains $\Omega^{(a)}$ and $\Omega^{(b)}$, as shown in Fig. 1.

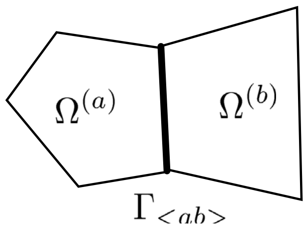

Fig. 1 Subdomain and its common boundary

Hence, the surface force is defined as follows:

$$\boldsymbol{\lambda}_{<ab>} = \boldsymbol{k}\cdot\boldsymbol{\delta}_{<ab>} \tag{2}$$

where $\boldsymbol{\delta}_{<ab>}$ denotes the relative displacement at the boundary $\Gamma_{<ab>}$, and $\boldsymbol{k}$ denotes the penalty function. The equation of motion, discretized with respect to space by substituting the abovementioned relationship in equation (1), is obtained as follows:

$$\boldsymbol{M}\ddot{\boldsymbol{U}} + \boldsymbol{K}\boldsymbol{U} = \boldsymbol{P} \tag{3}$$

The above equation of motion is expressed in the global coordinate system as follows:

$$\begin{bmatrix}\boldsymbol{M}_{dd} & \boldsymbol{M}_{d\varepsilon}\\ \boldsymbol{M}_{\varepsilon d} & \boldsymbol{M}_{\varepsilon\varepsilon}\end{bmatrix}\begin{Bmatrix}\ddot{\boldsymbol{d}}\\ \ddot{\boldsymbol{\varepsilon}}\end{Bmatrix} + \begin{bmatrix}\boldsymbol{K}_{dd} & \boldsymbol{K}_{d\varepsilon}\\ \boldsymbol{K}_{\varepsilon d} & \boldsymbol{K}_{\varepsilon\varepsilon}+\boldsymbol{D}\end{bmatrix}\begin{Bmatrix}\boldsymbol{d}\\ \boldsymbol{\varepsilon}\end{Bmatrix} = \begin{Bmatrix}\boldsymbol{P}_{d}\\ \boldsymbol{P}_{\varepsilon}\end{Bmatrix} \tag{4}$$

where $\boldsymbol{d}$ denotes rigid displacement and rigid rotation in the subdomain, and ε denotes constant strain in the subdomain. Assuming a rigid displacement field and a mass matrix that contains only diagonal elements, substitution of the above mentioned relationships into equation (4) yields the following:

$$\boldsymbol{M}_{dd}\ddot{\boldsymbol{d}} = \boldsymbol{P}_{d} - \boldsymbol{K}_{dd}\boldsymbol{d} \tag{5}$$

Here, as shown in Fig. 2, we expand element (1) and the adjoining elements. In this case, the integration on the boundary edge, with a focus on element (1), is relevant only to elements (2)–(4). Therefore, the other elements are not relevant in the simultaneous equations.

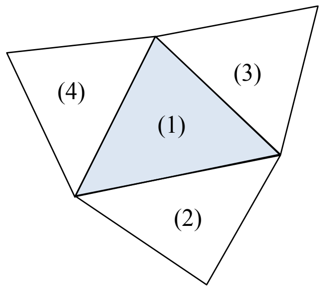

Fig. 2 Element (1) and adjoining elements

From the above relationship, the stress element is obtained using the surface forces at the element boundary, and can be expressed as follows:

$$\boldsymbol{M}^{(e)}\ddot{\boldsymbol{d}}^{(e)} = \boldsymbol{P}_d^{(e)} - \oint_{\Gamma^{(e)}} \boldsymbol{N}_d^{(e)}\boldsymbol{t}^{(e)} d\Gamma \tag{6}$$

Thus, the equation of motion can be computed for each element. The element acceleration can be obtained as the resultant of the contact forces, in a manner similar to the DEM approach, as follows:

$$\ddot{\boldsymbol{U}}^{n+1} = \boldsymbol{M}^{-1}\tilde{\boldsymbol{P}}^{n} \tag{7}$$

However, in DEM, the contact mechanism is based on point contact, whereas in RBSM, it is based on surface contact. Therefore, it is necessary to characterize the nature of the contact. We propose edge-to-edge contact, as shown in Fig. 3.

