Operational modal analysis using forced excitation and input-output autoregressive coefficients

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ABSTRACT

This study focused on the damage detection of reinforced concrete bridges. First, autoregressive coefficients (ARCs) are computed for the intact and the damaged structure. Autoregressive (AR) models are regarded between the most accurate methods to represent a stationary output time-series. The system is analyzed with different p-lags in each time interval and the ARCs of the output data are used to estimate the modal parameters. Second, a deep learning algorithm locates the possible damages and their extension based on the behavior of the ARC in the time domain. The addition of forced input data to the AR model analysis using a vector autoregressive (VAR) technique allows a more accurate identification. When there is a variation of the model parameters, either ARCs or modal parameters the system has been damaged. Damage detection is the most challenging part of operational modal analysis. This study takes advantage of the latest state of the art in neural network techniques, i.e. deep learning, and computational power to try to overcome these challenges.

1. INTRODUCTION

Each civil structure has its own modal properties: natural frequencies, mode shapes, and damping ratio. The modal properties are a function of the state of structural system: materials, boundary conditions, mass, stiffness. They are used as reference in Structural Health Monitoring (SHM) to assess the overall status of the structure. Many researches in civil engineering were performed on system identification. For example, fast Fourier transform (FFT) algorithm is the most straightforward algorithm in modal analysis due to its simplicity and effectiveness. However, FFT algorithm has limitations while processing stationary time series data set because it is dependent on a limited number of observation. Furthermore, when windows techniques are used the resulting modal parameters are an average of the time frame analyzed

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and sudden changes cannot be identified. AR models solve this data processing problem because they can take into account the evolution of stationary and non-stationary time-series (Overschee P. V. and Moor B. D. 1996).

The main advantage of AR models is the presence of parameters that evolve over time, i.e. ARCs. The AR model of stationary time series is used to identify the damage locations within a structure, which is one of the challenges of SHM in civil engineering. Many methods that take advantage of output only AR models have been developed. However, input-output AR models, which produce more accurate results than output only AR models, are still subject of research. The main issue is to either create or identify an input excitation. Civil structures, for example a reinforced concrete bridge, are massive in both scale and weight. Exciting such structure above white noise level, i.e. ambient vibration, requires a lot of energy, or a large equipment, or a mix of both. In this context, static actuator and linear exciter has been proposed to conduct vibration testing on structures (Underwood and Keller 2001). With a known input from an oscillator autoregressive exogenous (ARX) model can identify and track the modal properties and ARCs with greater accuracy than simple output-only model.

In this study, ARC and modal parameters obtained from experimental data are used together with a finite element model (FEM) of the target structure. The most challenging part of this study is the model updating technique used to match the computer model to the real model. In past studies, many model updating techniques have been proposed, such as: genetic algorithm, response surface methodology, and Monte Carlo optimization. Recent advances in neural network algorithm are promising new capabilities of system identification in many fields. This research takes full advantage of the parameters from both AR model and ARX model, and these new improvements to SHM.

The ARX model for input-output time series from the sensor data of a bridge is used to obtain AR coefficients. These ARCs are used to compute the modal properties of the structure through maximum entropy method, which uses the Burg's algorithm. The neural network algorithm developed is applied to the parameters obtained in order to identify the variation of modal parameters, and ARCs. Finally, the overall damage location is estimated by matching the parameters between the computer model and the actual measurement.

2. METHODS

2.1 AR model and VAR model

The output only time series AR model used in this study properly demonstrates the dynamic behavior of a target structure as shown in Eq. 1:

$$u(t) = \phi_{11}u(t-1) + \phi_{22}u(t-2) + \cdots + \phi_{pp}u(t-p), \tag{1}$$

where $u(t)$, ϕ_{ii} and p are respectively the output response, the ARC and the model order.

