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Dynamic condensation for design optimization and probability-based analysis

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ABSTRACT

Needs of large-scaled design optimization and probability-based analysis are increased. In these tasks, repeated analyses are required. These repeated analyses take a long calculation time. However, many commercial FEA programs are not s uitable to the repeated analyses. Thus, we propose a new condensation method for repeated analyses. The proposed method reuse reduced matrices obtained in previous analysis. The proposed method is implemented as the Matlab code combined with the commercial programs. Numerical examples demonstrate to show efficiency of the proposed method.

1. INTRODUCTION

Due to the increase of computational resource and development of computational techniques, Finite element model size in structural analysis is increased. In the past, dynamic analyses with a small component were performed. Nowadays, many people analyze an assembled overall structure. Moreover, many people want not only a simple analysis but also design optimization or probability-based analysis. These tasks are basically consisted of many repeated analyses. These repeated analyses take a large calculation time.

Unfortunately, many commercial finite element analysis (FEA) programs are not suitable for these repeated analyses, because initial FEA commercial programs were developed as analysis tool in almost fifty years ago. The few programs consider design optimization or probability-based analysis in initial version. Thus, if some part of structure is changed, commercial programs usually perform overall analysis processes again. Objective of this study is to improve the inefficient process of the commercial programs in repeated analyses.

We propose a new dynamic condensation method for repeated analyses. The proposed method reuse reduced matrices obtained in previous analysis. The proposed

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method is implemented as the Matlab code combined with the commercial programs. Finally, numerical examples are demonstrated to verify the proposed method.

2. PROPOSED CONDENSATION METHOD

In condensation method, all degrees of freedom (DOFs) are separated into master DOFs and slave DOFs as follows:

$$\begin{bmatrix} \mathbf{K}_{mm} & \mathbf{K}_{ms} \\ \mathbf{K}_{ms}^T & \mathbf{K}_{ss} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_m \\ \mathbf{u}_s \end{Bmatrix} = \lambda \begin{bmatrix} \mathbf{M}_{mm} & \mathbf{M}_{ms} \\ \mathbf{M}_{ms}^T & \mathbf{M}_{ss} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_m \\ \mathbf{u}_s \end{Bmatrix} \tag{1}$$

m and *s* means master DOFs and slave DODs, respectively. The transformation matrix that presents relation of master DOFs and slave DOFs is calculated with the second row of Eq. (1). The simplest method is static condensation method introduced by Guyan as follows (Guyan, 1965).

$$\mathbf{t}_S = \mathbf{K}_{ss}^{-1}\mathbf{K}_{ms}^T \tag{2}$$

To improve accuracy, the improved reduction method (Gordis, 1992) was suggested as follows:

$$\mathbf{t}_{IRS} = \mathbf{t}_S + \mathbf{K}_{ss}^{-1}\left(\mathbf{M}_{ms}^T + \mathbf{M}_{ss}\mathbf{t}_S\right)\left(\mathbf{M}_S\right)^{-1}\mathbf{K}_S \tag{3}$$

In this study, we apply Woodbury formula (Woodbury, 1950) in the improved reduction method to reuse the reduced matrix. The reduced mass matrix ($\mathbf{M}_S$) is changed during optimization process. The changed mass matrix ($\mathbf{M}_{S\_changed}$) is defined as Eq. (4).

$$\mathbf{M}_{S\_changed} = \mathbf{M}_S + U\mathbf{M}_{perturb}V \tag{4}$$

The inverse matrix of the changed mass matrix ($\mathbf{M}_{S\_changed}$) is expressed using Woodbury formula as follows:

$$\left[\mathbf{M}_{S\_changed}\right]^{-1} = \left(\mathbf{M}_S + U\mathbf{M}_{perturb}V\right)^{-1} = \mathbf{M}_S^{-1} - \mathbf{M}_S^{-1}U\left(\mathbf{M}_{perturb}^{-1} + V\mathbf{M}_S^{-1}U\right)^{-1}V\mathbf{M}_S^{-1} \tag{5}$$

Due to the Eq. (5), inverse matrix is efficiently calculated. Figure 1. Shows the flowchart of optimization process in previous method and proposed method.