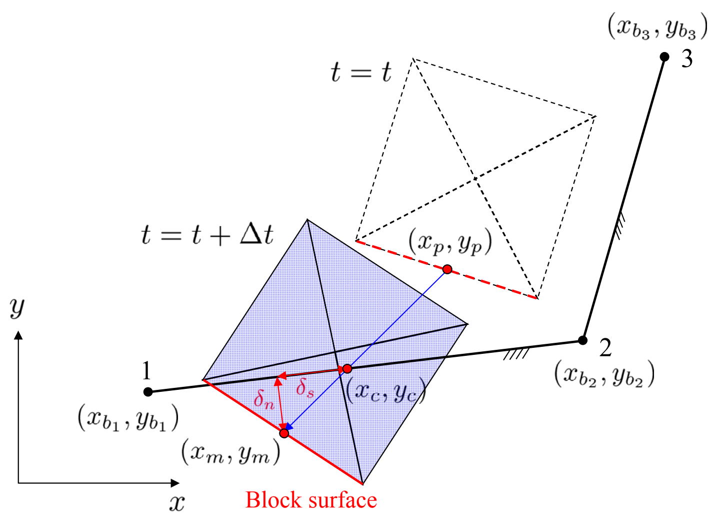

Fig. 3 Contact mechanism

When in a contact state, the amount of penetration in the normal and shear directions, δ_n and δ_s, respectively, are defined as follows:

$$\boldsymbol{\delta}_n = (y_m - y_c) \frac{x_{b_2} - x_{b_1}}{L_b} - (x_m - x_c) \frac{y_{b_2} - y_{b_1}}{L_b} \tag{8}$$

$$\boldsymbol{\delta}_s = (x_m - x_c) \frac{x_{b_2} - x_{b_1}}{L_b} - (y_m - y_c) \frac{y_{b_2} - y_{b_1}}{L_b} \tag{9}$$

Thus, we determine the contact force per unit length, applied in equation (2), as follows:

$$\begin{Bmatrix} \boldsymbol{\lambda}_n \\ \boldsymbol{\lambda}_s \end{Bmatrix} = \begin{bmatrix} \boldsymbol{k}_n & 0 \\ 0 & \boldsymbol{k}_s \end{bmatrix} \begin{Bmatrix} \boldsymbol{\delta}_n \\ \boldsymbol{\delta}_s \end{Bmatrix} \tag{10}$$

The slip along the boundary edge can be modeled with the Mohr–coulomb failure criterion.

$$f = \tau^2 - (c - \sigma_n \tan\phi)^2 = 0 \tag{11}$$

where ϕ is the internal friction angle, and c is the joint cohesion. If the normal stress acting on the boundary is tensile, the two faces of the joint can separate.

3. NUMERICAL EXAMPLE

As a numerical example, to confirm the applicability of the stability analysis of landslides, we evaluated the dynamic behavior in multiple block models. Table 1 shows the analytical conditions and material properties, and Fig. 4 shows the model of a slope in a gravitational field.

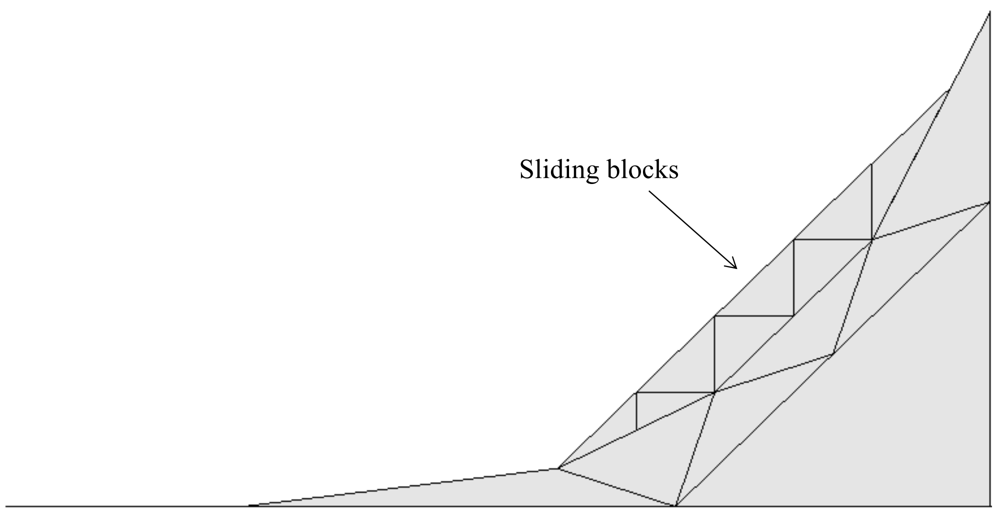

Fig. 4 RBSM model of slope

Table 1 Block properties and calculation parameters

Parameter	Value
Young's modulus (MPa)	200
Poisson's ratio	0.2
Density (kg/m^3)	245
Time increment (s)	0.003
Friction angle	30°
Cohesion of joint (kN/m^2)	0.0
Tensile strength of joint (kN/m^2)	0.0