In output only AR model, Burg's algorithm is reducing the sums of the squares of the forward and the backward prediction errors and it computes the ARCs (Brockwell

and Davis 2002). Based on the computed ARCs the system matrix [Eq. 2] of the model is assembled.

$$
\Pi = \begin{pmatrix}
\phi_{11} & \phi_{22} & \cdots & \phi_{p-1\,p-1} & \phi_{pp} \\
I & 0 & \cdots & 0 & 0 \\
0 & I & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & I & 0
\end{pmatrix},
\tag{2}
$$

The eigenvalue decomposition of the system matrix [Eq. 3] is used to find the dynamic properties of the model.

$$
\Pi = L \begin{pmatrix}
\hat{\pi}_1 & 0 & \cdots & 0 \\
0 & \hat{\pi}_2 & 0 & 0 \\
0 & 0 & \ddots & \vdots \\
0 & 0 & \cdots & \hat{\pi}_p
\end{pmatrix} L^{-1},
\tag{3}
$$

Natural frequencies:

$$
f_i = \frac{\sqrt{\mathrm{Re}^2(\lambda_i) + \mathrm{Im}^2(\lambda_i)}}{2\pi}; \text{ where } \lambda_i = \frac{\ln(\hat{\pi}_i)}{T_s},
\tag{4}
$$

Mode shapes:

$$
\Psi = \begin{pmatrix} \Psi_1 & \Psi_2 & \cdots & \Psi_p \end{pmatrix} = \begin{pmatrix} I & 0 & \cdots & 0 \end{pmatrix} L,
\tag{5}
$$

Damping ratios:

$$
\zeta_i = -\frac{\mathrm{Re}(\lambda_i)}{2\pi f_i}
\tag{6}
$$

2.2 ARX and eigen-system realization algorithm
The ARX model is one of the evolutions of the output only AR model. ARX takes input signals into account (7):

$$
y(k) + \alpha_1 y(k-1) + \cdots + \alpha_p y(k-p) = \beta_0 u(k) + \beta_1 u(k-1) + \cdots + \beta_p u(k-p),
\tag{7}
$$

where $y(k)$, $u(k)$ and p are respectively: the output response, the input excitation, and the model order.

Given the relation of the input and output signals in time domain the model parameters: α and β are computed by solving two linear differential equations in a state space:

$$x(k+1) = Ax(k) + Bu(k), \tag{8}$$

$$y(k) = Cx(k) + Du(k), \tag{9}$$

For the state space functions (8) and (9), the eigen-system realization algorithm (Juang and Pappa 1985) computes the Markov parameters using equation (10):

$$Y(k) = CA^{k-1}B, \tag{10}$$

These Markov parameters are assembled in the Hankel matrix shown in Eq. (11):

$$H(k-1) = \begin{pmatrix} Y_k & Y_{k+1} & \cdots & Y_{k+\beta-1} \\ Y_{k+1} & Y_{k+2} & \cdots & Y_{k+\beta} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{k+\alpha-1} & Y_{k+\alpha} & \cdots & Y_{k+\alpha+\beta-2} \end{pmatrix}, \tag{11}$$

The singular value decomposition (SVD) of the Hankel matrix results in the matrices *S, U and V^T*. The diagonal element of matrix *S* are the eigenvalues while the columns of U and the row of V^T are the eigenvectors Eq. (15):

$$H(0) = R_n \Sigma_n S_n^{\ T} = [R_n \Sigma_n^{\frac{1}{2}}][\Sigma_n^{\frac{1}{2}} S_n^{\ T}], \tag{12}$$

$$H(1) = [R_n \Sigma_n^{\frac{1}{2}}]A[\Sigma_n^{\frac{1}{2}} S_n^{\ T}], \tag{13}$$

$$A = \Sigma_n^{-\frac{1}{2}} R_n^{\ T} H(1) S_n \Sigma_n^{-\frac{1}{2}}, \tag{14}$$

$$A = USV^T, \tag{15}$$

2.3 Time domain variation

If a structure is not subject to any damage and no degradation occurs its stiffness and damping do not change overtime. The stationary property of the ARC can be used to find whether a structure has been damaged or not. In a stationary time series process a system generates AR coefficients with a mean and a variance that are constant over time, taking into account variation in the stiffness of the structure due to temperature change, either daily or seasonal. A change in the mean and variance of the ARC means that something happened within the structure, such as: degradation of the materials, change in the boundary conditions, or damage in one or more structural elements. The direct comparison between the ARC of the intact and current structure cannot identify what exactly happened. We know that something happened but we do not know what It is. A more sophisticated algorithm is required to identify the source of these property changes. While many algorithms exist to perform this task neural

network was chosen because of the recent advance in its performance, computational efficiency, and computational power.

