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The dynamic responses of a viscoelastic conical helical rod

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ABSTRACT

The effects of viscosity parameters on the dynamic behavior of a cantilevered viscoelastic conical helicoidal rod having square cross-section are investigated by using the mixed finite element method. It is assumed that, the linear viscoelastic material exhibits standard type distortional behavior while having elastic Poisson's ratio and the material properties are implemented into the formulation through the use of the correspondence principle. The rod is subjected to a dynamic load acting at the tip of it. The element matrix is based on the Timoshenko beam theory including the rotary inertia in the formulation. The solutions are carried out into the Laplace domain and the results are transformed back to time space numerically by the modified Durbin's transformation algorithm. Numerical results for the cantilevered viscoelastic conical helicoidal rod which is subjected to impulsive type loads (sinusoidal, rectangular, triangle, right triangle) and step type load are presented. New and original problems are handled and interesting results are obtained for the literature.

1. INTRODUCTION

The theoretical foundations for viscoelasticity are well established by many researches, among them, (Fung 1965, Flügge 1975, Christensen 1982) can be referred. The viscoelastic behaviors of straight and circular rods were examined extensively (Chen and Lin 1982, Chen 1995, Wang et al. 1997, Aköz and Kadıoğlu 1999, Kadıoğlu and Aköz 2003, Kocatürk and Şimşek 2006a-b, Payette and Reddy 2010), however, few studies exist in literature that investigates the viscoelastic behavior of helicoidal rods. By using the complementary functions method and the ordinary differential equations based on the Timoshenko beam theory, Temel (2004) and Temel et al. (2004) studied quasi-static and dynamic analysis of viscoelastic cylindrical helicoidal rods subjected to time dependent loads in the Laplace space. The dynamic behavior of viscoelastic helixes with non-circular cross-sections based on the Timoshenko beam theory with the inclusion of the rotary inertia, using the mixed finite element method is

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investigated by Eratlı et al. (2014).

In this study, the dynamic behavior of cantilevered linear viscoelastic conical helicoidal rod subjected to different external dynamic loads is investigated by employing the mixed finite element method. The viscoelastic material exhibits the standard type of distortional behavior. The viscoelastic properties of a body may be accounted for through the use of the correspondence principle (Shames and Cozarelli 1997), which states that the equations for viscoelastic case in the Laplace space may be obtained from those for elastic case by replacing the elastic constants by complex moduli according to the chosen viscoelastic model. The field equations of the helicoidal rod are based on the Timoshenko assumptions. The solution of the structural problem is carried out in the Laplace space. The results of the dynamic problem obtained in the Laplace space are transformed back to the time domain numerically by means of the Modified Durbin's transformation algorithm (Dubner and Abate 1968, Durbin 1974, Narayanan 1980). The verification of the present formulation exists in Eratlı et al. (2014). To investigate the influence of the impulsive type (sinusoidal, rectangular, right triangle, triangle) and step type loads on the dynamic behavior, the cantilevered linear viscoelastic conical helicoidal rod having square cross-section is handled.

2. FORMULATION

2.1 The Helix Geometry

The geometry of a helix can be defined in the Cartesian coordinate system in terms of the helix parameters as: $x = R(\varphi)\cos\varphi$, $y = R(\varphi)\sin\varphi$ and $z = p(\varphi)\varphi$, where $p(\varphi) = R(\varphi)\tan\alpha$. For the conical helix, the centerline radius $R(\varphi)$ is expressed as $R(\varphi) = R_1 + (R_2 - R_1)\varphi/2n\pi$ where n is the number of active turns, R_1, R_2 are the bottom and top radii of the helix, respectively. $p(\varphi)$ is a function of the horizontal angle φ and defines the step for unit angle of the helix.

2.2 The Field Equations in the Laplace Space

The field equations based on the Timoshenko beam theory in the Laplace space can be listed as follows (Eratlı et al. 2014):

$$\begin{aligned} -\bar{\mathbf{T}}_{,s} - \bar{\mathbf{q}} + \rho A z^2\,\bar{\mathbf{u}} &= \mathbf{0} \\ -\bar{\mathbf{M}}_{,s} - \mathbf{t}\times\bar{\mathbf{T}} - \bar{\mathbf{m}} + \rho \mathbf{I} z^2\,\bar{\mathbf{\Omega}} &= \mathbf{0} \\ \bar{\mathbf{u}}_{,s} + \mathbf{t}\times\bar{\mathbf{\Omega}} - \bar{\mathbf{C}}_{\gamma}\bar{\mathbf{T}} &= \mathbf{0} \\ \bar{\mathbf{\Omega}}_{,s} - \bar{\mathbf{C}}_{\kappa}\bar{\mathbf{M}} &= \mathbf{0} \end{aligned} \qquad (1)$$

