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Topology optimization of laminated composite plates with isotropic and orthotropic material

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ABSTRACT

The goal of this study is to evaluate continuous material topology optimization of laminated composite plates of isotropic and orthotropic material. This research presents an extension of the study by Sigmund with a published 99 line topology optimization code written in Matlab. The element stiffness matrix was written for the plane stress problem with isotropic materials in the lines of 86 - 99 is changed to the application of orthotropic and laminated composite materials. Matlab code for stiffness matrix of laminated composite is given and some numerical examples are presented to show the change from the isotropic material to orthotropic and laminated composite plates for topology optimization.

Keywords: Laminated composite plate, Topology optimization, 99-line Matlab code, Maximal stiffness, Isotropic material

1. INTRODUCTION

In recent years, the topology optimization has been received numerous attentions as an innovative numerical method due to its significant advantages. In engineering it is often desired to apply some optimization techniques to the design of a structure, component or device. Other than sizing (Bendsee, 1983) and shape optimization techniques (Haber, 1996; Mohammadi, 2010; Sokolowski, 2003; Sarfraz, 2014; Berzoy and Strefezza, 2009), a significant contribution is given by topology optimization (Bendsee and Kikuchi, 1988; Bendsee and Haber, 1993; Bendsee and Sigmund, 2003; Bletzinger et al., 1993), which represents the fundamental form of optimization; indeed, topology optimization aims at finding the optimal distribution of a material in a design domain such that an objective functional is minimized under certain constraints. The minimum compliance case represents the most common topology optimization problem,

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for which the goal is to generate the globally stiffest structure by distributing only a limited amount of material in the design domain (Bischoff et al., 2004) additionally, another interesting problem consists in generating the lightest structure under stress constraints, see among the others. Historically, topology optimization has been used principally for structural static problems based on a linear elastic model, but many other cases have also been successfully considered. Topology optimization has been used for structures and with different material models as for elastoplastic structures (Sigmund, 2004).

With rapid growth of the use of composite materials (Bondi et al., 2013) in many commercial products ranging from sports equipment to high-performance aircraft, literature on composite materials has proliferated. A simple search of a popular web site for the words "composite materials" yielded more than 250 entries. Many of these titles are published papers on mechanics of composite materials and have been adopted by educational institutions for introductory courses. As the application of composites to commercial products has increased, so has the need for literature that focuses on the design aspects of these materials. However, the number of titles that focus on the mechanics of composites far outnumbers those dealing with design. In particular, papers that focus on topology optimization of composite materials virtually could be found in as (Lund, 2009; Pedersen, 2002).

In this paper, the topology optimization is studied for orthotropic beams and laminated composite plates based on the educational article by Sigmund (Sigmund, 2001).

2. TOPOLOGY OPTIMIZATION OF LAMINATED COMPOSITE PLATE IN PLANE STRESS STATE

The educational article by Sigmund (Sigmund, 2001) provides 99 lines of Matlab code for topology optimization of isotropic plates (Chamkha, 2014; Foti, 2013) in the plane stress state. The aim of this section is to analyze this Matlab code for further study on topology optimization for the class of topology optimization. Specifically, the code is divided into 4 parts:

- Main program (lines 1-36)
- Optimality criteria based optimizer (lines 37-48)
- Mesh-independency filtering (lines 49-64)
- Finite element code (lines 65-99)

The finite element code is focused to study the element stiffness matrix. The element stiffness matrix is calculated in the lines 86-99.

First, the general element stiffness matrix is derived for the plane stress problem in parametric space (ξ,η) as follow (Reddy, 2006)

$$[K]=\begin{bmatrix}[K^{11}] & [K^{2}]\\ [K^{1}] & [K^{2}]\end{bmatrix} \tag{1}$$

where,

$$K_{ij}^{11} = \int_{\Omega^e} \left[\left(A_{11} \frac{\partial \psi_i}{\partial \xi} \frac{\partial \psi_j}{\partial \xi} + A_{66} \frac{\partial \psi_i}{\partial \eta} \frac{\partial \psi_j}{\partial \eta} \right) + A_{16} \left(\frac{\partial \psi_i}{\partial \xi} \frac{\partial \psi_j}{\partial \eta} + \frac{\partial \psi_i}{\partial \eta} \frac{\partial \psi_j}{\partial \xi} \right) \right] d\xi d\eta \tag{2a}$$