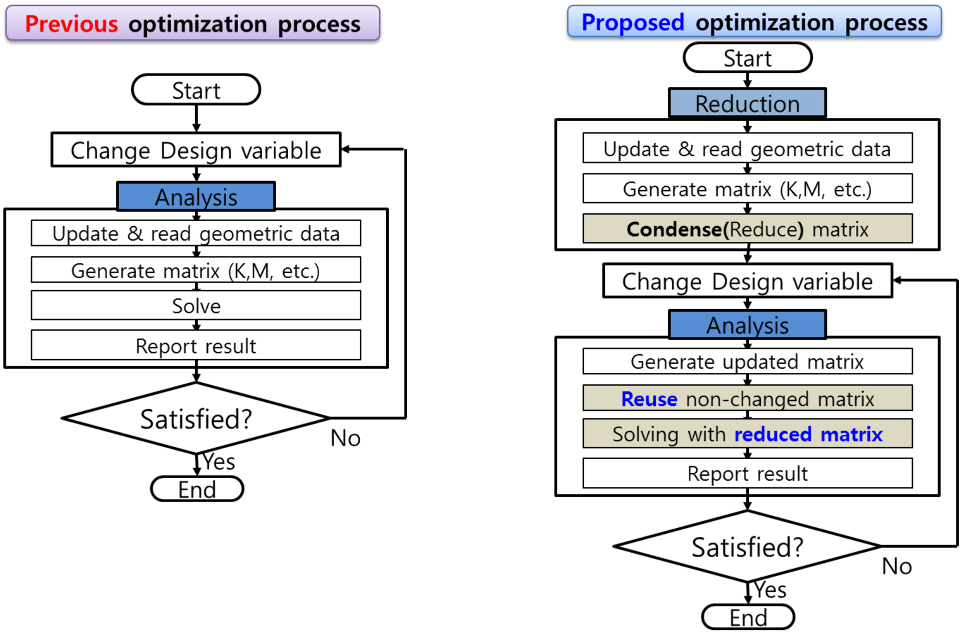
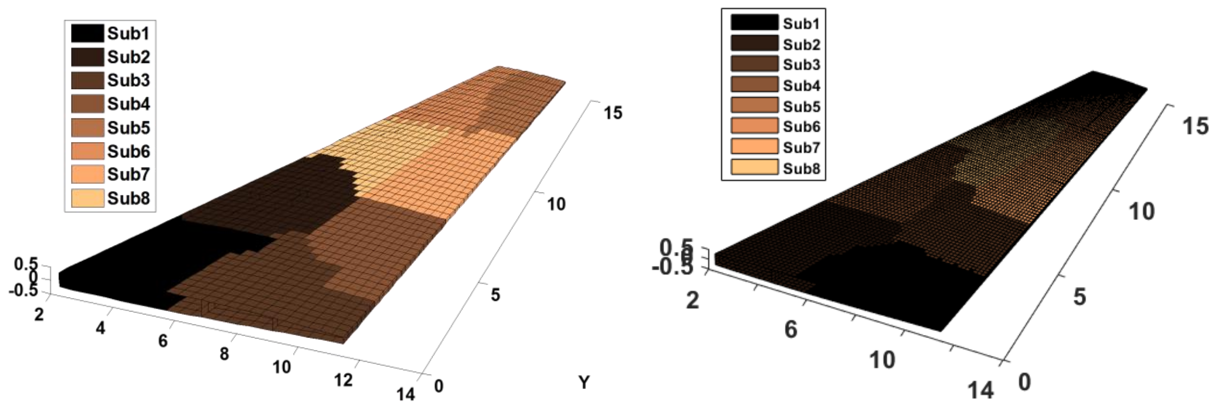


Fig. 1 Flowchart of optimization process

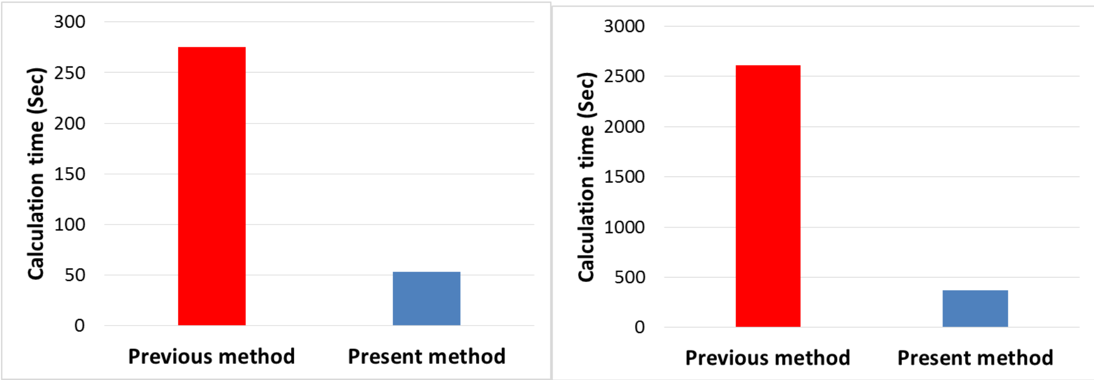
3. Numerical example

To verify the proposed method, the Eigen value analysis is performed in numerical examples. The numerical models are wing box models constituted to rib, spar and skin as shown Figure 2. Each model is separated into eight sub-domains. The calculation time of the Eigen value analysis is compared, when the first sub-domain is only changed by the optimization algorithm. Figure 3 shows the calculation time in the previous method and the proposed method. As shown in figure 3, the proposed method shows good performance compared to the previous method.



(a) 1st Model (b) 2nd Model

Fig. 2 Configuration of numerical model



(b) Model 1 (b) Model 2

Fig. 3 Calculation time

4. CONCLUSIONS

We propose a new dynamic condensation method that is suited to design optimization and probability-based analysis. The proposed method reuse reduced matrices obtained in previous analysis using Woodbury formula. Due to the numerical examples, it is verified that the proposed method shows good performance in repeated analysis. It is hoped that the proposed method help to perform the large-scaled design optimization problem and probability-based analysis.

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