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Modeling Capability of Continuum Mechanics Based Beam Elements

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ABSTRACT

In this paper, we present the excellent modeling capability of the continuum mechanics based beam elements. Since the elements have embedded cross-sectional discretization, individual handling of cross-sectional elements is enabled. Due to this novel feature, many complicated beam structures such as beams with discontinuously varying cross-sections, arbitrary composite cross-sections, and functionally graded cross-sections, and layered beams with incomplete shear connections can be easily analyzed. Furthermore, the geometric and material nonlinear analysis is also available, because the beam elements are formulated from continuum mechanics. The various applications of the continuum mechanics based beam elements are demonstrated through representative numerical examples.

1. INTRODUCTION

Beam elements are abundantly used for analysis of various scientific and engineering structures. Recently, demands of beam elements with high modeling capability is increasing due to the advent of new and complex applications. A number of studies have been presented to improve the modeling and analysis capabilities of beam elements. In particular, the continuum mechanics based beam elements can represent highly coupled mechanical behaviors under complicated 3D geometries. The element can also provide ability of individual handling of cross-sectional elements. In this paper, the brief concept of the continuum mechanics based beam elements and their various applications are introduced.

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2. Continuum mechanics based beam element

We here provide a brief formulation of the continuum mechanics based beam elements. The detailed formulation is well illustrated in Yoon et. al. 2012 and Yoon and Lee 2014. The geometry interpolation of the entire beam is obtained by a material position of the sub-beam element m (see Fig. 1),

$$\mathbf{x}^{(m)} = \sum_{k=1}^{q} h_k(r)\mathbf{x}_k + \sum_{k=1}^{q} h_k(r)\bar{y}_k^{(m)}\mathbf{V}_{\bar{y}}^k + \sum_{k=1}^{q} h_k(r)\bar{z}_k^{(m)}\mathbf{V}_{\bar{z}}^k \quad (1)$$

$$\text{with } \bar{y}_k^{(m)} = \sum_{j=1}^{p} h_j(s,t)\bar{y}_k^{j(m)} \text{ and } \bar{z}_k^{(m)} = \sum_{j=1}^{p} h_j(s,t)\bar{z}_k^{j(m)} \quad (2)$$

From the geometry interpolation in Eq. (1), the displacement interpolation of the sub-beam m is derived as

$$\mathbf{u}^{(m)} = \sum_{k=1}^{q} h_k(r)\mathbf{u}_k + \sum_{k=1}^{q} h_k(r)\bar{y}_k^{(m)}\left\{\boldsymbol{\theta}_k \times \mathbf{V}_{\bar{y}}^k\right\} + \sum_{k=1}^{q} h_k(r)\bar{z}_k^{(m)}\left\{\boldsymbol{\theta}_k \times \mathbf{V}_{\bar{z}}^k\right\} \quad (3)$$

$$\text{with } \mathbf{u}_k = \begin{bmatrix} u_k \\ v_k \\ w_k \end{bmatrix} \text{ and } \boldsymbol{\theta}_k = \begin{bmatrix} \theta_x^k \\ \theta_y^k \\ \theta_z^k \end{bmatrix} \quad (4)$$

