The 2015 World Congress on
Advances in Structural Engineering and Mechanics (ASEM15)
Incheon, Korea, August 25-29, 2015

Prestress Force and Moving load Identification on Prestressed Concrete beam based on Virtual Distortion Method

*Ziru Xiang[1)], T. H. T Chan[2)], D.P. Thambiratnam[3)] and Theanh Nguyen[4)]

[1), 2), 3) 4)] *School of Civil Engineering & Built Environment, QUT, Brisbane, Australia*
[1)] *ziru.xiang@hdr.qut.edu.au*

ABSTRACT

Prestress force (PF) is one of the most important parameters in prestressed concrete bridges as it determines the load carrying capacity of a bridge. Unexpected loss of PF may cause failure of a bridge which makes prestress force identification (PFI) critical to evaluate the bridge safety. No method at present could identify PF effectively. Vibration based methods have been used but require a determined exciting force which is inconvenient for in-service bridges. The best excitation for in-service bridges is normal traffic, but the load caused by vehicles is difficult to measure. Hence it prompts the need to investigate whether PF and moving load could be identified together.

This paper presents a synergic identification (SI) method combined by the virtual distortion method (VDM) and Duhamel Integral to determine PF and moving load simultaneously, on a prestressed concrete beam. The PF is transformed into an external pseudo-load and this load is localized in each beam element as a pair of shear forces and a moment at its nodes via VDM. Then these local pseudo-loads could be identified together with the moving load via Duhamel Integral. The time consuming problem during the inversion of Duhamel Integral is overcome by the load-shape function method (LFM) which could decrease the size of the system matrix and improve computational efficiency.

This SI method determines PF in bridges using their dynamic responses due to unknown moving loads caused by passing vehicles which makes PFI practical for in-service bridges. Moreover, the moving load identified during the process could benefit bridge safety evaluation.

[1)] PhD candidate
[2), 3)] Professor
[4)] Doctor

Introduction

Prestressed concrete bridges (PCBs) have become a preferred type in bridge construction globally, for reasons of economy and savings in life-cycle costs. However, several bridge failures in the prestressed systems have caused large losses such as the collapse of Koror-Babeldaob Bridge (Burgoyne 2006) that killed two people and cost more than $5.2 million loss. It is necessary to develop an effective method to evaluate the existing prestress force (PF) in those bridges, not only to ensure the structural and operational safety, but also to warn of unexpected hazards.

Prestressed concrete is defined as concrete in which internal stress is introduced to counteract the stresses resulting from external load to a desired degree (Kim 2003). The given external loading, which is called PF, is one of the most important parameters to control crack formation in concrete, reduce deflection and add strength to the prestressed members to reduce concrete tensile stress. As a result of elastic shortening, creep and shrinking of concrete, steel relaxation and frictional loss between tendon and concrete, prestressed concrete may lose its PF which in turn would lead to catastrophic failures of the prestressed concrete bridge. The significance of prestress force identification (PFI) is therefore obvious.

However, existing PF cannot be estimated directly unless a detecting system has been instrumented at the time of construction (Lu 2006). As a result since middle of 1990s, researchers have begun to use indirect identification methods based on vibration test data. These methods can be categorized into the following two main approaches.

One approach to detect PF is to use natural frequencies of PCBs but the implementation is very difficult and sometimes almost impossible (Abraham 1995). Saiidi et al. (1994) inferred that the practical range of PF has little influence on the natural frequencies of prestressed concrete members. Miyamoto et al. (2000) established a natural frequency equation of a girder subjected to an external prestressed tendon and showed that the natural frequency decreases because of the dominance of the axial PF. Materazzi et al. (2009) argued that in prestressed concrete beams, bonded tendons provided an increase in frequencies of the bending vibration modes, leaving almost unaffected the torsional modes. Those studies indicate the difficulties of PFI via modal characteristics as being 1) almost impossible to identify PF through the natural frequencies because they are not sensitive to the PF change and vary conversely due to internal and external prestressed tendon; 2) difficult to identify PF from the measured mode shapes because they remain almost identical under different PFs.

