The 2015 World Congress on
Advances in Civil, Environmental, and Materials Research (ACEM15)
Incheon, Korea, August 25-29, 2015

Optimal Control of Cable Vibration using MR Damper based on Nonlinear Modeling

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ABSTRACT

The highly nonlinear feature of the MR material leads to a relatively complex representation of its mathematical model, and resulting in difficulties of carrying out effective control strategies on cable vibration. In this paper, a simple and efficient semi-active control scheme is proposed for tuning the MR damper real-time for optimal cable vibration control based on the nonlinear damper forces. The nonlinear Bouc-wen model is first employed to accurately portray the salient characteristics of the MR damper, and a piecewise linear interpolation scheme is used to determine the input current of MR damper corresponding to the real-time damping force requirement. Simulation study has been carried out to validate the effectiveness of the proposed semi-active control scheme for free cable vibration of the first three modes.

Keywords: Stayed cable, semi-active control, MR damper, modified Bouc-wen model

1. INTRODUCTION

Mechanical dampers have been proved to be one of the most effective countermeasures for vibration mitigation of stay cables in various cable-stayed bridges. The performance of passive linear viscous dampers has been widely studied. The optimal tuning of a linear viscous damper attached close to the cable support was numerically studied by Pacheco (1993), introducing a universal curve for evaluating the efficiency of the damper. An analytical solution to the problem of a linear viscous damper acting on a taut cable was provided by Krenk (2000), where a simple expression for the optimal tuning was obtained using complex vibration modes. These studies indicated that the maximum supplemental damping ratio achieved with passive linear viscous dampers was approximately $a/2L$, where a was the distance from the

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cable's lower anchorage to the damper and L was the length of the cable. However, for long stay cables, as the installation height of the damper is restricted due to the aesthetic concern, passive dampers may not provide sufficient supplemental damping for suppressing multi-mode cable vibrations. Therefore, semi-active MR dampers have been proposed for the vibration mitigation of long stay cables. Johnson (2007) has investigated and compared several different control strategies via simulations, concluded that far more superior damping could be achieved with semi-active dampers to the cables than that with the optimal passive linear viscous dampers. Weber (2010) applied the energy equivalent approach to a model MR damper as an equivalent linear viscous damper or a nonlinear friction damper in their theoretical and experimental studies on cable vibration control using MR dampers. Wang (2013) proposed a strategy for cable control with friction dampers, which provides the additional modal damping ratio higher than that obtained by optimal linear viscous dampers. In addition, as the damper force has to be adjusted in proportion to the cable amplitude at damper location, the modal damping ratio of cable with friction damper is not a constant like that for a linear viscous damper.

For modeling MR damper, several idealized mechanical models are proposed for predicting the response of the prototype MR damper, such as Bingham model (Stanway 1987) and Bouc-wen model (Wen 1976). Bingham model does not exhibit the nonlinear force-velocity response observed in the data for the case when the acceleration and velocity have opposite signs and the magnitude of the velocity is small. Bouc-Wen model predicts the force-displacement behavior of the damper well, and it possesses force-velocity behavior that more closely resembles the experimental data. However, similar to Bingham model, the nonlinear force-velocity response of the Bouc-Wen model does not roll off in the region where the acceleration and velocity have opposite signs and the magnitude of the velocity is small. To better predict the damper response in this region, a modified version of the system is proposed (Spencer 1997).

This paper aims to propose a simple and efficient semi-active control scheme for tuning MR damper real-time for optimal cable vibration control based on the optimal force of friction damper proposed by Wang (2013) and using an approximated linear relation between damper force and input voltage developed from the modified Bouc-wen model of MR damper. Simulation study will be carried out to validate the effectiveness of the proposed semi-active control scheme for free cable vibration of the first three modes.

2. PIECEWISE LINEAR INTERPOLATION SCHEME BASED ON MODIFIED BOUC-WEN MODEL

2.1 Modified Bouc-wen model

The modified Bouc-wen model as shown in Fig.1 will be used in this paper to portrait the physical behavior of MR damper.