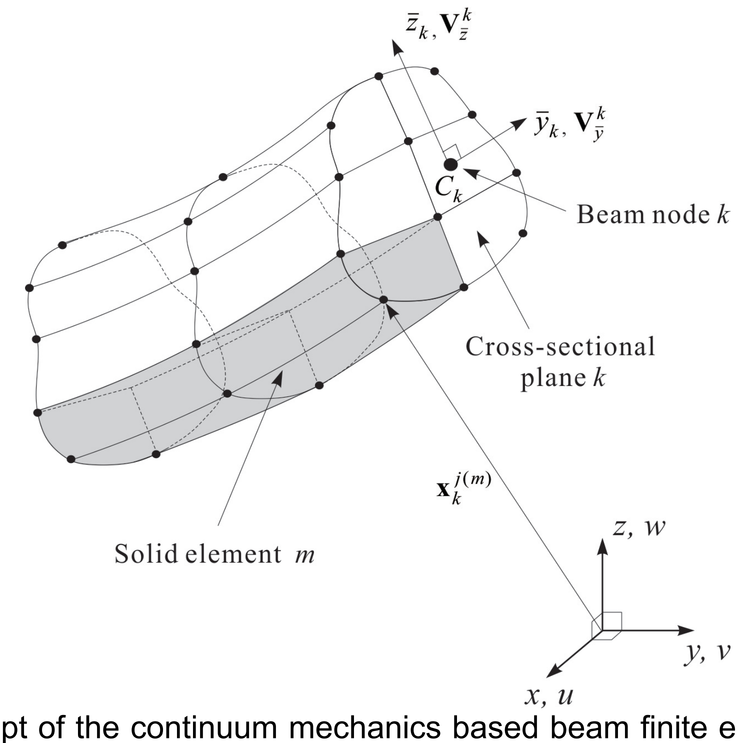

Fig. 1 The concept of the continuum mechanics based beam finite element with cross-sectional discretization.

The continuum mechanics based beam elements can be easily enriched by using appropriate displacement functions with the corresponding DOFs to obtain a generalized displacement field as

$$\mathbf{u}_g^{(m)} = \mathbf{u}^{(m)} + \mathbf{u}_a^{(m)} \tag{5}$$

in which $\mathbf{u}_g^{(m)}$ is the generalized displacement field and $\mathbf{u}_a^{(m)}$ is the additional displacement field such as warping displacements, slip displacements, and displacements for cross-sectional distortions. Some examples of the enriched beam formulation are introduced with its numerical applications in the following section.

3. Application of the continuum mechanics based beam elements

In this section, we demonstrate the performance and modeling capability of the continuum mechanics based beam elements through various applications.

3.1 Discontinuously varying beam problem

A discontinuously varying beam that consists of three different thin-walled cross-sections illustrated in Fig. 2, is considered. The additional displacement field is employed as

$$\mathbf{u}_a^{(m)}(r,s,t) = \sum_{k=1}^{q} h_k(r)\left[f_k^{(m)}(s,t)\alpha_k + f_L^{(m)}(s,t)\beta_L^k + f_R^{(m)}(s,t)\beta_R^k\right]\mathbf{V}_r^k , \tag{6}$$

where $f_k^{(m)}$ is the free warping function, and $f_L^{(m)}$ and $f_R^{(m)}$ is interface warping functions. The detailed derivation is well illustrated in Yoon and Lee 2014. Angle of twist and displacement v along the beam length is shown in Fig. 3. The numerical results show good agreement with the reference solution obtained from the corresponding shell element model.

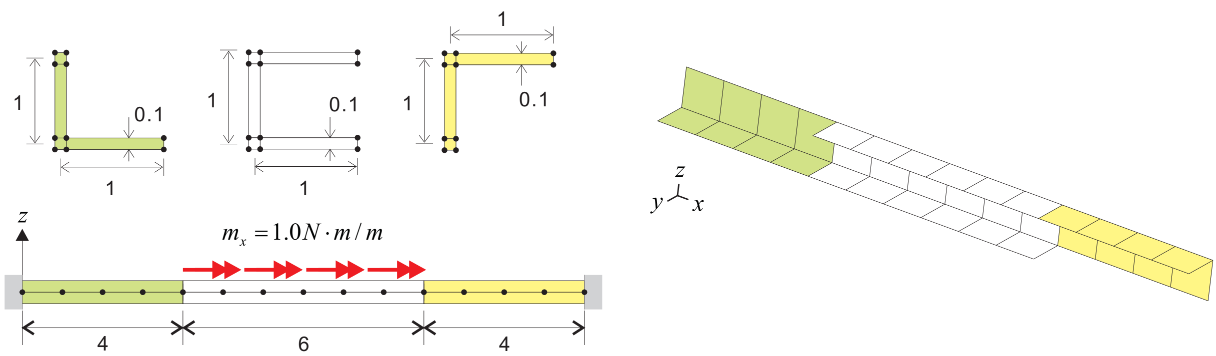

Fig. 2 A discontinuously varying beam with three different thin-walled cross-sections. (Yoon and Lee 2014)