where the Laplace transformed variables are denoted by the over bars, comma as a subscript under the variable designates the differentiation with respect to s and z is the Laplace transformation parameter. Using the Frenet coordinate system, in Eq. (1), $\bar{\mathbf{u}} = \bar{u}_t\,\mathbf{t} + \bar{u}_n\,\mathbf{n} + \bar{u}_b\mathbf{b}$ is the displacement vector, $\bar{\mathbf{\Omega}} = \bar{\Omega}_t\,\mathbf{t} + \bar{\Omega}_n\,\mathbf{n} + \bar{\Omega}_b\mathbf{b}$ is the rotation vector, $\overline{\mathbf{T}} = \overline{T}_t\,\mathbf{t} + \overline{T}_n\,\mathbf{n} + \overline{T}_b\mathbf{b}$ is the force vector, $\overline{\mathbf{M}} = \overline{M}_t\,\mathbf{t} + \overline{M}_n\,\mathbf{n} + \overline{M}_b\mathbf{b}$ is the moment vector, ρ is the density of material, A is the area of the cross section, $\mathbf{I} = I_t\,\mathbf{t} + I_n\,\mathbf{n} + I_b\,\mathbf{b}$ is the moment of inertia of the cross section, $\overline{\mathbf{q}}$ and $\overline{\mathbf{m}}$ are the distributed external force and moment vectors in the Laplace space, $\overline{\mathbf{C}}_\gamma$ and $\overline{\mathbf{C}}_\kappa$ are the compliance matrices in the Laplace space (Eratlı et al. 2014).

2.3 The Functional in the Laplace Space

The field equations in Eq. (1) are written in operator form $\mathbf{Q} = \mathbf{Ly} - \mathbf{f}$. After proving this operator to be potential, the functional of the structural problem is obtained in the Laplace space as follows:

$$\begin{aligned}\mathbf{I}(\overline{\mathbf{y}}) = &-[\overline{\mathbf{u}},\overline{\mathbf{T}}_{,s}] + [\mathbf{t}\times\overline{\mathbf{\Omega}},\overline{\mathbf{T}}] - [\overline{\mathbf{M}}_{,s},\overline{\mathbf{\Omega}}] - \tfrac{1}{2}[\mathbf{C}_\kappa\overline{\mathbf{M}},\overline{\mathbf{M}}] - \tfrac{1}{2}[\mathbf{C}_\gamma\overline{\mathbf{T}},\overline{\mathbf{T}}] \\ &+ \tfrac{1}{2}\rho A z^2\left[\overline{\mathbf{u}},\overline{\mathbf{u}}\right] + \tfrac{1}{2}\rho\, z^2\left[\mathbf{I}\overline{\mathbf{\Omega}},\overline{\mathbf{\Omega}}\right] - \left[\overline{\mathbf{q}},\overline{\mathbf{u}}\right] - \left[\overline{\mathbf{m}},\overline{\mathbf{\Omega}}\right] \\ &+ [(\overline{\mathbf{T}} - \hat{\overline{\mathbf{T}}}),\overline{\mathbf{u}}]_\sigma + [(\overline{\mathbf{M}} - \overline{\mathbf{M}}),\overline{\mathbf{\Omega}}]_\sigma + [\hat{\overline{\mathbf{u}}},\overline{\mathbf{T}}]_\varepsilon + [\hat{\overline{\mathbf{\Omega}}},\overline{\mathbf{M}}]_\varepsilon\end{aligned} \tag{2}$$

The terms with hats in Eq. (2) define the known values on the boundary. The subscripts ε and σ , represent the geometric and dynamic boundary conditions, respectively. The details of the variational formulation and functional exist in Eratlı et al. (2014).

2.4 Mixed Finite Element Formulation

The curvilinear elements are two-nodded are used to discretize the beam domain. Linear shape functions $\phi_i = (\varphi_j - \varphi)/\Delta\varphi$ and $\phi_j = (\varphi - \varphi_i)/\Delta\varphi$ are employed in the mixed finite element formulation where $\Delta\varphi = (\varphi_j - \varphi_i)$. The exact nodal curvatures are approximated over the elements by linear shape functions that are used for interpolating field variables. Each node has 12 degrees of freedom namely, $\{\overline{\mathbf{u}}, \overline{\mathbf{\Omega}}, \overline{\mathbf{T}}, \overline{\mathbf{M}}\}$.