$$K_{ij}^{12} = K_{ji}^{21} \int_{\Omega^e} \left(A_{16} \frac{\partial \psi_i}{\partial \xi} \frac{\partial \psi_j}{\partial \xi} + A_{26} \frac{\partial \psi_i}{\partial \eta} \frac{\partial \psi_j}{\partial \eta} + A_{12} \frac{\partial \psi_i}{\partial \xi} \frac{\partial \psi_j}{\partial \eta} + A_{66} \frac{\partial \psi_i}{\partial \eta} \frac{\partial \psi_j}{\partial \xi} \right) d\xi d\eta \tag{2b}$$

$$K_{ij}^{22} = \int_{\Omega^e} \left[\left(A_{66} \frac{\partial \psi_i}{\partial \xi} \frac{\partial \psi_j}{\partial \xi} + A_{22} \frac{\partial \psi_i}{\partial \eta} \frac{\partial \psi_j}{\partial \eta} \right) + A_{26} \left(\frac{\partial \psi_i}{\partial \xi} \frac{\partial \psi_j}{\partial \eta} + \frac{\partial \psi_i}{\partial \eta} \frac{\partial \psi_j}{\partial \xi} \right) \right] d\xi d\eta \tag{2c}$$

in which A_{ij} are the extensional stiffnesses which are defined in terms of the lamina stiffnesses $\bar{Q}_{ij}^{(k)}$ as (Reddy, 1997)

$$A_{ij} = \sum_{k=1}^{N} \bar{Q}_{ij}^{(k)} \left(z_{k+1} - z_k \right) \tag{3}$$

where N is number of lamina and details calculation of the lamina stiffnesses $\bar{Q}_{ij}^{(k)}$ could be found in the book by Reddy. And ψ_i are linear Lagrange interpolation functions and c_{ij} given for the constitutive equation as

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{21} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix}. \tag{4}$$

in which N_{xx}, N_{xy}, N_{yy} are the in-plane forces, $\varepsilon_{xx}, \varepsilon_{yy}, \gamma_{xy}$ are in-plane strains. The parent element in parametric space is defined as shown in Fig. 1 and the linear Lagrange interpolation functions associated with rectangular elements can be obtained as

$$\psi_1 = \frac{1}{4}(1-\xi)(1-\eta) \tag{5a}$$

$$\psi_2 = \frac{1}{4}(1+\xi)(1-\eta) \tag{5b}$$

$$\psi_3 = \frac{1}{4}(1+\xi)(1+\eta) \tag{5c}$$

$$\psi_4 = \frac{1}{4}(1-\xi)(1+\eta) \tag{5d}$$

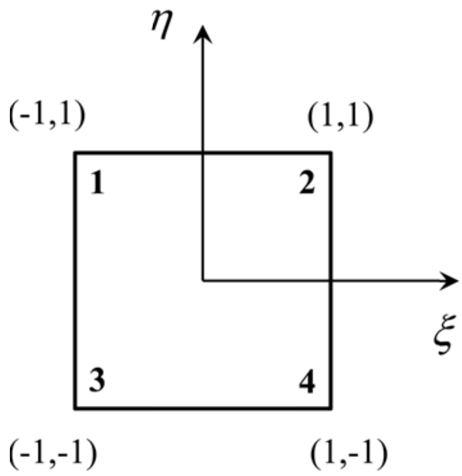

Figure 1. Bi-linear parent element in parametric space

When the number of lamina is one and the oriented direction is 0 degree, we have the orthotropic plate cases. And in the case of one lamina, $E_1 = E_2,\ G_{12} = \dfrac{E}{2(1+\nu)}$, we have the isotropic cases.

3. NUMERICAL APPLICATIONS AND DISCUSSION

The The 99 line Matlab code of topology optimization by Sigmund is modified to orthotropic and laminated composite material.

3.1 Initial definition for Isotropic testing

After generating numerical examples for laminated composite plates, added Matlab code for new element stiffness matrix is tested first for isotropic plates. The Matlab code given in “atluu1.m” for the MBB-beam with half domain and symmetric boundary conditions with top(60,20,0.5,3.0,1.5). In order to have the same results as those by Sigmund (Sigmund, 2001), the input parameters are

```
E0=1;
E1=1*E0;
E2=1*E0;
v12=0.3;
G12=E1/(2*(1+v12));
v21=v12*E2/E1;
N=1;
theta_degree(1)=0;
t(1)=1;
```

Present results are in excellent agreements with those obtain by the published 99 lines Matlab code.