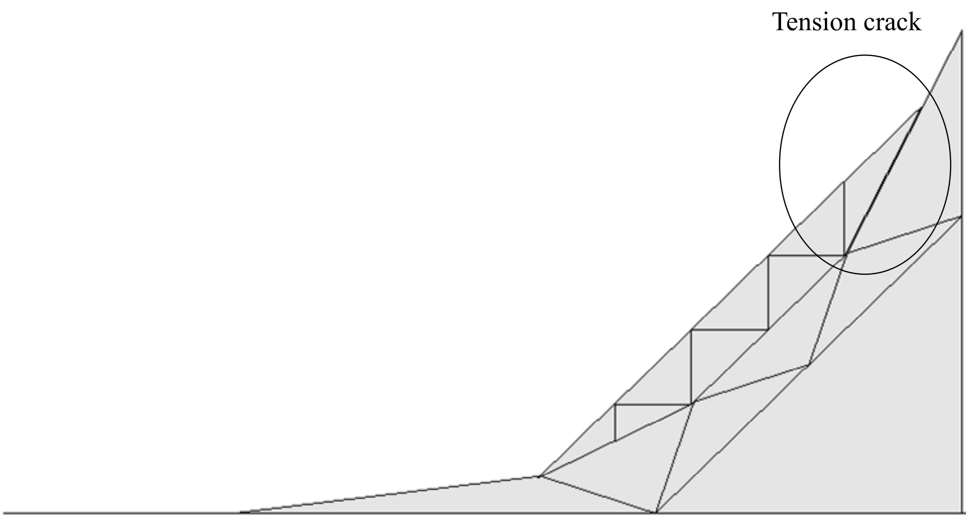

Fig. 5 Result of RBSM (slope with an existing crack)

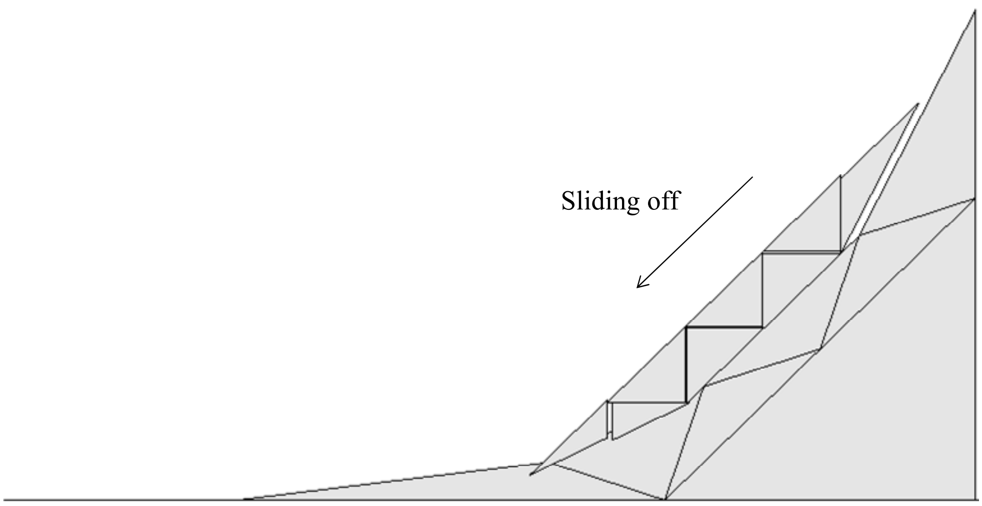

Fig. 6 Result of RBSM (Crack grew, and blocks of the slope were sliding off)

As per the result of evaluation, landslide movement and collapse on account of the tension cracks were observed on the upper part of the blocks, as shown in Fig. 5; subsequently, the blocks started to move and slide down, as shown in Fig. 6.

Table 2 Calculation factor of safety

	Shear Stress (kN/m)	Normal Stress (kN/m)	Factor of Safety
Present method (RBSM)	217	271	0.72
Limit equilibrium analysis (FELLENIUS)	231	253	0.63

The factors of safety obtained from present method (RBSM) and the limit equilibrium method are listed in Table 2. From the present method, it is 0.72, whereas from the limit equilibrium analysis, it is 0.63. In the limit equilibrium analysis, the normal and shear forces acting on the slip surface can be obtained only when the limit equilibrium conditions are satisfied, while in the present method, as the kinematics of the block system is considered, the actual interaction force between blocks can be obtained, indicating that this method is more stable.

4. CONCLUSIONS

This paper proposed an explicit dynamic model to solve sequentially large deformation behavior; this model is expanded to include RBSM that is suitable for analysis of progressive failures, such as landslides. First, we defined the explicit method based on the hybrid principle of virtual work, and developed a new RBSM approach for contact mechanism and friction characteristics. Moreover, we confirmed the applicability of the stability analysis of landslide by observing the simulated behavior of multiple block models.

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