2.5 The Standard Model

If the viscoelastic material exhibits the standard type of distortional behavior then the complex shear modulus can be expressed in the form (Mengi and Argeso 2006, Baranoğlu and Mengi 2006)

$$\overline{G} = G\left[\frac{1+\beta^G \tau_r^G z}{1+\tau_r^G z}\right] \tag{3}$$

where

$$G = \frac{G_1 G_2}{(G_1 + G_2)} \tag{4}$$

$$\tau_r^G = \frac{\eta^G}{(G_1 + G_2)} \tag{5}$$

$$\beta^G = \frac{G_1 + G_2}{G_2} > 1 \tag{6}$$

τ_r^G is the retardation time of relaxation function associated with shear modulus, η^G is the damping parameter and β^G is the ratio of the instantaneous value of relaxation function to the equilibrium value of relaxation function (Eratlı et al. 2014).

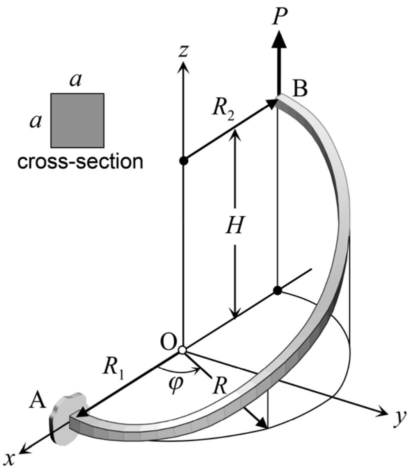


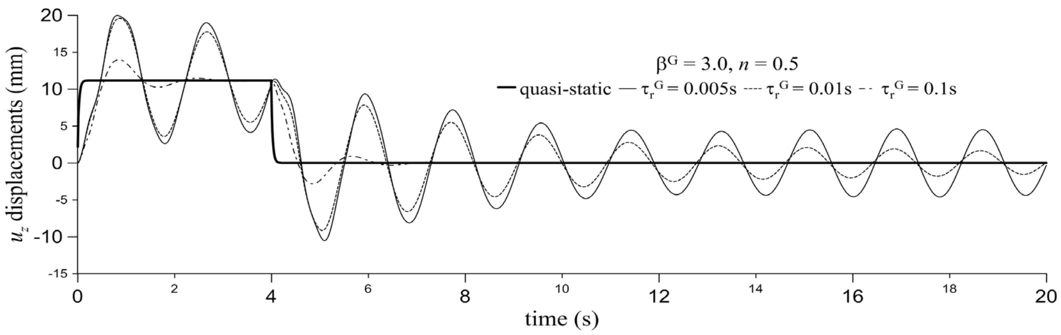
Fig. 1 The cantilevered conical helicoidal rod

3. NUMERICAL EXMPLES

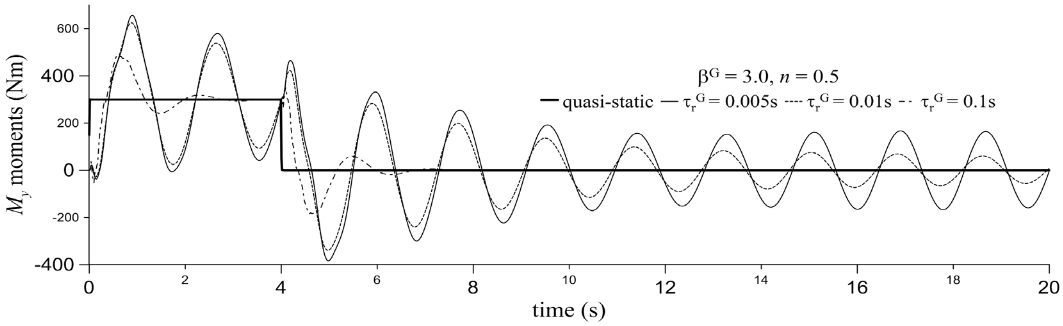
A viscoelastic cantilever conical helicoidal rod is solved (see Fig. 1). The helix geometry has $n = 0.5$ number of active turns, the height of the rod is $H = 3\,\mathrm{m}$ and the minimum radius of helix to maximum radius of helix ratios $R_{\min} / R_{\max} = 0.5$ where $R_{\max} = 2\,\mathrm{m}$. The square cross-sectional area of the rod is $A = 400\,\mathrm{cm}^2$. The viscoelastic material exhibits the standard type of distortional behavior while having elastic Poisson's ratio $\bar{\upsilon} = \upsilon = 0.3$. The material parameters are $G = 8 \times 10^8\,\mathrm{N/m}^2$, $\tau_r^G = 0.005\mathrm{s}, 0.01\mathrm{s}, 0.1\mathrm{s}$, $\beta^G = 3$ and the material density $\rho = 7850\ \mathrm{kg/m}^3$. The form of the complex shear modulus can be obtained by using Eq. (3). The rod is subjected to an external dynamic load $P = P(t)$ acting from the free end B (see Fig.1). The quasi-static and dynamic responses of the rod are determined within $0 \le t \le 20\,\mathrm{s}$ by considering five different time histories of the point load P, namely, impulsive types (rectangular, right triangle, triangle, sinusoidal) with $t_{load} = 4\mathrm{s}$ and step type having an intensity of $P = 100\,\mathrm{N}$. The analyses are carried out in the Laplace space and the results are transformed back to the time space numerically by modified Durbin's algorithms (Dubner and Abate 1968, Durbin 1974, Narayanan 1980). The parameters which are used in the analysis for inverse Laplace transformation algorithm are $N = 2^{11}$ and $aT = 6$ which are verified by Eratlı et al. (2014).