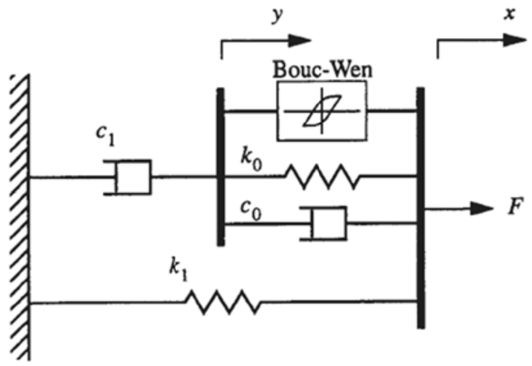


Fig.1 Modified Bouc-wen Model of MR Damper

As proposed in Spencer (1997), to obtain the governing equations, consider only the upper section of the model. The forces on either side of the rigid bar are equivalent, therefore

$$c_1 \dot{y} = \alpha z + k_0(x-y) + c_0(\dot{x}-\dot{y}) \tag{1}$$

where the evolutionary variable z is governed by

$$\dot{z} = -\gamma\left|\dot{x}-\dot{y}\right| z\left|z\right|^{n-1} - \beta(\dot{x}-\dot{y})\left|z\right|^{n} + A(\dot{x}-\dot{y}) \tag{2}$$

The total force generated by the system is then obtained by summing up the forces in the upper and lower sections of the system in Fig. 1, which yields

$$F = \alpha z + \mathrm{c}_0(\dot{x}-\dot{y}) + k_0(x-y) + k_1(x-x_0) \tag{3}$$

To determine a model that is valid for fluctuating magnetic fields, the functional dependence of the parameters on the applied voltage (or current) must be determined, and the following relations are proposed as

$$\alpha = \alpha(u) = \alpha_a + \alpha_b u \tag{4}$$

$$\mathrm{c}_1 = \mathrm{c}_1(u) = \mathrm{c}_{1a} + \mathrm{c}_{1b} u \tag{5}$$

$$\mathrm{c}_0 = \mathrm{c}_0(u) = \mathrm{c}_{0a} + \mathrm{c}_{0b} u \tag{6}$$

where the dynamics involved in the MR fluid reaching rheological equilibrium and in driving the electromagnet in the MR damper are accounted for through the first-order filter as

$$\dot{u} = -\eta(u-v) \tag{7}$$

and v is voltage applied to current driver. Optimal values of a total of 14 parameters (c_{0a}, c_{0b}, k_0, c_{1a}, c_{1b} k_1, x_0, α_a, α_b, γ, β, A, n, η) determined for the prototype MR damper (Spencer 1997) are as follows. The prototype MR damper (shown in Fig.2) is a small-scale MR damper with a maximum damping force of 3000N, which was conducted by the research group of University of Notre Dame and Washington University. The damper is 21.5 cm long in its extended position, and the main cylinder is 3.8 cm in diameter. The main cylinder houses the piston, the magnetic circuit, and accumulator and 50 ml of MR fluid, and the damper has $a \pm 2.5$ cm stroke, this prototype MR damper will be used in the following simulation studies.

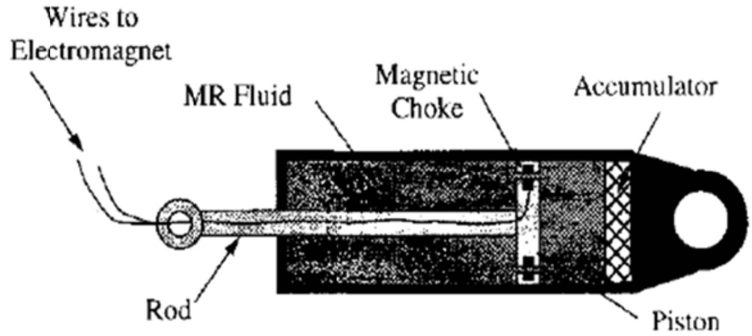
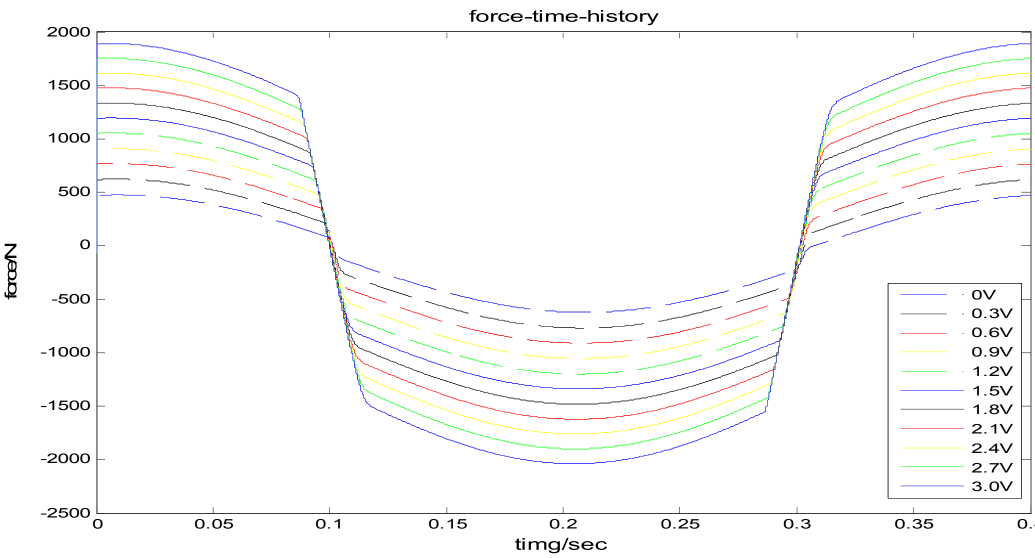