2.4 Neural network

Neural network (Lee, Grosse et al. 2011), which is inspired to biological neural networks, leads the advance in damage identification techniques for civil structures. The primary model that is built in advance is used as the initial condition of the neural network algorithm. Given the ARCs and the dynamic properties of a target structure the observed output response is directly related to the specific outcome of an update computer model.

A FEM based on Opensees (McKenna 2011) is used for this study. However, any computer model can be implemented in the neural network algorithm as long as it supports parametrization. For example, ANSYS with its APDL scripting (Wang, He et al. 2012) can be an option. Modern improvement of computer performance enables faster analysis of the computer model. After a series of iterative structural analysis, our neural network algorithm can detect the possible locations where a change occurred.

3. RESULTS

3.1 System Identification of a reinforced concrete bridge with remote sensing vibrometer

The first test-bed is a reinforced concrete bridge on campus. A remote sensing vibrometer (RSV) is pointed at the structure and the time domain of the velocity is recorded. The data of this experiment were used as baseline to develop and test the algorithm of VAR model of single output system so that ARCs are properly extracted from time series output data. The algorithm is written in MATLAB and Burg's algorithm is used to compute ARCs from raw data.

The experimental setup is shown in Fig. 1. The laser pointer is placed 200 m from the target, the experiment has been carried out 3 times and the recorded data are each 5 minutes long. The sampling frequency was 480 Hz. The output response data is single velocity data of the target.

Fig.1 Experimental Setting up

Fig. 2 shows the ARCs of the target system. X domain stands for sampling trial of time-series output data and Y domain stands for ARC. Each ARC has its own pattern and distinct mean and variation. Therefore, the stationary time-series output data generates a satisfactory representation of ARCs at the different time lag.

After obtaining the appropriate ARCs from Burg's algorithm, the VAR model, which is composed of ARCs, is used to compute the modal parameters, as shown in Fig. 3.
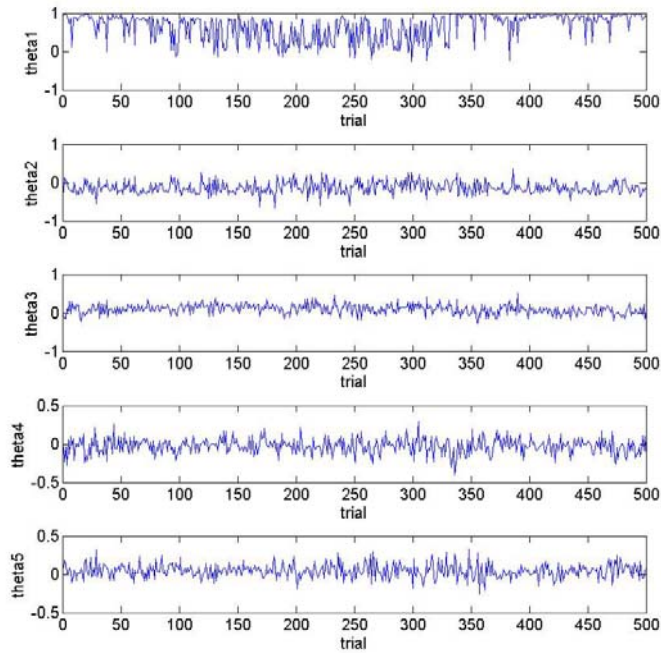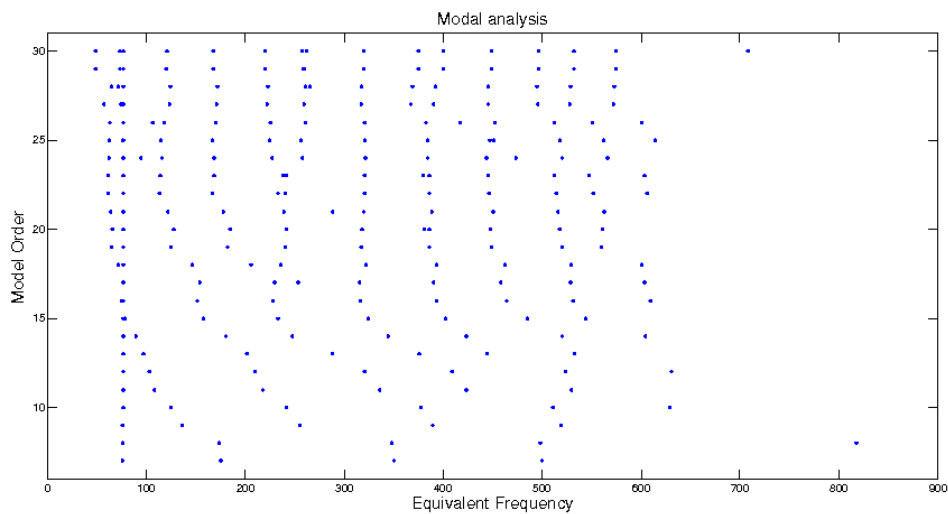