3.2 Optimal topology of orthotropic beams

For orthotropic beams, the input parameters are set as follows.

```
E0=1;
E1=50*E0;
E2=1*E0;
v12=0.3;
G12=E0*0.5;
v21=v12*E2/E1;
N=1;
theta_degree(1)=0;
t(1)=1;
```

The example of MBB-beam with half design domain and symmetric boundary conditions is used for studing of topology optimization of orthotropic plates: top(60,20,0.5,3.0,1.5). In the case of orthotropic plates, topology optimization of the MBB-beam is investigated with respect to the orthotropy ratio E_1 / E_2 as shown in Fig. 2. It can be seen from the Fig. 2 that when the orthotropy ratio is increased, the topology optimization of the MBB-beam is changed significantly. The upper-right conner is dissapeared as high orthotropy ratio. Assume that $G_{12} / E_2 = 0.5$ and $\nu_{12} = 0.3$.

(a)

(b)

(c)

(d)

Figure 2. Topology optimization of the orthotropic MBB-beam: (a) isotropic, (b) $E_1 / E_2 = 4$, (c) $E_1 / E_2 = 10$, (d) $E_1 / E_2 = 50$.

For the example of a cantilever beam with one load as shown in Fig. 3, the same manner is carried out by using various orthotropy ratio ranging from 4 to 50. The results are depicted in Fig. 4. It can be seen from the Fig. 4 that the density distribution is reduces as the orthotropy ratio E_1 / E_2 increases. It should be noted that E_1, E_2 are in *x, y*-directions, recpectively. The Matlab code for example of the cantilever beam with one load given in “atluu2.m”.

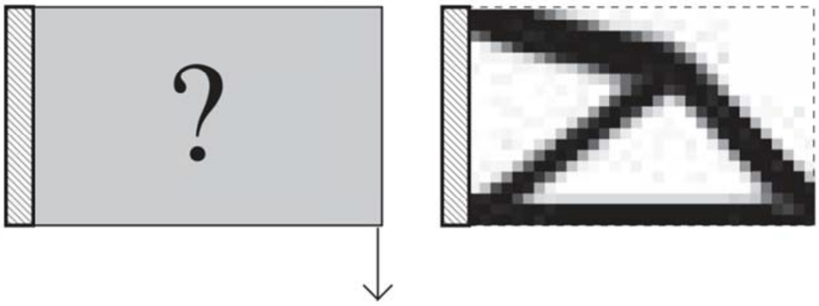

Figure 3. Schematic of a cantilever beam with one load and topology optimization for isotropic beam

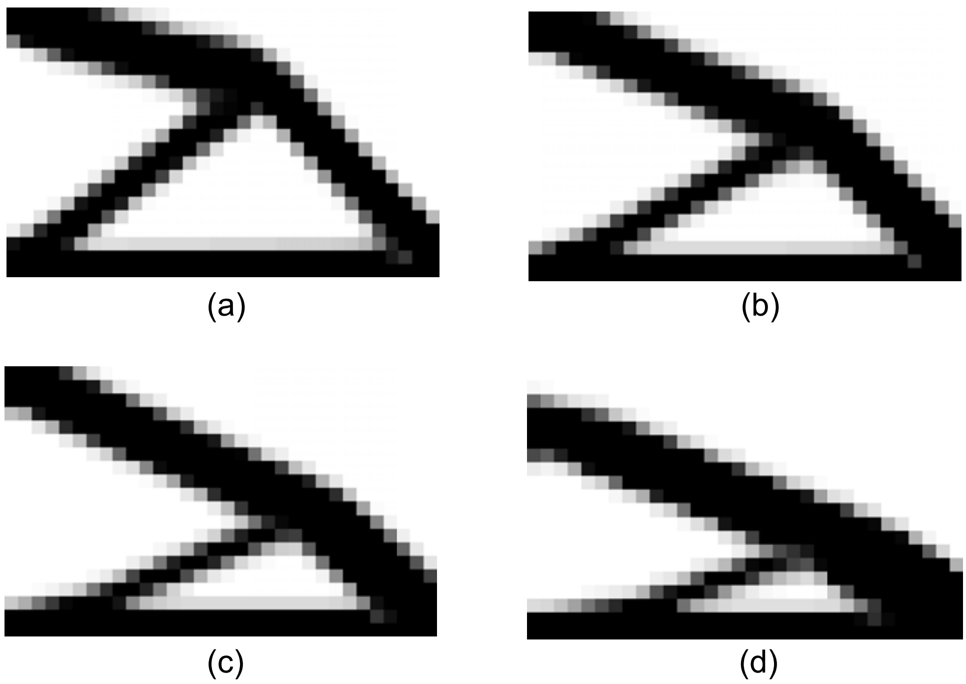

Figure 4. Topology optimization of an orthotropic cantilever beam with one load: (a) isotropic, (b) $E_1 / E_2 = 4$, (c) $E_1 / E_2 = 10$, (d) $E_1 / E_2 = 50$.