Fig.2 Schematic of MR Damper

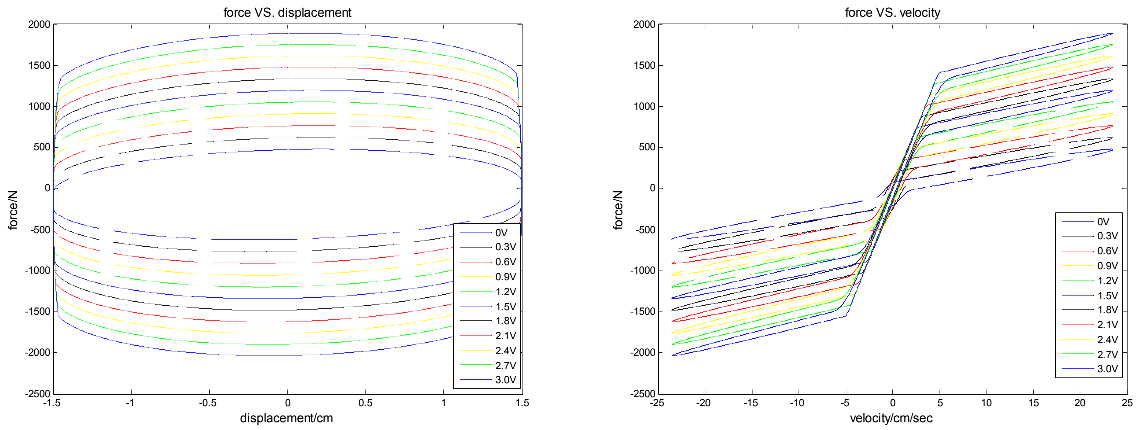
Table1. Parameters for Generalized Bou-wen Model

Parameter	Value	Parameter	Value
c_{0a}	$21.0 N \cdot s/cm$	α_a	$140 N/cm$
c_{0b}	$3.50 N \cdot s/cm \cdot V$	α_b	$695 N/cm \cdot V$
k_0	$46.9 N/cm$	γ	$363 cm^{-2}$
c_{1a}	$283 N \cdot s/cm$	β	$363 cm^{-2}$
c_{1b}	$2.95 N \cdot s/cm \cdot V$	A	301
k_1	$5.00 N/cm$	n	2
x_0	$14.3 cm$	η	$190 s^{-1}$

Simulation studies were performed in SIMULINK of Matlab. The damper force of the modified Bouc-wen model is computed based on the displacement at damper location and the input current/voltage of the MR damper. The response of the damper under a 2.5 Hz sinusoid vibration with an amplitude of 1.5 cm is shown in Fig.3 with 11 constant voltage levels 0 V, 0.3 V,..., 3 V. It can be seen that, at 0 V, the MR damper primarily exhibits the characteristics of a purely viscous device. However, as the voltage increases, the damper force increases and produces behavior associated with a plastic material in parallel with a viscous damper. Also, notice that the increase in damper force for a given increase in the applied voltage is approximately linear for voltages between 0 and 3V.