The vertical displacements u_z at the free end B and the moments M_y at the clamped end A (see Fig. 1) are determined using 40 finite elements for all type of loads. Figs. 2-6 are depicted to investigate the influence of different type time-dependent loads on the dynamic response of viscoelastic conical helicoidal rod for the vertical displacements u_z and the moments M_y. An interpretive discussion of each figure is as follows:

- As the viscous behavior of the material increases (τ_r^G increases), the dynamic behavior of the viscoelastic conical helicoidal rods dissipates and approaches to the quasi-static case (see Figs. 2-6). This approximation is impressive in the case of triangle impulsive type of load (see Fig. 5).
- The amplitudes of u_z and M_y are the highest for step type load and the lowest for triangle impulsive load case.
- From the time variation curves of u_z, if the first values of extrema that occur within the sampling time interval in each figure are determined and compared with the results that correspond to the step type load (see Fig. 6(a) for $\tau_r^G = 0.005\text{s}$), the percent reductions in the case of rectangular, right triangle, sinusoidal and triangle impulsive load are approximately 0.3%, 11.1%, 37.6% and 86.8%, respectively.

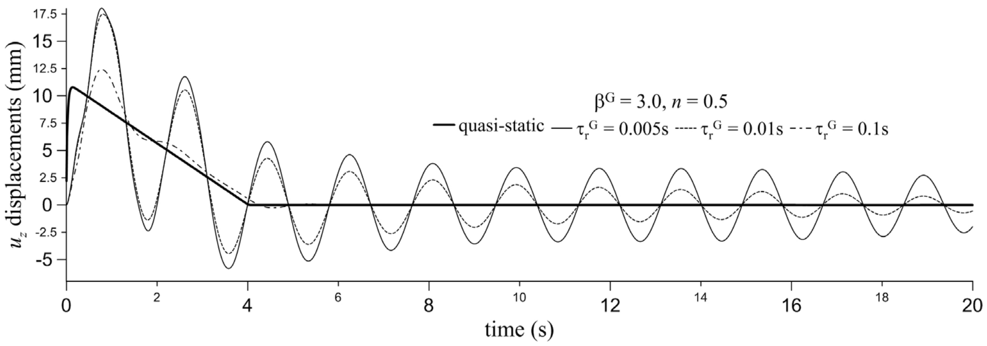


(a) u_z

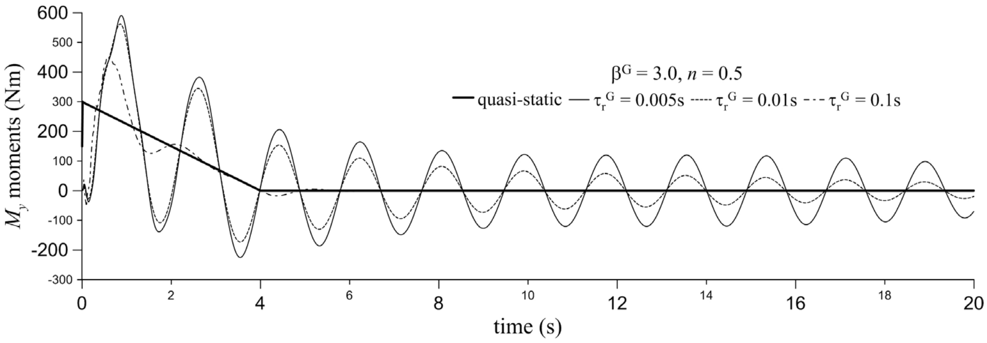


(b) M_y

Fig. 2 Time histories of viscoelastic conical helicoidal rod subjected to the rectangular impulsive type load