Another approach to identify PF is to use vibration based methods via dynamic responses. Law (2005) stated it is feasible to measure PF through vibrational responses, and a laboratory experiment found results sensitive to the combination of the model error and measurement noise (Lu 2006). The eccentricity of the prestressed tendon was not considered in that method but was later includes in the study by Xu (2011). Li (2013) conducted a model updating approach via a measured response from moving vehicle loads to identify the magnitude of PF in a highway bridge and the simulation showed good results. Although large error was observed under a rough road condition, this work provided an innovative strain-displacement relationship of a plate shell element to identify the PF in a box girder bridge model.

Most of vibration based methods require a determined exciting force which in practice is inconvenient because bridges need to be closed during testing, or passing vehicles may affect the excitation. The best excitation for in-service bridges is the traffic loads, but these loads are usually difficult to measure. The methods were either based on given external excitation (Law 2005; Lu 2006; Xu 2011), or required a known moving force (Li 2013), which means these methods are difficult to be applied practically.

This analysis of the most relevant literature revealed that vibration based methods have promising potential in PFI, but there is plenty of room for improvement such as extending the model to approximate practical structures and detecting the PF under an undetermined moving load. In this paper, a synergic identification (SI) method will be developed to determine the PF in a prestressed beam using its dynamic responses due to an unknown moving load.

Method

This section presents a SI method to identify the moving load and PF in a simply supported beam. The proposed method is the combination of three methods: Virtual Distortion Method (VDM) is used to transform PF into an external pseudo-load, thus the identification of PF and moving load turns into a multi-force identification of pseudo-load and moving load; Duhamel integral is used to determine these loads; and to overcome the time consuming problem, the load-shape function (LFM) will be introduced to decrease the size of system matrix and improve the detecting efficiency (Wang 2012).

Virtual distortion method (VDM)

VDM is a quick reanalysis method applicable for statics and dynamics of structures which has been used in structural damage identification (Kolakowski 2008). The variations in structure (including structural damage) are modelled in forms of the related responses-coupled virtual distortions imposed on the original (undamaged) structure. Then repeating modal updating work of the damaged structure is avoided. Based on VDM, PF is modelled as an equivalent pseudo-load. Thus the prestressed structure is modelled by a non-prestressed structure (called as an original structure here) subjected to the pseudo-load. This pseudo-load is related to the PF and the structural dynamic response. Hence based on the principle of superposition, the response of the prestressed structure due to an external excitation can be expressed as a sum of the responses of the original structure due to the same excitation and the pseudo-load, respectively. In this way, the response of the prestressed structure can be expressed solely in the terms of certain characteristics of the original structure.

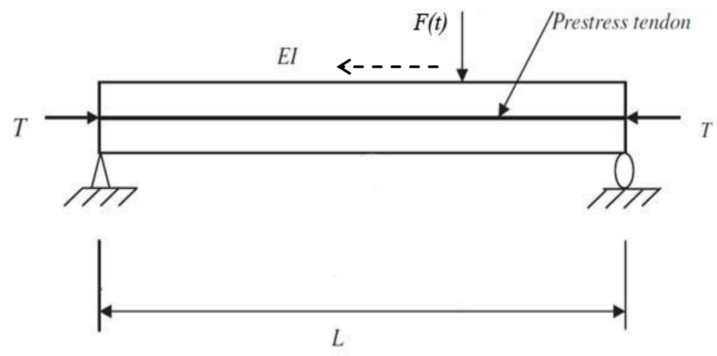

Fig.1 A simply supported prestressed beam

As Fig. 1 shows, a simply supported prestressed concrete beam with an internal prestressed tendon is established. The PF is represented by an axial load applied on the beam. With a moving force applied along the beam, the equation of motion of the prestressed beam can be written as