(a) Force versus Time



(b) Force versus Displacement

(c) Force versus Velocity

Fig.3 Measured Force for 2.5 Hz Sinusoidal Vibration with Amplitude of 1.5 cm (voltage linearly increases between 0-3V)

2.2 Piecewise linear interpolation scheme

Although the modified Bouc-wen model describes well the highly nonlinear feature of MR damper, it has difficulties forming a simple relationship between the damping force and the input voltage applied to the current driver. In this paper, a piecewise linear interpolation scheme (Weber 2013) based on modified Bouc-wen model is used to determine the input voltage of MR damper corresponding to the requirement of real-time damping force.

A series of MR damper forces are computed using the modified Bouc-wen model by inputting different constant voltages and with actual MR damper displacement and velocity. The desired damper force is determined by the selected control laws. Then, a piecewise linear interpolation scheme, as shown in Fig. 4, is employed to estimate the desired input voltage based on the computed forces of constant voltages and the desired damper force. Finally, the actual damper force is computed with the desired voltage and actual MR damper displacement through the modified Bouc-wen model.

The efficiency of the piecewise linear interpolation scheme is validated by simulations. It can be noted that the actual damper force differs slightly from the desired control force if the desired force is not constraint by $f(V_{\max})$ due to the choice of desired control force.