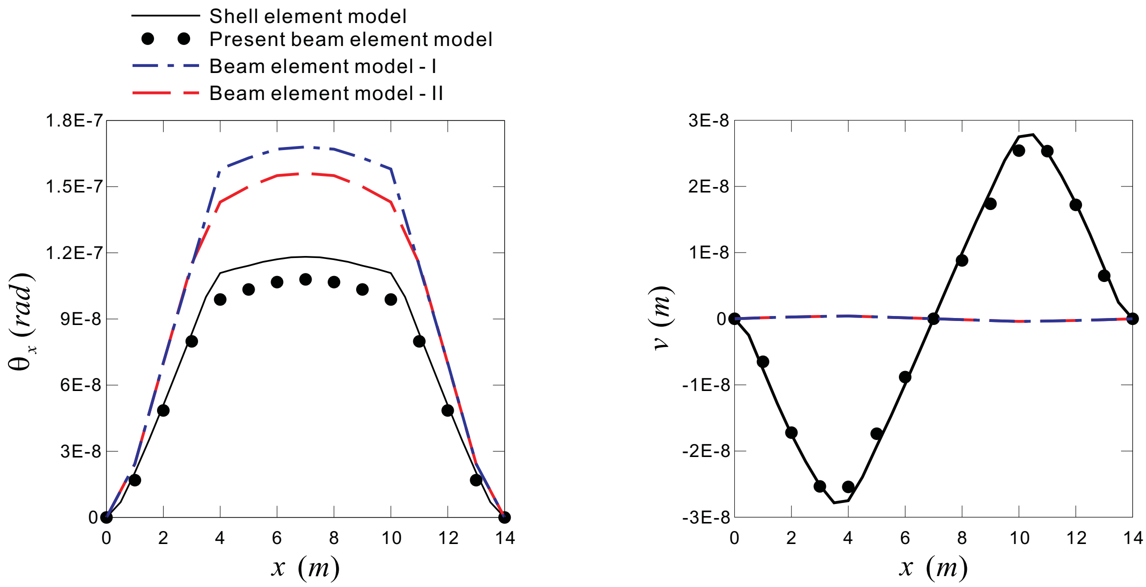

Fig. 3 Angle of twist and displacement v along the beam length (Yoon and Lee 2014)

3.2 Three-layer beam problem

The slip displacement field $\mathbf{u}_s^{(m)}$ is included into the basic beam displacement as

$$\mathbf{u}_s^{(m)} = \sum_{k=1}^{q} h_k(r) \sum_{i=1}^{l} f_k^{i(m)}(s,t)\alpha_k^i \mathbf{V}_{\bar{x}}^k , \tag{7}$$

in which $f_k^{i(m)}(s,t)$ are the slip functions and α_k^i are the slip DOFs. There are two slip functions, which is the bending slip function $\mathbf{x}^b$ and the axial slip function $\mathbf{x}^a$. The two slip functions can be calculated as follows.

$$\mathbf{x}^b = \mathbf{\Psi}^{-1}\mathbf{r}_b \quad \text{and} \quad \mathbf{x}^a = \mathbf{\Psi}^{-1}\mathbf{r}_a \tag{8}$$

in which

$$\mathbf{r}_b = \begin{bmatrix} d_1 & d_2 & \cdots & d_{n-1} & 0 \end{bmatrix}^{\mathrm{T}} \quad \text{and} \quad \mathbf{r}_a = \begin{bmatrix} E_1^{-1} - E_2^{-1} & E_2^{-1} - E_3^{-1} & \cdots & E_{n-1}^{-1} - E_n^{-1} & 0 \end{bmatrix}^{\mathrm{T}} \tag{9}$$

$$\mathbf{\Psi} = \begin{bmatrix} \mathbf{Q}_n \\ \mathbf{A}_n \end{bmatrix}, \tag{10}$$

$$\text{with} \quad \mathbf{Q}_n = [\mathbf{I}_n \quad \mathbf{0}] + [\mathbf{0} \quad -\mathbf{I}_n] \quad \text{and} \quad \mathbf{A}_n = [A_1 \quad A_2 \quad \cdots \quad A_n]. \tag{11}$$

where d_i is the length between the centroid of sub-beam $i+1$ and the centroid of sub-beam i, E_i is Young's modulus of the ith sub-beam and A_i is cross-sectional area of the ith sub-beam. The constant slip modulus K_s is employed to model the partial interaction effect between layers and only the load-slip curve is considered to be linear

$$V_s = K_s \cdot u_s . \tag{12}$$

As a simple verification, the three-layer T-section simply supported beam as shown in Fig. 4, referred previously by Chui and Barclay 1998, is analyzed and is compared with its reference solution. The z-directional concentrated load P_z is applied at the mid-span of the beam. Slip moduli at the interlayer are $K_1 = 11\,\text{MPa}$ and $K_2 = 6\,\text{MPa}$ and Young's moduli of layer are $E_1 = 18{,}000\,\text{MPa}$, $E_2 = 6{,}000\,\text{MPa}$ and $E_3 = 10{,}000\,\text{MPa}$.