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[\overline{K}]\{x\}=[B]\{F\} \tag{1}$$

$$\overline{K}=K-K_g \tag{2}$$

where x is the displacement vector, and $\dot{x}$, $\ddot{x}$ are the velocity and acceleration vectors respectively. M is the mass matrix; $\overline{K}$ is the global stiffness matrix of the prestressed girder. $\{F\}$ is an moving excitation force vector, and $[B]$ is the mapping matrix . K is the global stiffness matrix without PF (stiffness matrix of beam), i.e. the original structure, and K_g is the stiffness matrix contributed by PF which is named as global geometrical stiffness matrix (Lu 2006). Rayleigh damping is used, C is the damping matrix, and is represented by a linear combination of the system mass and stiffness matrices,

$$C=\alpha[M]+\beta[\overline{K}] \tag{3}$$

where α and β are the Rayleigh damping coefficients.

Eq. (2) can be transformed into the equation of motion of the original structure subjected to the same external excitation $\{F\}$ and a response-coupled pseudo-load $\{P\}$

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=[B]\{F\}+\{P\} \tag{4}$$

$$\{P\}=[K_g]\{x\} \tag{5}$$

where $\{P\}$ models the influence of PF and is related to the global geometrical stiffness matrix and the displacement response.

Therefore, SI of moving load and PF turns into the identification of moving load and pseudo-load.

The global geometrical stiffness matrix K_g can be expressed as

$$[K_g] = \sum_{1}^{N} L_i^T R_i^T K_{g,i}^e T_i L_i \tag{6}$$

L_i and R_i are the mapping and transformation matrix respectively, $K_{g,i}^e$ is the local geometrical stiffness matrix of the *ith* element and N is the number of the elements. The local geometrical stiffness matrix of each element can be written as

$$K_{g,i}^e = T\begin{bmatrix} \frac{2l}{15} & & & sym. \\ -\frac{l}{30} & \frac{2l}{15} & & \\ -\frac{1}{10} & -\frac{1}{10} & \frac{6}{5l} & \\ \frac{1}{10} & \frac{1}{10} & -\frac{6}{5l} & \frac{6}{5l} \end{bmatrix} \qquad i = 1,2,3,....,N \tag{7}$$

where T is PF. Also, the nodal displacement of *ith* element in the local coordinate

$$\{x\}_i^e = R_i L_i \{x\} \tag{8}$$

Substituting Eq. (6), (8) to Eq. (5), presents the pseudo-load in the global coordinate via local nodal displacements, $\{P\}_i^e$ is the local pseudo-load,

$$\{P\} = \sum_{i}^{N} L_i^T R_i^T \{P\}_i^e \tag{9}$$

$$\{P\}_i^e = T\begin{bmatrix} \frac{2l}{15} & & & sym \\ -\frac{l}{30} & \frac{2l}{15} & & \\ -\frac{1}{10} & -\frac{1}{10} & \frac{6}{5l} & \\ \frac{1}{10} & \frac{1}{10} & -\frac{6}{5l} & \frac{6}{5l} \end{bmatrix}\{x\}_i^e \tag{10}$$

$\{P\}_i^e$ can be obtained via force identification method, then PF can be easily calculated.

In VDM, local pseudo-load $\{P\}_i^e$ which causes virtual distortion presents the modification of elements. For a two-dimensional beam element there are three components of virtual distortions that have to be applied. The three distortion components correspond to three states of deformation (Fig. 2) in the orthogonal base, obtained through the solution of the eigen-problem of the two-dimensional beam element stiffness matrix. These states of deformation are axial compression or tension, pure bending and bending plus shear.