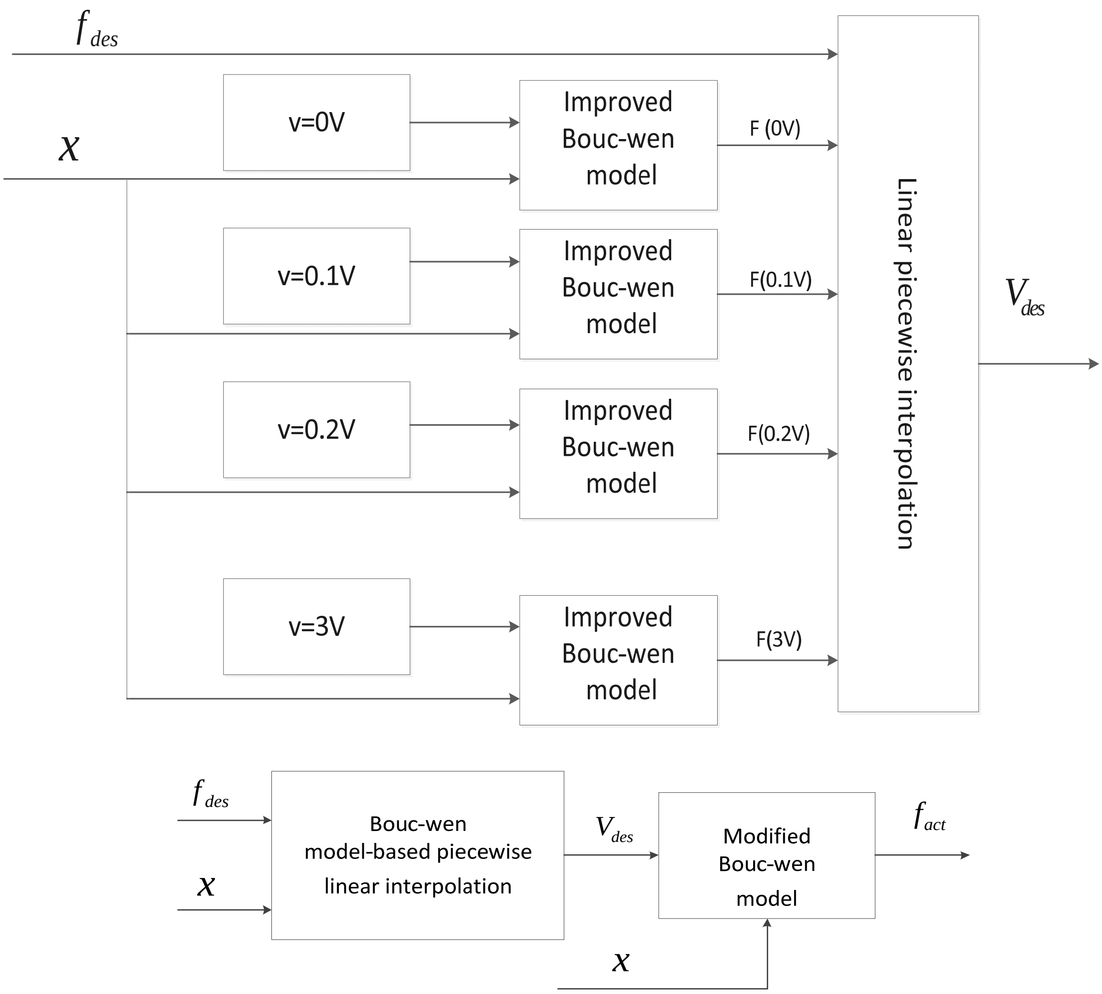


Fig.4 Block diagram of piecewise linear interpolation scheme

3. SEMI-ACTIVE CONTROL OF CABLE

3.1 Equation of motion of a taut cable

The cable and the MR damper are shown in Fig.5. The length of the cable is L, and the damper is located at the distance of a from the left end. The cable tension is T, and the mass per unit length is m. The force in the MR damper is F_d, and the inherent cable damping is assumed to be zero. The bending stiffness and the sag of the cable are negligible.

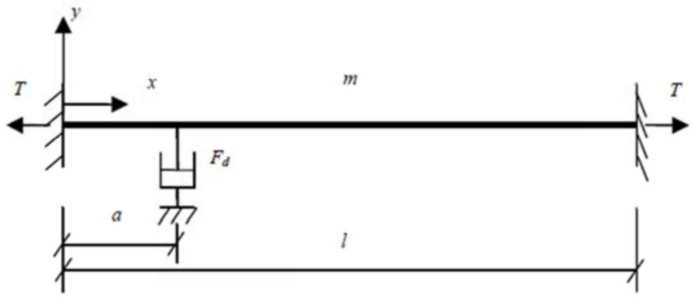


Fig.5. Taut cable with MR damper

The motion of the taut cable in the linear range is described by the following partial differential equation:

$$M\frac{\partial^2 y}{\partial t^2} - T\frac{\partial^2 y}{\partial x^2} = F(x,t) + F_d\delta(x-a) \tag{8}$$

where $y(x,t)$=transverse deflection of the cable; $F(x,t)$= distributed load on the cable; F_d = transverse damper force at $x=a$ location; and $\delta(\cdot)$=Dirac delta function. The partial differential equation above is to be solved with the boundary conditions

$$y(0,t) = y(l,t) = 0 \tag{9}$$

The transverse deflection is approximated by a finite series in the form (Pacheco, B. M.1993) of

$$\mathrm{y}(x,t) = \sum_{i=1}^{n} q_i(t)\sin(\pi\,\mathrm{i}\,x/\mathrm{l}) \tag{10}$$

where n = number of degrees of freedom, $q_i(t)$= generalized displacements, $\varphi_i(x)$ = set of shape functions, which is selected to be sinusoidal shape functions as

$$\varphi_i(x) = \sin(\pi\,\mathrm{i}\,x/\mathrm{l}) \tag{11}$$

in which l = span of the cable.

As $\varphi_i(x)$ is proportional to the i th undamped mode of the cable, by Galerkin method, substituting Eq. (11) into the equation of motion (9), multiplying by $\varphi_i(x)$ and integrating over the length of the cable, one obtains the following governing matrix equation (Johnson 2007)

$$M\ddot{q} + Kq = F_q(t) + \varphi(a)F_d(t) \tag{12}$$

where

$$m_{ij} = m\int_0^l \frac{\sin \pi ix}{l}\frac{\sin \pi jx}{l}dx = \frac{ml}{2}\delta_{ij} \tag{13}$$

$$k_{ij} = -T\int_0^l \frac{\sin \pi ix}{l}\left(\frac{\sin \pi jx}{l}\right)'' dx = \frac{T\pi^2 i^2}{2l}\delta_{ij} \tag{14}$$

$$F_{qi}(t) = \int_0^l F(x,t)\sin(\mathrm{i}\ \pi x/l)dx \tag{15}$$

$$\varphi(a) = \sin\left(\mathrm{i}\ \pi a/l\right) \tag{16}$$

As these sinusoidal shape functions are mutually orthogonal, the mass $M = [m_{ij}]$ and stiffness $K = [k_{ij}]$ matrices are diagonal. For control design, the system dynamics may be equivalently written in state-space form with input/output relations:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \tag{17}$$

where $x(t) = \begin{Bmatrix} q(t) \\ \dot{q}(t) \end{Bmatrix}_{2n\times 1}$ is the state vector,

$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ M^{-1}Bss \end{bmatrix}$, $C = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$, $D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $u = \begin{bmatrix} F(t) \\ F_d(t) \end{bmatrix}$, $Bss = \begin{bmatrix} Ds & Bs \end{bmatrix}$.

