The 2015 World Congress on
Advances in Civil, Environmental, and Materials Research (ACEM15)
Incheon, Korea, August 25-29, 2015

The effects of the viscosity parameters on the barrel type helical rod

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ABSTRACT

The objective of this study is to investigate the dynamic behavior of a linear viscoelastic barrel type helical rod having a hollow-circular cross-section. The proposed viscoelastic model has a standard type of distortional behavior and standard type of bulk compressibility. In order to convert the time space problem into the frequency space, the linear viscoelastic material properties are implemented into the formulation through the use of the correspondence principle. Field equations are based on the Timoshenko beam theory. Mixed finite element dynamic analysis is performed in the Laplace space and the results obtained are transformed back to time domain numerically by Modified Durbin's algorithm. The viscoelastic barrel type helical rod fixed at both ends is forced by dynamic rectangular impulsive type of uniformly distributed load. The effect of the viscous shear modulus and the bulk compressibility on the dynamic behavior of linear viscoelastic barrel type helical rod is investigated.

1. INTRODUCTION

It is a common practice to analyze the dynamic response of engineering structures by assuming that the material is linearly elastic. However, there may be some cases that, the viscous effects arising from the internal friction within these structures could not be neglected. Many researchers investigated the theoretical foundations of viscoelasticity (Fung 1965, Flügge 1975, Christensen 1982). Various numerical applications exist in the literature for solving viscoelastic straight and circular bars (Chen and Lin 1982, Chen 1995, Wang et al. 1997, Aköz and Kadıoğlu 1999, Kadıoğlu and Aköz 2003, Kocatürk and Şimşek 2006, Payette and Reddy 2010, Tehrani and Eipakchi 2012). However, the number of researches concerning dynamic analysis of

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viscoelastic helicoidal rods is limited. Temel (2004) and Temel et al. (2004) studied quasi-static and dynamic analysis of viscoelastic cylindrical helicoidal rods subjected to time dependent loads in the Laplace space by using the complementary functions method and the ordinary differential equations based on the Timoshenko beam theory. By using the mixed finite element method, Eratlı et al. (2014) investigated the dynamic behavior of viscoelastic helixes having non-circular cross-sections based on the Timoshenko beam theory with the inclusion of the rotary inertia.

In this study, dynamic behavior of a linear viscoelastic barrel type helical rod having circular hollow cross section is examined by using mixed finite element method. The formulation is based on Timoshenko beam theory and the analyses are performed in Laplace space by adopting correspondence principle (Shames and Cozarelli 1997). A viscoelastic model is proposed that takes standard type of distortional behavior and standard type of bulk compressibility into account. The exact nodal curvature values are inserted into the finite element matrix which is generated by the linear shape functions. The solutions are carried out in Laplace transform space and the results are transformed back to time domain numerically by Modified Durbin algorithm (Dubner and Abate 1968, Durbin 1974, Narayanan 1980). The linear viscoelastic barrel type helical rod fixed at both ends which is subjected to a dynamic rectangular impulsive type of uniformly distributed load is handled. The effect of the viscous shear modulus and the bulk compressibility on the dynamic behavior of the helical rod is investigated.

2. FORMULATION

2.1 The Helix Geometry

In the Cartesian coordinates, the geometrical properties of the helix are $x = R(\varphi)\cos\varphi$, $y = R(\varphi)\sin\varphi$ and $z = p(\varphi)\varphi$, $p(\varphi) = R(\varphi)\tan\alpha$ where α denotes the pinch angle, $R(\varphi)$ and $p(\varphi)$ signify the centerline radius and the step for unit angle of the helix as a function of the horizontal angle φ. The infinitesimal arc length $\mathrm{d}s = \sqrt{R^2(\varphi) + p^2(\varphi)}\mathrm{d}\varphi = c(\varphi)\mathrm{d}\varphi$. In the case of barrel helix, the radius at any point on the helix geometry is $R(\varphi) = R_{\max} + (R_{\min} - R_{\max})(1-\varphi/2n\pi)^2$, where n is the number of active turns, $R_{\min}$ and $R_{\max}$ are the radii of the helix.