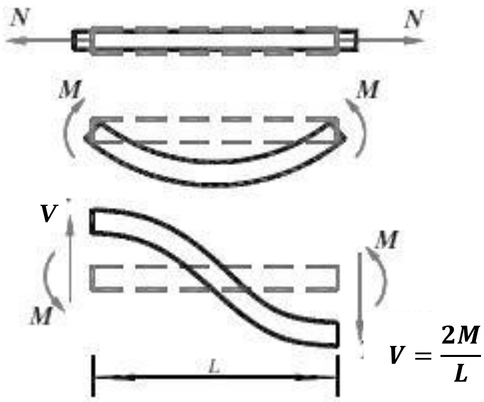

Fig. 2 Three deformation states of a beam element

In this paper, the beam is subjected to a moving force, which means the distortion is bending plus shear. Thus, the local pseudo-load is applied as a pair of shear force and a moment at its node. For the 2D beam elements, the local displacement includes only the deflection and rotation of each node.

Load-shape function method (LFM)

With the assumption of zero initial conditions, the dynamic response caused by the external moving load $\{F\}$ and the local pseudo-load $\{P\}_i^e$ can be obtained by Duhamel integral:

$$y_j(t) = \int_0^t h_i^j(t-\tau) f_i(\tau) d\tau + \sum_i^N \int_0^t d_i^j(t-\tau) p_i(\tau) d\tau \tag{11}$$

where $h_i^j(t-\tau)$ is the *jth* response of a unit moving force on the original structure (without prestress force), $d_i^j(t-\tau)$ donates the *jth* dynamic response of a unit pseudo-load (a pair of shear force or a moment) on the original structure, in the scope of VDM it is called the dynamic influence function. $f_i(t)$, $p_i(t)$ are the moving load and local pseudo-load under detection.

Transform the integral in matrix form:

$$Y_j = H^j F + \sum_i^N D^j P \tag{12}$$

where Y_j is the response vector of the *jth* sensor, F and P are the moving load vector and local pseudo-load vector respectively, H^j is the *jth* impulse response matrix and D^j is the *jth* dynamic influence matrix, which is a Toeplize matrix.

The system in Eq. (12) is dense. The system matrix could be very large in the cases of long sampling time or high sampling rate. LFM is introduced to help lessen the computation effort. In finite element method (FEM), the shape function of beam element can be written as(Zhang 2008):

$$n(\xi) = \left[1 - 3\xi^2 + 3\xi^3, \xi - 2\xi^2 + \xi^3, 3\xi^2 - 2\xi^3, \xi^3 - \xi^2\right] \tag{13}$$

where $\xi = 0:1/l:1$, l is the time step of each element.

Formulating the time history of moving force as a 'time beam', the force is simulated by the node deformation of the 'time beam' elements. When the 'time beam' is divided into *m* elements, F can be written as

$$F = N(\xi)\alpha \tag{14}$$

$\alpha = [\alpha_1, \alpha_2, ..., \alpha_{2m+2}]^T$ is the coefficient of the function. $N(\xi) = [N_1, N_2, N_3, ..., N_{2m+2}]$ is the load-shape function matrix. For example, when *m=2*, $N(\xi)$ is presented as follow,

$$N(\xi) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1-3(1/l)^2+2(1/l)^3 & (1/l)[1-(1/l)^2] & (1/l)^2[3-2(1/l)] & -(1/l)^2[1-(1/l)] & 0 & 0 \\ 1-3(2/l)^2+2(2/l)^3 & (2/l)[1-(2/l)^2] & (2/l)^2[3-2(2/l)] & -(2/l)^2[1-(2/l)] & ... & ... \\ ... & ... & ... & ... & ... & ... \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1-3(1/l)^2+2(1/l)^3 & (1/l)[1-(1/l)^2] & (1/l)^2[3-2(1/l)] & -(1/l)^2[1-(1/l)] \\ ... & ... & ... & ... & ... & ... \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{15}$$