3.3 Optimal topology of laminated composite plates

The laminated composite plate with two-layer cross-ply $0^{\circ}/90^{\circ}$ stacking sequence is tested for topology optimization of plate with biaxial load (or cantilever beam with two load) as shown in Fig. 5. Assume each lamina has the same material properties, which are $G_{12}/E_2 = 0.5$ and $\nu_{12} = 0.3$. The optimized topology for the plate are present in Fig. 6 with various orthotropy ratios E_1/E_2 = 10, 20, 50 and also compared with isotropic case to depict the change from isotropic material to cross-ply laminated composite. It can be seen from the Fig. 6 that the change is significant when the orthotropy ratio is high. The Matlab code is given in "atluu3.m".

```
E0=1;
E1=50*E0;
E2=1*E0;
v12=0.3;
G12=0.5*E0;
v21=v12*E2/E1;
N=2;
theta_degree(1)=0;
t(1)=0.5;
theta_degree(2)=90;
t(2)=0.5
```

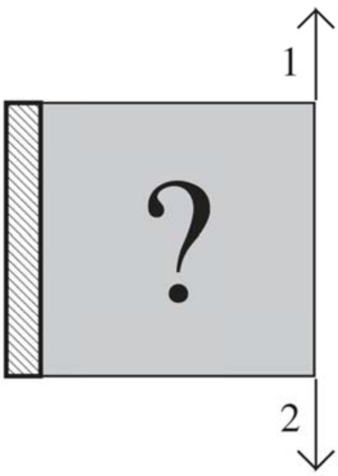

Figure 5. a cantilever beam with two load-cases or a plate with bi-axial load

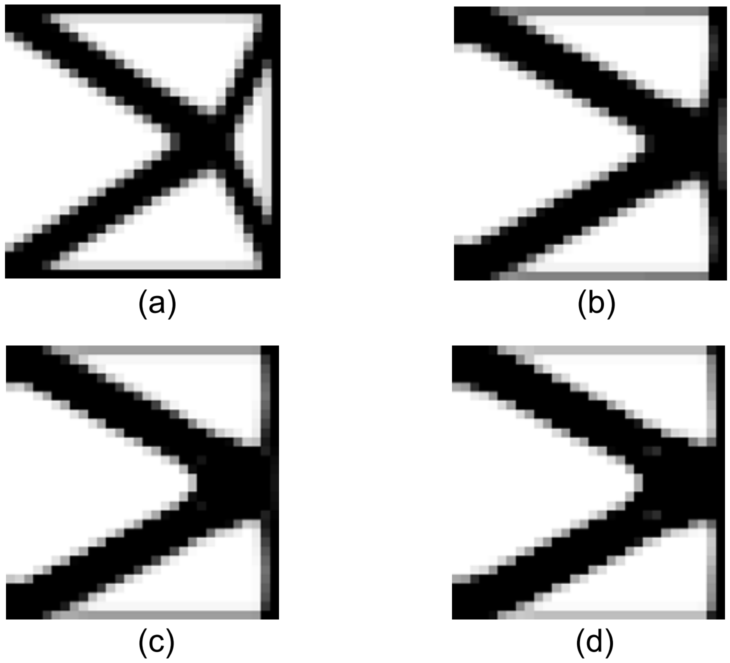

Figure 6. Topology optimization of an orthotropic cantilever beam with one load: (a) isotropic, (b) $E_1 / E_2 = 10$, (c) $E_1 / E_2 = 20$, d) $E_1 / E_2 = 50$

4. Conclusions

The topology optimization for laminated composite plates under states of plane stress is studied based on the 99 lines of Matlab code by Sigmund. In this study, the change from isotropic material to orthotropic material and laminated composite is shown for different examples. It is found that when the orthotropy ratio increases, the density distribution is changed to smaller. However the limitation of this study is that the results are restricted to given numerical examples by Sigmund and the comparison for topology optimization of the laminated composite material is lacked.

Acknowledgments

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