2.2 The Field Equations and the Functional in the Laplace Space

Field equations based on the Timoshenko beam theory in Laplace space are given by referring to Frenet coordinate system as (Eratlı et al. 2014),

$$\begin{aligned} -\overline{\mathbf{T}}_{,s} - \overline{\mathbf{q}} + \rho A z^2\, \overline{\mathbf{u}} &= \mathbf{0} \\ -\overline{\mathbf{M}}_{,s} - \mathbf{t}\times\overline{\mathbf{T}} - \overline{\mathbf{m}} + \rho \mathbf{I}\, z^2\, \overline{\mathbf{\Omega}} &= \mathbf{0} \end{aligned} \tag{1}$$

$$\begin{aligned} \overline{\mathbf{u}}_{,s} + \mathbf{t}\times\overline{\mathbf{\Omega}} - \overline{\mathbf{C}}_{\gamma}\overline{\mathbf{T}} &= \mathbf{0} \\ \overline{\mathbf{\Omega}}_{,s} - \overline{\mathbf{C}}_{\kappa}\overline{\mathbf{M}} &= \mathbf{0} \end{aligned} \tag{2}$$

where the Laplace transformed variables are denoted by the over bars, comma as a subscript under the variable designates the differentiation with respect to s, and, z is the Laplace transformation parameter. $\bar{\mathbf{u}}(\bar{u}_t, \bar{u}_n, \bar{u}_b)$ is the displacement vector, $\bar{\boldsymbol{\Omega}}(\bar{\Omega}_t, \bar{\Omega}_n, \bar{\Omega}_b)$ is the rotation vector, $\bar{\mathbf{T}}(\bar{T}_t, \bar{T}_n, \bar{T}_b)$ is the force vector, $\bar{\mathbf{M}}(\bar{M}_t, \bar{M}_n, \bar{M}_b)$ is the moment vector in the Laplace space, ρ is the density of homogeneous material, A is the area of the cross section, $\mathbf{I}(I_t, I_n, I_b)$ is the moment of inertia of the cross section, $\bar{\mathbf{q}}$ and $\bar{\mathbf{m}}$ are the distributed external force and moment vectors in the Laplace space, $\bar{\mathbf{C}}_\gamma$ and $\bar{\mathbf{C}}_\kappa$ are the compliance matrices in the Laplace space. The functional of the structural problem in the Laplace space is given by

$$
\begin{aligned}
\mathbf{I}(\bar{\mathbf{y}}) = & -[\bar{\mathbf{u}}, \bar{\mathbf{T}}_{,s}] + [\mathbf{t} \times \bar{\boldsymbol{\Omega}}, \bar{\mathbf{T}}] - [\bar{\mathbf{M}}_{,s}, \bar{\boldsymbol{\Omega}}] - \tfrac{1}{2}[\mathbf{C}_\kappa \bar{\mathbf{M}}, \bar{\mathbf{M}}] - \tfrac{1}{2}[\mathbf{C}_\gamma \bar{\mathbf{T}}, \bar{\mathbf{T}}] \\
& + \tfrac{1}{2}\rho A z^2 \left[\bar{\mathbf{u}}, \bar{\mathbf{u}}\right] + \tfrac{1}{2}\rho z^2 \left[\mathbf{I}\bar{\boldsymbol{\Omega}}, \bar{\boldsymbol{\Omega}}\right] - \left[\bar{\mathbf{q}}, \bar{\mathbf{u}}\right] - \left[\bar{\mathbf{m}}, \bar{\boldsymbol{\Omega}}\right] \\
& + [(\bar{\mathbf{T}} - \hat{\bar{\mathbf{T}}}), \bar{\mathbf{u}}]_\sigma + [(\bar{\mathbf{M}} - \bar{\mathbf{M}}), \bar{\boldsymbol{\Omega}}]_\sigma + [\hat{\bar{\mathbf{u}}}, \bar{\mathbf{T}}]_\varepsilon + [\hat{\bar{\boldsymbol{\Omega}}}, \bar{\mathbf{M}}]_\varepsilon
\end{aligned}
\tag{3}
$$

The terms with hats in Eq. (3) define the known values on the boundary. The subscripts ε and σ, represent the geometric and dynamic boundary conditions, respectively. The detailed formulation of the functional is well documented in Eratlı et al. (2014).