The local pseudo-load could be regarded as a time consequent force which is able to apply load-shape function as well. By comparing Eq. (12) and (14), the LFM equation for Duhamel integral is:

$$Y_j = H^j N\alpha_F + \sum_j D^j N\alpha_P^j = B_F^j \alpha_F + \sum_j B_P^j \alpha_P^j \quad j = 1,2,3... \tag{16}$$

α_F is the relevant coefficient of the moving load and α_P^j is the coefficient for the local pseudo-load which is represented as the form of a pair of shear force and a moment in each beam element. That makes the number of α_P^j twice the number of beam elements. To guarantee the uniqueness of the solution, there should be at least as many independent sensors as the total number of unknown coefficients. If the beam is divided into *N* elements, *j* should be more than *2N+1*.

$$\begin{bmatrix} \alpha_F \\ \alpha_P^1 \\ : \\ : \\ \alpha_P^{2N} \end{bmatrix} = \begin{bmatrix} B_F^1 & B_P^1 & .. & .. & B_P^{2N} \\ : & : & & & \\ : & & : & & \\ : & & & : & \\ B_F^{2N+1} & & & & B_P^{2N} \end{bmatrix}^+ \begin{bmatrix} Y_1 \\ Y_2 \\ : \\ : \\ Y_{2N+1} \end{bmatrix} \tag{17}$$

The dimensions of B_F^j and B_P^j are much smaller than H^j and D^j, hence the calculation of generalized inverse matrixs B_F^{j+} and B_P^{j+} are reduced. Moreover, LFM could make the inversion robust to noise and is helpful to identify smooth loads with higher accuracy (Zhang 2008).

After identifying the coefficient, the moving load and PF can be determined through Eq. (14) and (10).

SIMULATION AND RESLUTS

The prestressed beam

A simply supported prestressed concrete beam is established. The span length is 20 m. The second moment of area of concrete cross section is 0.372 m^4 while the tendon cross section area is 1.39×10^{-4} m^2. The Young's modulus of the beam and the tendon are 3.45×10^{10} N/m^2 and 2.95×10^{11} N/m^2, the densities are 2.3×10^3 kg/m^3 and 7.9×10^3 kg/m^3, and the Poisson's ratios are 0.2 and 0.3 respectively. This girder is divided into 10 elements, 11 nodes and the displacement and rotation responses of each node should be measured which means 21 sensors are placed at the position of each node (only 1 sensor of rotation is arranged on the left end because it is simply supported and the deflection is 0). A time-varying moving load *F(t)* is crossing the beam with $v = 80$ km/h. Rayleigh damping is applied. The sampling rate is 200Hz. The case setting is based on the works by Huang (2013) and is shown in Table 1.

Table 1 Case setting

Case no.	PF (KN)	Moving load (KN)
1	**2100**	F(t)=2[30+2sin(10πt)+4cos(15πt)+cos(3πt+0.2π)]
2	**5000**	
3	**18000**	

Applying load shape-function method (LFM)

In equation (14), the load-shape function (LSF) $N(\xi)$ is introduced to decrease the size of impulse response matrix H . Define the frequency of LSF f_{LSF} as

$$f_{LSF} = f_s / 2l \tag{20}$$

$$m = Zf_s / l \tag{21}$$

where f_s is the sampling rate, l and m are the time step and the number of the 'time beam' elements. Z is time period. To ensure that LSF can model the force accurately, the frequency of LSF should not be smaller than the frequency of the moving force. For an unknown force, its frequency distribution can be determined by the Fourier transformation of measured response. Fig. 3 shows the mid-span displacement response in case 2 and Fig. 4 is its Fourier transformation.