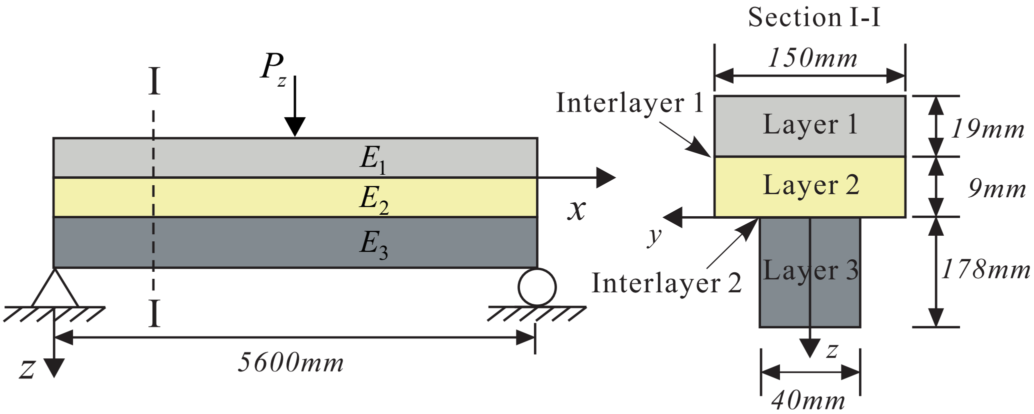

Fig. 4 Three-layer T-section simple supported beam

The layered-beam is modeled by 10 beam elements with cross-section discretized by 3 sub-beam elements. The reference solutions in Chui and Barclay 1998 are obtained by 8-node plane stress elements in ANSYS (Swanson Analysis Systems Inc. 1994). The deflection curves along the beam length is depicted in Fig. 5. The numerical results show good agreement with the reference solution obtained from shell element model.

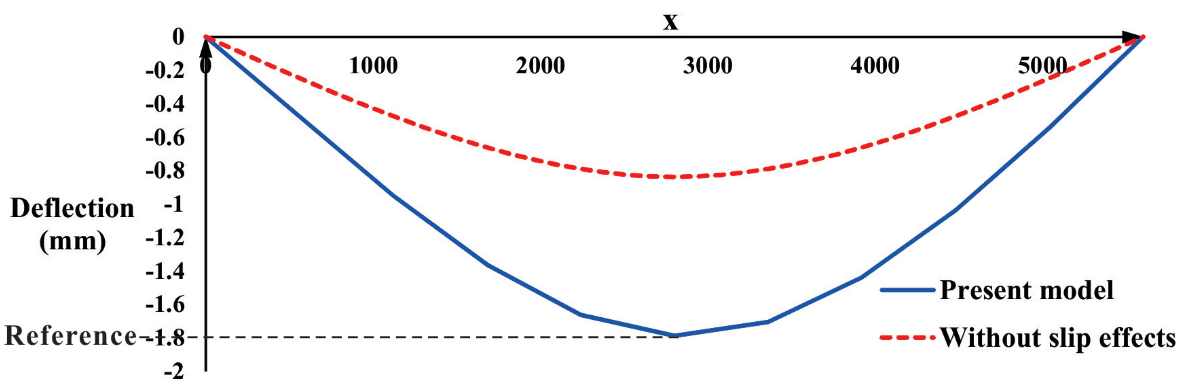

Fig. 5 Deflection curves along the beam length

4. Conclusions

In this paper, the concept of the basic continuum mechanics based beam formulation and the enriched beam formulation including additional displacement fields are introduced. Afterward, several numerical applications are illustrated to verify the enriched beam formulations. Discontinuously varying beam and layered beam with deformable shear connection are solved with the corresponding additional displacement fields. As a result, the numerical results are presented graphically and good agreement with the reference solution can be found. Clearly, the continuum mechanics based beam element shows its excellent modeling capability.

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