2.3 Mixed Finite Element Formulation

The two-nodded curvilinear elements based on Timoshenko beam theory are generated in Laplace space. Using the subscripts i, j to represent the node numbers of the bar element, the linear shape functions $\phi_i = (\varphi_j - \varphi)/\Delta\varphi$ and $\phi_j = (\varphi - \varphi_i)/\Delta\varphi$ are employed in the mixed finite element formulation where $\Delta\varphi = (\varphi_j - \varphi_i)$. The non-cylindrical helix geometry is introduced into the mixed finite element formulation by considering the variable nodal curvatures of a two nodded curved element. Each node has 12 degrees of freedom namely, $\{\bar{\mathbf{u}}, \bar{\boldsymbol{\Omega}}, \bar{\mathbf{T}}, \bar{\mathbf{M}}\}$.

2.4 The Viscoelastic Model

It is assumed that, the linear viscoelastic material exhibits standard type of distortional behavior and standard type of bulk compressibility. In that case, the forms of complex shear modulus (Mengi and Argeso 2006, Baranoğlu and Mengi 2006, Eratlı et al. 2014) and complex bulk modulus $\bar{K}$ can be expressed as

$$\bar{G} = G\left[\frac{1 + \beta^G \tau_r^G z}{1 + \tau_r^G z}\right]; \ \beta^G = G_g / G > 1 \tag{4}$$

and

$$\bar{K} = K\left[\frac{1 + \beta^K \tau_r^K z}{1 + \tau_r^K z}\right]; \ \beta^K = K_g / K > 1 \tag{5}$$

In Eq. (4), τ_r^G, G and G_g are, respectively, the retardation time, the equilibrium value and the instantaneous value of relaxation function associated with shear modulus. Similarly, in Eq. (5), τ_r^K, K and K_g are, respectively, the retardation time, the equilibrium value and the instantaneous value of relaxation function associated with bulk modulus.

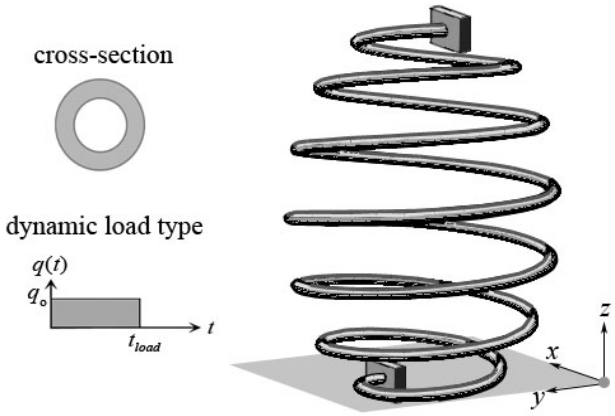
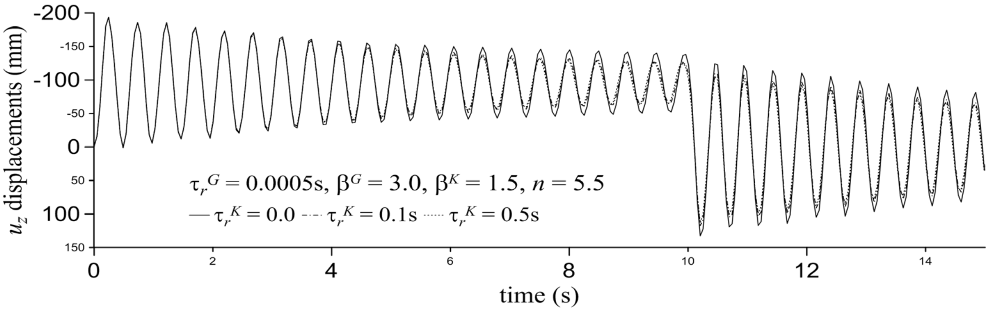