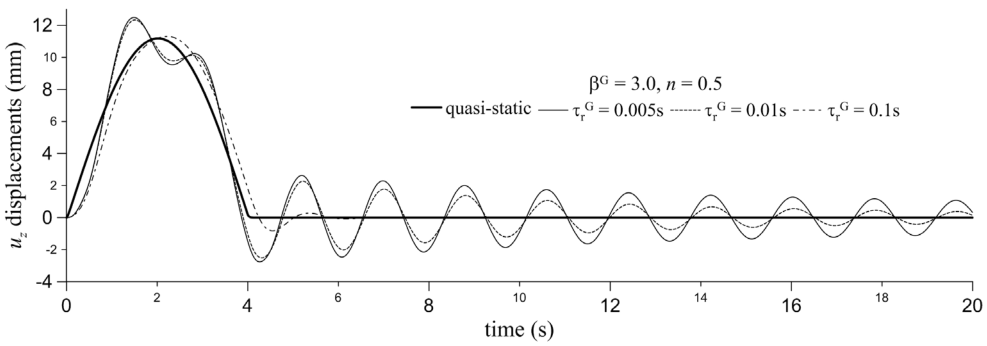


(a) u_z

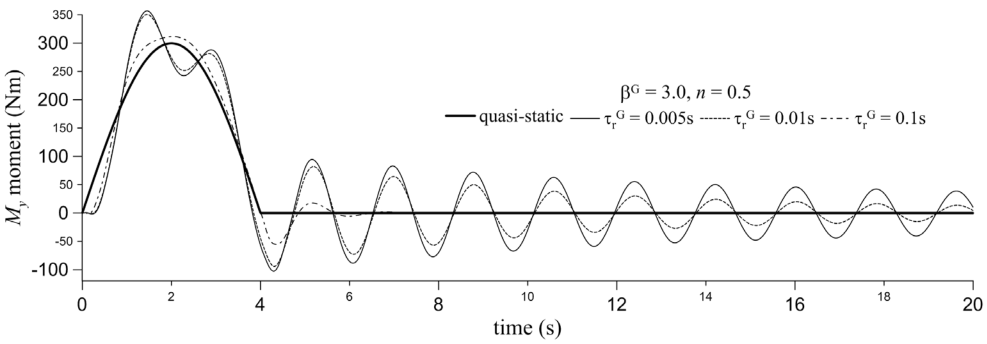


(b) M_y

Fig. 3 Time histories of viscoelastic conical helicoidal rod subjected to the right triangle impulsive type load