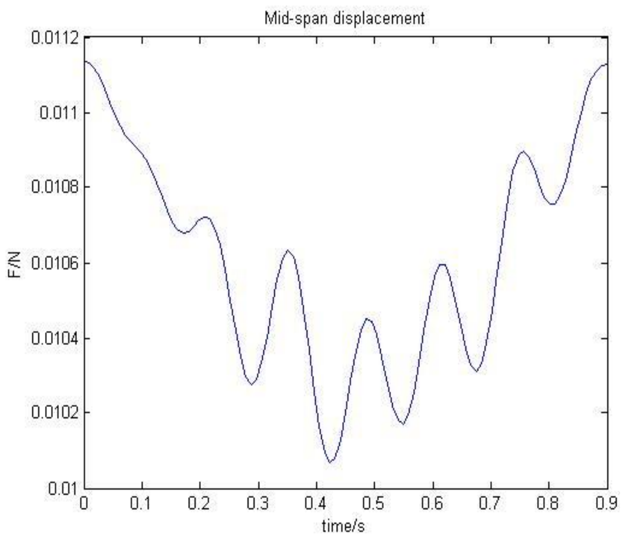

Fig.3 Mid-span displacement response

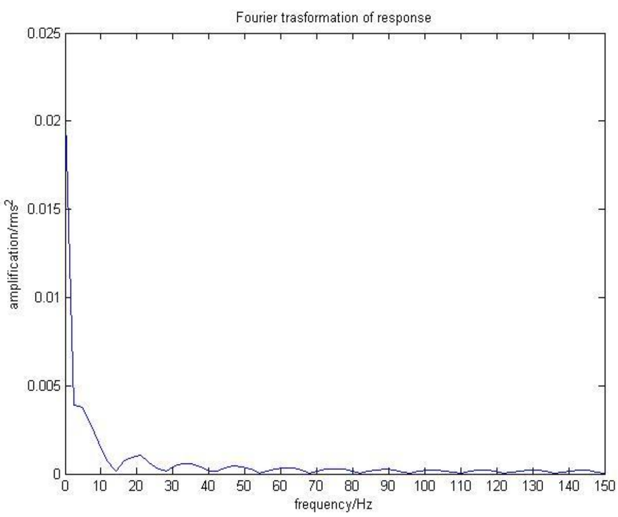

Fig. 4 Frequency analysis of mid-span displacement response

From Fig.4 it can be observed that the main frequency is less than 10Hz. Taking $f_{LSF} = 10Hz$ $l = f_s / 2 f_{LSF} = 10$, the time period $Z = 20/22.2 = 0.9s$ is divided into $m = Zf_s / l = 18$ elements. Thus there are 18 nodes and 36 load shape-functions, and each function has 10 steps.

In the Duhamel integral, the dynamic influence matrix D^j can be calculated by impulse responses excited by a unit shear force or a moment respectively at each node. However, during the calculation of H^j , the excitation load is moving, which means the unite force in each moment should be applied at different position. In this case, it should be calculated 181 times which is time consuming.

Therefore, a new approach is proposed to calculate $B^j{}_F$ without calculating H^j. Taking $N(\xi)$ as a moving load, the response could be than obtained as the elements of relevant column in $B^j{}_F$. Considering the $1st$ load-shape function N_1, it is applied as a moving force on the same position which the moving load $\{F\}$ placed at that moment. The displacement measured by the jth sensor is the $1st$ column of $B^j{}_F$ (Wang 2012).

In this case, 21 sensors are placed. Before LFM is introduced, the impulse response matrix H^{21} and dynamic influence matrix D^{21} both have dimensions of (3801×3801). After LFM is adopted, the dimension of B^{21} is only (3801×22), comparing Eq. (12) and (16), the calculation of H^+ and D^+ are $O(3801\times 3801^2)$ while the calculation of B^+ is only $O(3801\times 22^2)$, which is 30000 times faster, LFM can hence improve the computational efficiency.

Result and discussion

Fig. 5 shows the SI results of both moving load and PF in each case (the red number ($\times 10^7$) is pointing the true value). Both moving load and PF could be identified. For the moving load, only large errors are found close to the start and end of the time histories which is due to instability of vibration responses at the beginning and end of the period, while those in middle show good agreement with the true value. The same problem occurs at the start of PFI.