Fig. 1 The barrel type helical rod having fixed at both ends

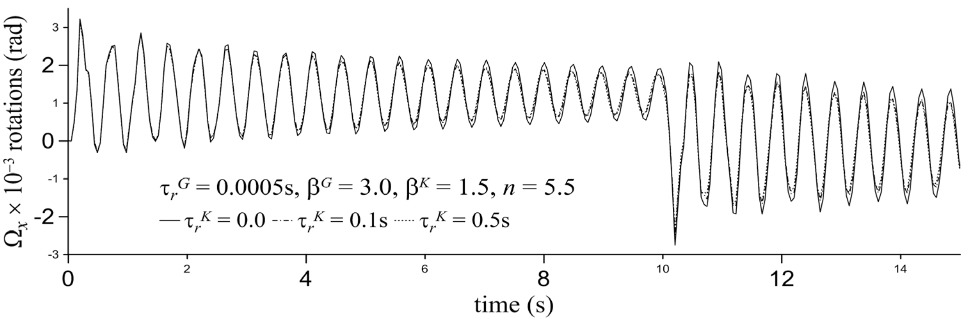
3. NUMERICAL EXMPLES

A barrel type helical rod fixed at both ends which is subjected to dynamic rectangular impulsive type of distributed load is considered (see Fig.1). The helix geometry has $n = 5.5$ number of active turns, the height of the rod is $H = 6\,\mathrm{m}$. The ratio of the maximum over the minimum radii of the helix is $R_{\min} / R_{\max} = 0.5$ with $R_{\max} = 3\,\mathrm{m}$. The inner and outer radii of circular hollow cross section are $24\,\mathrm{cm}$ and $30\,\mathrm{cm}$, respectively. The material parameters are $G = 80\,\mathrm{GPa}$, $K = 175\,\mathrm{GPa}$, $\tau_r^G = 0.0005\,\mathrm{s}, 0.005\,\mathrm{s}, 0.01\,\mathrm{s}$, $\beta^G = 3$ and $\tau_r^K = 0.0, 0.1\,\mathrm{s}, 0.5\,\mathrm{s}$, $\beta^K = 1.5$, the material density $\rho = 7850\ \mathrm{kg/m^3}$. The complex shear and bulk modules are obtained by using Eqs. (4)-(5). The rod is subjected to a dynamic rectangular impulsive type of vertical uniformly distributed load. The intensity and the duration of the loading are $q_o = 10\,\mathrm{kN/m}$ and $t_{load} = 10\,\mathrm{s}$, respectively. The dynamic response of the rod is determined within $0 \le t \le 15\,\mathrm{s}$. The analyses are carried out in the Laplace space and the results are transformed back to the time space numerically by modified Durbin's algorithms (Dubner and Abate 1968, Durbin 1974, Narayanan 1980). The parameters which are used in the analysis for inverse Laplace transformation algorithm are $N = 2^9$ and $aT = 6$.

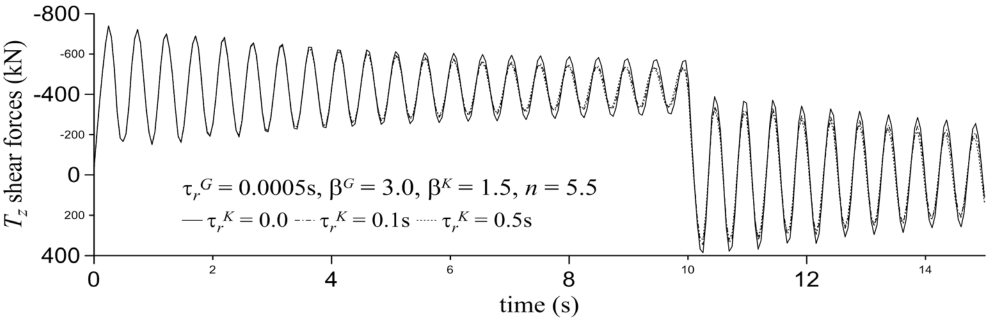
The vertical displacement u_z and the rotation Ω_x at the middle of the barrel type rod, and, the shear force T_z and the moment M_y at the fixed end are determined using 100 finite elements. Fig. 2 are plotted to examine the effects of viscous dilatational behavior on the dynamic response of the rod. As seen from these figures, time histories of the selected quantities do not differ significantly as the viscous dilatational behavior of the