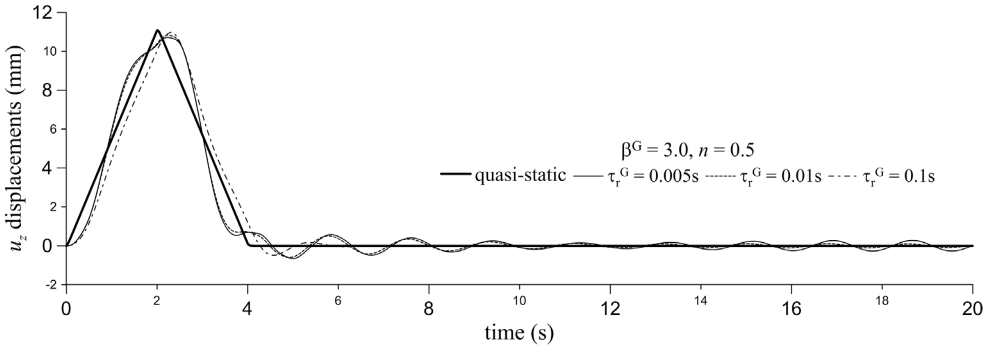


(a) u_z

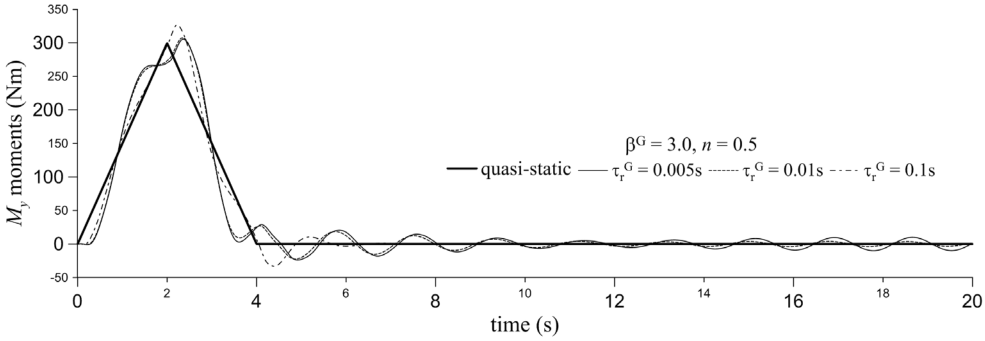


(b) M_y

Fig. 4 Time histories of viscoelastic conical helicoidal rod subjected to the sinusoidal impulsive type load

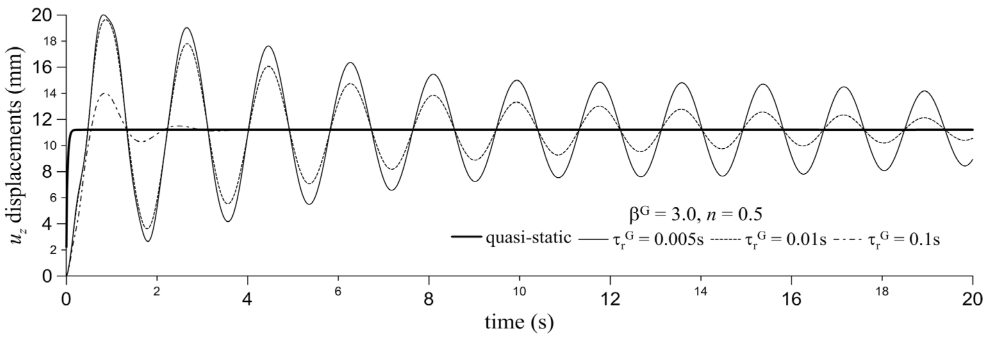


(a) u_z

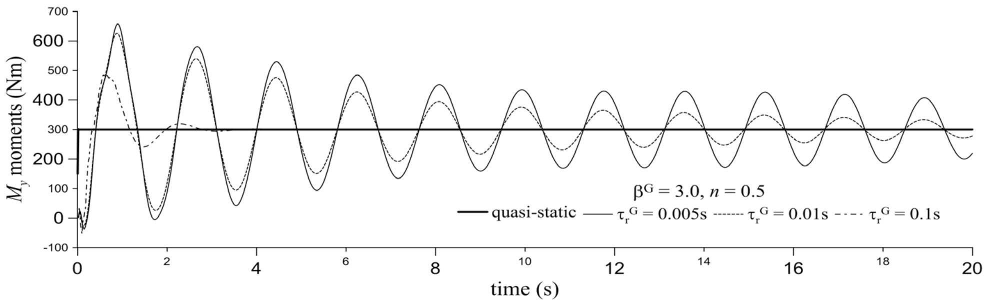


(b) M_y

Fig. 5 Time histories of viscoelastic conical helicoidal rod subjected to the triangle impulsive type load



(a) u_z



(b) M_y

Fig. 6 Time histories of viscoelastic conical helicoidal rod subjected to the step type load

- From the time variation curves of M_y, if the first values of extrema that occur within the sampling time interval in each figure are determined and compared with the results that correspond to the step type load (see Fig. 6(b) for $\tau_r^G = 0.005\,\text{s}$), the percent reductions in the case of rectangular, right triangle, sinusoidal and triangle impulsive load are approximately 0.1%, 11.3%, 45.7% and 115.2%, respectively.

4. CONCLUSION

The dynamic viscoelastic response of conical helicoidal rod subjected to the different external dynamic loads is analyzed by using the mixed finite element method. For this aim, the viscoelastic material behavior is simulated by using the standard model and the viscoelastic properties are accounted using the correspondence principle. The finite element solutions are carried out in the Laplace space. The results obtained in transform space are inverted back to time space using modified Durbin's algorithm. The influence of different external dynamic loads on the dynamic viscoelastic response of conical helicoidal rod is discussed extensively.

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