3.2 optimal amplitude dependent friction force control

A semi-active control scheme (see in Fig.6) is proposed for tuning the MR damper real-time for the optimal control of cable vibration. The desired control force is amplitude dependent friction force proposed by Wang (2013) as

$$F_d = -\mathrm{sgn}(\dot{y}_d)\frac{T}{a}Y_d \tag{18}$$

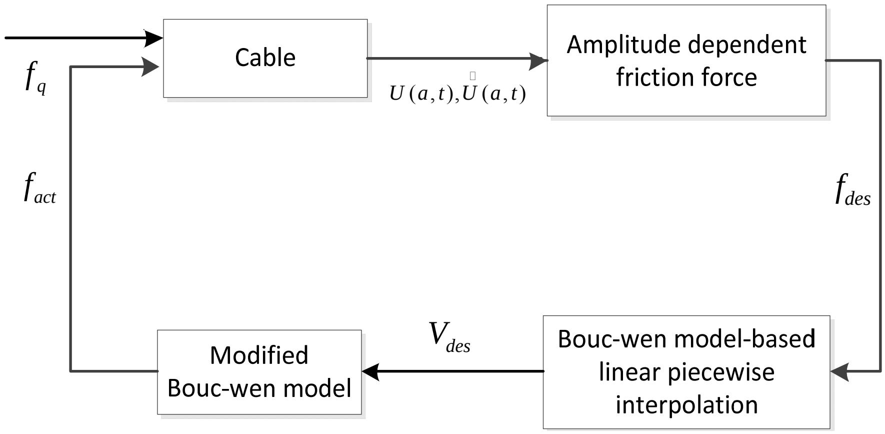


Fig.6 Block diagram of the semi-active control system

A 80 m long stay cable is used for numerical simulation on the mitigation of cable vibration. Table2. lists the cable parameters where m is the mass per unit length of the cable and T is the cable tension. The damper is located at the distance $a=3.2m$ from the left end.

Table2. Main parameters of cable

L (m)	T (KN)	m (kg/m)	a (m)
80	1226	10	3.2

First three modes free vibration of the cable were simulated. The input current of MR damper corresponding to the required real-time damping force was determined through the piecewise linear interpolation scheme as introduced in section 2.2.

For the first mode vibration, $F(x,t)$ in Eq. (8) is taken as $\sin\frac{\pi x}{l}F(t)$, where $F(t)=5\sin(2\pi f_1 t)$, the time histories of cable vibrations at the mid-span, with and without MR damper, are shown in Fig.7. For the second mode vibration, $F(t)=5\sin(2\pi f_2 t)$ $f_2=2f_1$, the time histories of cable vibrations at one fourth of the cable length are shown in Fig.8. For the third mode, $F(t)=10\sin(2\pi f_3 t)$, $f_3=3f_1$, the time histories of cable vibrations at one sixth of the cable length are shown in Fig.9. In Figs.7-9, solid lines represent cable vibrations without damper, while dash lines demonstrate vibration with desired damper force and dotted lines correspond to vibration with actual damper force. Comparisons of the RMS value of displacements under different vibration modes and different control schemes are shown in Table 3.

It can be observed from Figs.7-9 and Table3. that with the installation of MR damper, the reduction of first mode vibration is about 75%, that of second mode vibration is around 78%, and the reduction for the third mode vibration is 84%. Also, the performance of MR damper with actual damper forces approximated by the linear interpolation scheme is as good as that with desired damper forces. This proved the effectiveness and applicability of the proposed semi-active control strategy.