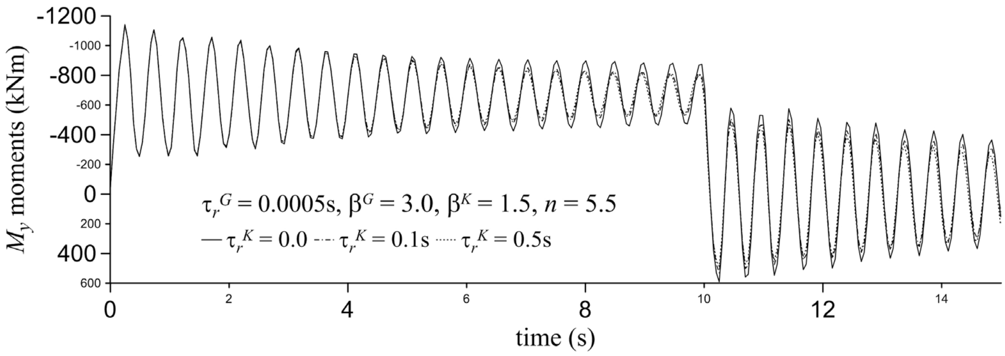
(a) u_z



(b) Ω_x

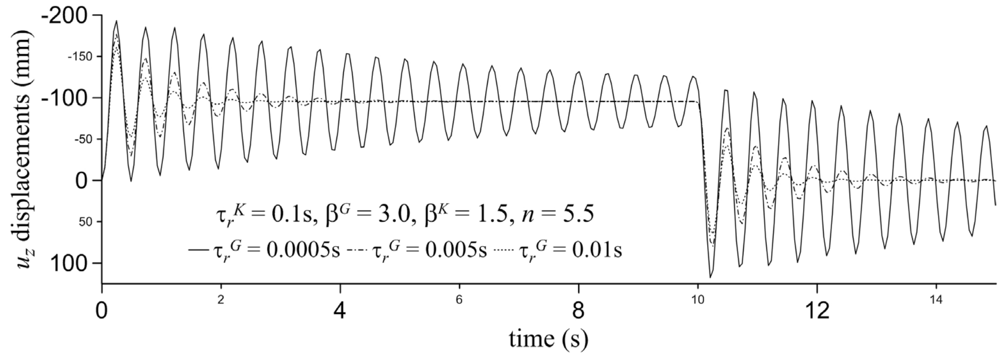


(c) T_z

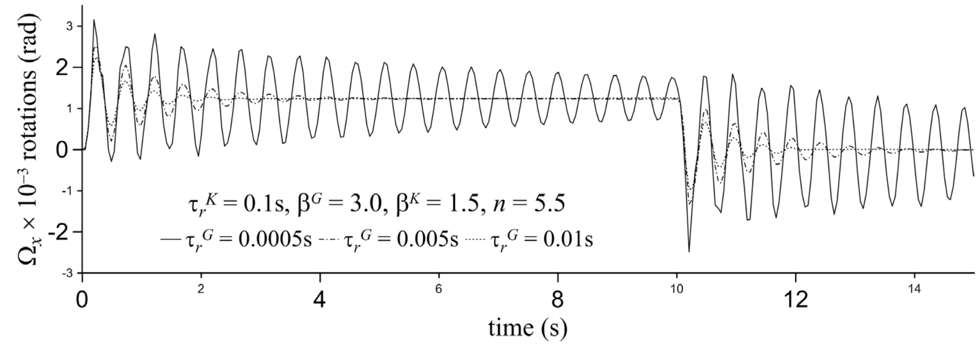


(d) M_y

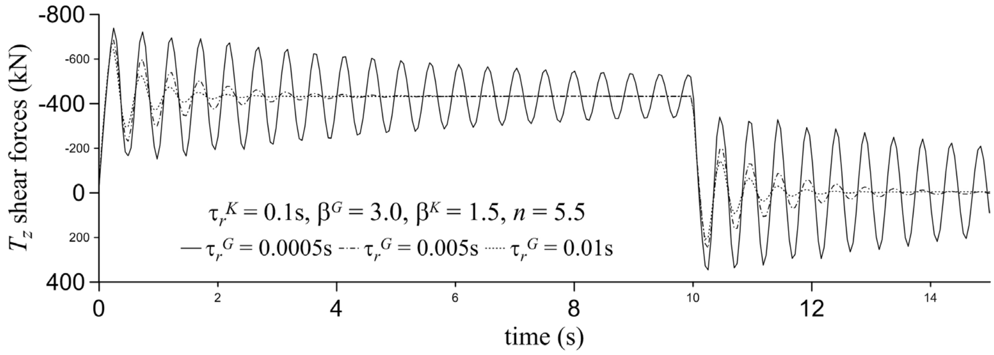
Fig. 2 The time histories of four nodal variables



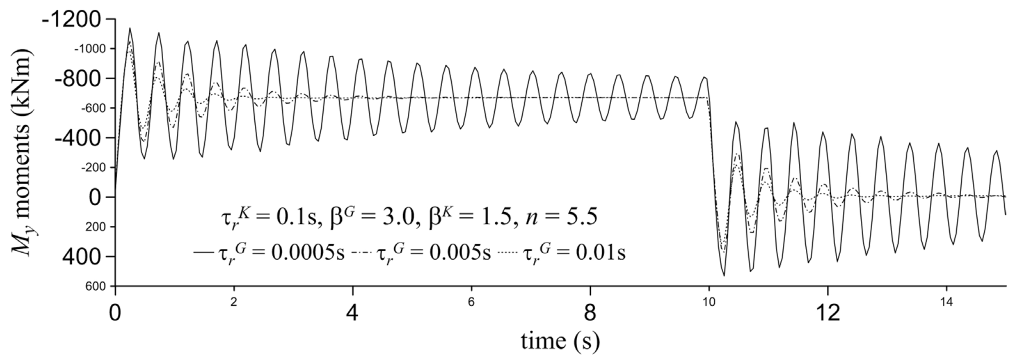
(a) u_z



(b) Ω_x



(c) T_z



(d) M_y

Fig. 3 The time histories of four nodal variables material increases (as values of τ_r^K increases). A similar analysis is performed to see the effects of viscous distortional behavior on the dynamic response of the rod, in which the results are depicted in Fig. 3. We may observe that as values of τ_r^G increases (the distortional viscous behavior of the material increases) the selected nodal time histories dissipate rapidly and there is a significant change in the dynamic responses.

4. CONCLUSION

The effect of the viscous distortional behavior and the bulk compressibility on the dynamic response of viscoelastic barrel type helical rod is investigated. For this aim, a viscoelastic model is proposed that takes standard type of distortional behavior and standard type of bulk compressibility. The viscoelastic material properties are implemented into the formulation through the use of the correspondence principle. The analysis is carried out in the Laplace space and the results are transformed back to time space numerically by using the modified Durbin's algorithm. As a sample problem, a barrel type helical rod fixed at both ends which is subjected to dynamic rectangular impulsive type of uniformly distributed load is considered. The results of the dynamic analysis reveal that, the distortional viscous behavior dominates the total viscous behavior. However, it is also seen that, for the materials which exhibit viscous bulk compressibility the inclusion of this effect may improve the dynamic analysis.

ACKNOWLEDGMENT

This research is supported by The Scientific and Technological Research Council of Turkey under project no 111M308 and by the Research Foundation of ITU under project no 38078. These supports are gratefully acknowledged by the authors.

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