Table 2 shows the identified PF in each element when $t = 0.9s$. From the table, it could be observed that the error percentage between average and true value decreases with the increase of PF. The reason is that the local pseudo-load, due to its small geometrical stiffness matrix K_g, is much smaller than the real nodal force caused by the moving load (see Eq. (10)). Thus the bigger the PF is, the larger influence it will have on the responses which makes it easier to distinguish. Except the elements at the two ends, PF in all elements are appraised within acceptable accuracy.

Table 2 Identified prestress force in each element

Element no.	Case 1 (KN)	Case 2 (KN)	Case3 (KN)
1	3296.8	6342.8	12638
2	3777.8	9449.3	23457
3	1334.8	4006.0	13349
4	2178.8	4359.3	21778
5	1264.3	5058.9	25281
6	1293.5	5142.9	19264
7	771.85	5176.7	18570
8	1546.2	6242.3	15603
9	3282.1	3341.9	16678
10	6995.1	8002.8	29998
Average	2498.3	5712.3	19662
True	2100.0	5000.0	18000
Error percentage	18.95%	14.24%	9.23%

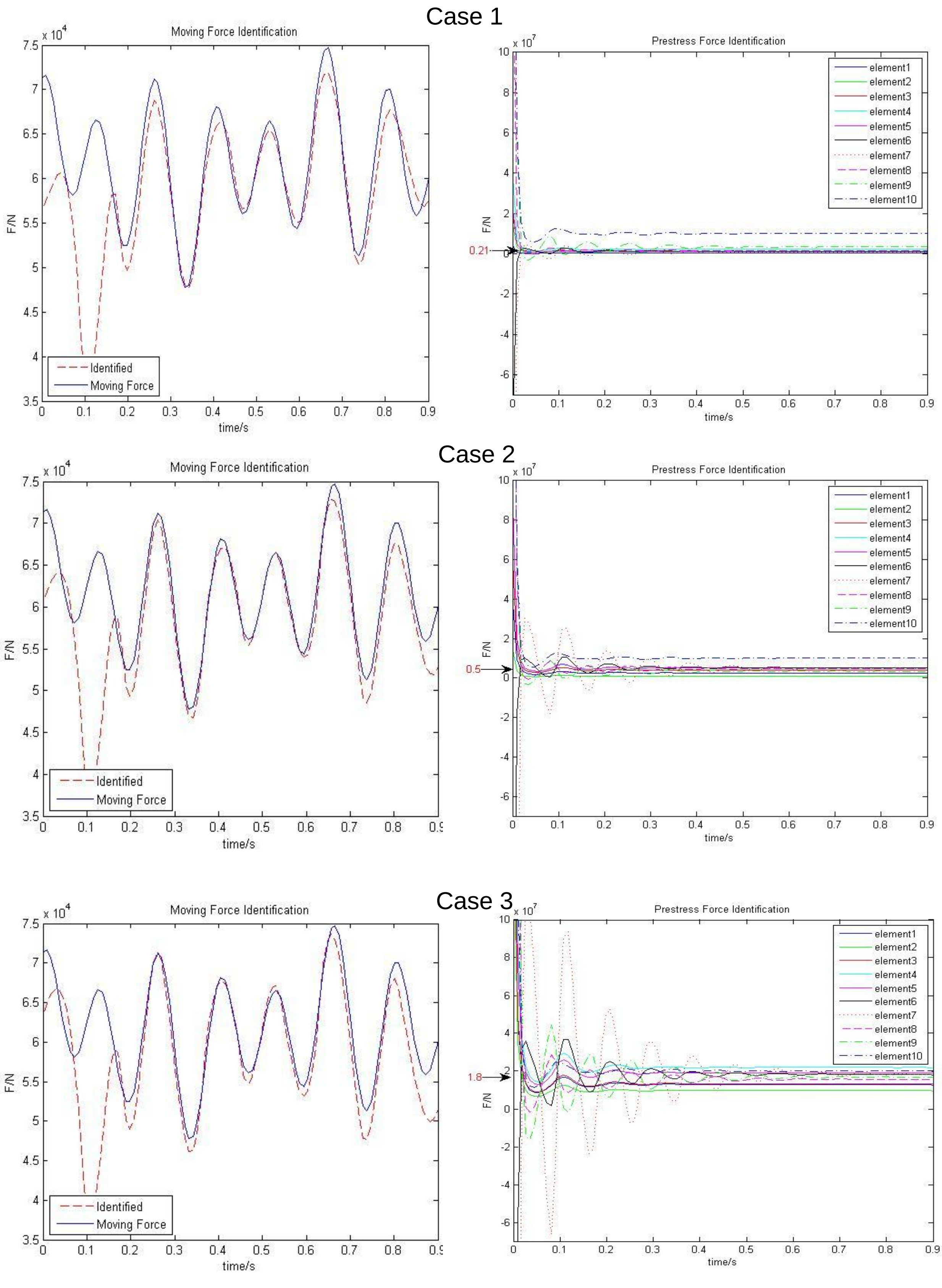

Fig. 5 Synergic identification results

CONCLUSION

A SI method to assess PF and moving load in a prestressed concrete beam is proposed. The vibration of a prestressed system due to a moving load is modified to a vibration of an original system (not prestressed) subjected to the moving load and a pseudo-load which is caused by PF and structure dynamic response. LFM is introduced to solve the inverse problem and its efficiency has been shown.

A numerical simulation example is conducted. Results show that different PFs as well as moving load could all be determined with good accuracy. This method has shown its great potential in determining PF in beams, and extending it for prestressed concrete box girder will be a future research topic for the authors.

REFERENCES