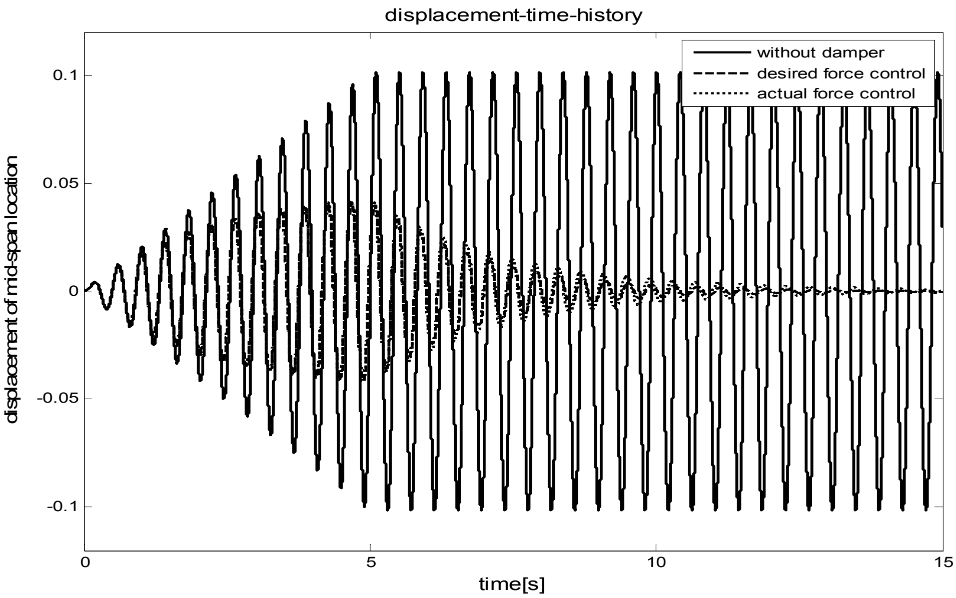


Fig.7 Displacement-time-history at mid-span

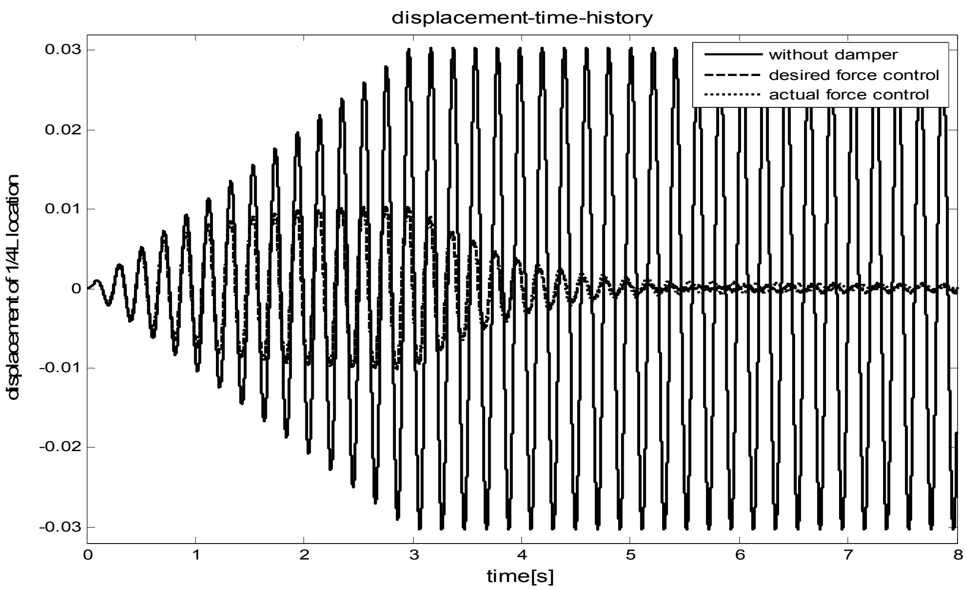


Fig.8 Displacement-time-history at one fourth of the cable length

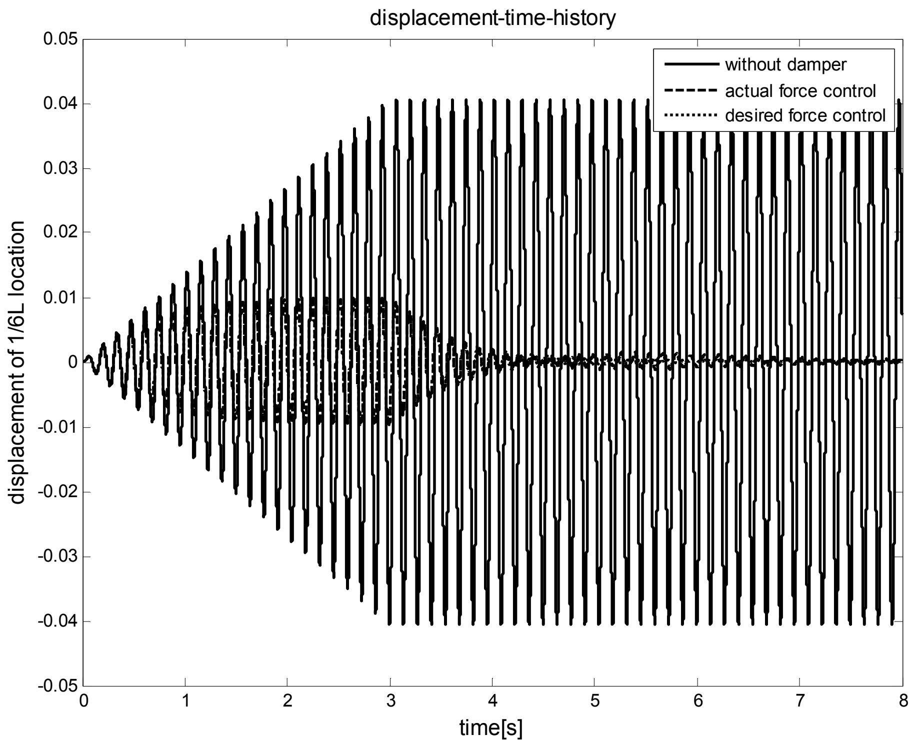


Fig.9 Displacement-time-history at one sixth of the cable length

Table3. Comparison of RMS displacement for the first three modes with/without damper

	1/2L location (the first mode)		1/4L location (the second mode)		1/6L location (the third mode)	
	σ_{rms}	Decay ratio ζ	σ_{rms}	Decay ratio ζ	σ_{rms}	Decay ratio ζ
Without control σ_{rms-ud}	63.362	--	18.631	--	24.883	--
Desired force control σ_{rms-1}	15.464	75.24%	4.124	77.86%	3.937	84.18%
Actual force control σ_{rms-2}	15.799	75.07%	4.104	77.97%	3.901	84.32%

Decay ratio: $\zeta = (\sigma_{rms-ud} - \sigma_{rms-i}) / \sigma_{rms-ud}$

4. CONCLUSIONS

This paper proposed a simple and efficient semi-active control scheme for tuning MR damper real-time for optimal cable vibration control based on the optimal force of friction damper proposed by Wang (2013) and using an approximated linear relation between damper force and input current developed from the modified Bouc-wen model of MR damper. Simulation study has been carried out to validate the effectiveness of the proposed semi-active control scheme for free cable vibration of the first three modes. The results indicated that with the installation of MR damper, the reduction of the first three modes of cable vibration is all greater than 75%. Also, the performance of MR damper with actual damper forces approximated by the linear interpolation scheme has been shown to be as good as that with desired damper forces determined from the optimal friction force control law.

Therefore, the proposed semi-active control strategy using MR damper is efficient and applicable for mitigating cable vibrations.

References

Wen, Y. K. (1976), "Method of random vibration of hysteretic systems", J ENG MECH, ASCE, 102(2), 249-263.

Stanway, R., Sproston, J. L., and Stevens, N. G. (1987), "Non-linear modeling of an electro-rheological vibration damper", J. Electrostatics, 20, 167-184.

Pacheco, B., Fujino, Y. and Sulekh, A. (1993), "Estimation Curve for Modal Damping in Stay Cables with Viscous Damper", J STRUCT ENG, 119(6), 1961-1979.

Spencer, B.F., Dyke, S.J., Sain, M.K. and Carlson, J.D. (1997), "Phenomenological model for magnetorheological dampers", J ENG MECH, ASCE, 123(3), 230-238.

Krenk, S. (2000), "Vibrations of a taut cable with an external damper", J APPL MECH-T ASME, 67(4), 772-776

Johnson, E.A., Baker, G.A., Spencer, B.F. and Fujino, Y. (2007), "Semiactive Damping of Stay Cables", J ENG MECH, 133(1), 1-11.

Weber, F., Hogsberg, J. and Krenk, S. (2010), "Optimal Tuning of Amplitude Proportional Coulomb Friction Damper for Maximum Cable Damping", J STRUCT ENG, ASCE, 136(2), 123-134.

Wang, H.P. and Sun, L.M. (2013), "Semi-active control of stay cables using nonlinear friction damper", Proceedings of SPIE Conference on Sensors and Smart Structures Technologies for Civil, Mechanical, and Aerospace Systems, San Diego, CA.

Weber, F. (2013), "Bouc-Wen model-based real-time force tracking scheme for MR dampers", SMART MATER STRUCT, 22(0450124)