Abraham, M.A., S. Park and N. Stubbs. 1995. "Loss of prestress prediction based on nondestructive damage location algorithms." In *Smart Structures & Materials' 95*, edited, 60-67: International Society for Optics and Photonics.

Burgoyne, C. and R. Scantlebury. 2006. "Why did Palau bridge collapse?" *The Structural Engineer* 84: 30-37.

Huang, Y. and Z. Fang. 2013. "Finite Element Analysis and Research of the Static and Dynamic Properties of Pre-stress Simply-supported Beam." *Journal of Wuhan University of Technology(Transportation Science & Engineering)* 37 (1): 123-126.

Kim, J.-T., Y.-S. Ryu and C.-B. Yun. 2003. "Vibration-based method to detect prestress loss in beam-type bridges, edited, 559-568. http://dx.doi.org/10.1117/12.484638.

Law, S.S. and Z. Lu. 2005. "Time domain responses of a prestressed beam and prestress identification." *Journal of sound and vibration* 288 (4): 1011-1025.

Li, H., Z. Lv and J. Liu. 2013. "Assessment of prestress force in bridges using structural dynamic responses under moving vehicles." *Mathematical Problems in Engineering* 2013.

Lu, Z. and S.S. Law. 2006. "Identification of prestress force from measured structural responses." *Mechanical systems and signal processing* 20 (8): 2186-2199.

Wang, L., J.-L. Hou and J.-P. Ou. 2012. "Moving force identification based on load shape function for a long-span bridge structure." *Chinese Journal of Computational Mechanics* 29 (2).

Xu, J. and Z. Sun. 2011. "Prestress force identification for eccentrically prestressed concrete beam from beam vibration response." *Structural Longevity* 5 (2): 107-115.

Zhang, Q., Ł. Jankowski and Z. Duan. 2008. "Identification of coexistent load and damage based on virtual distortion method." In *Proceedings of the 4th European workshop on structural health monitoring, Kraków, Poland*, edited, 1121-1128.