Fig. 2 ARCs of the target system

Fig. 3 Modal analysis using ARCs

3.2. Cable stayed bridge

This experiment was carried out on a cable stayed bridge, with a main span length of 270 m, and it includes multiple sensors. In this case, the size of the vector autoregressive matrix is much larger than the matrix from single output data. However, as shown in Fig. 6, it shows more defined ARCs.
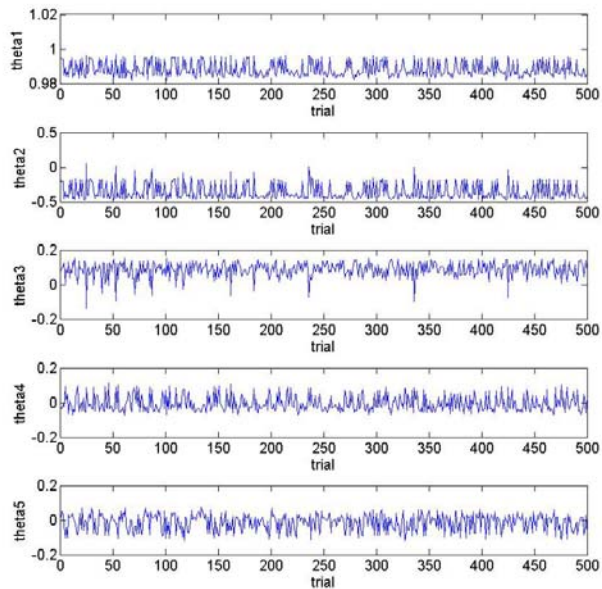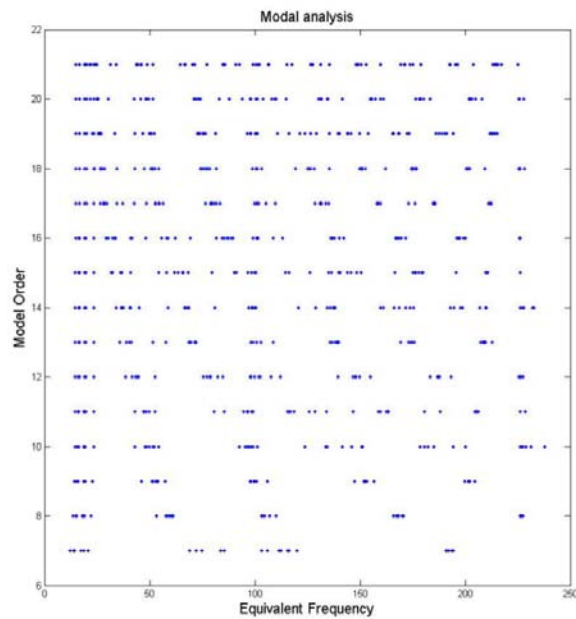

Fig. 4. ARCs of the bridge
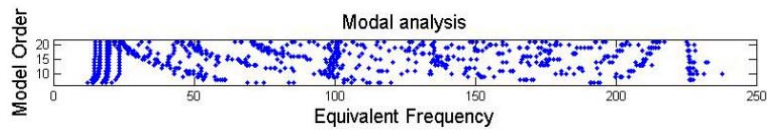

Fig.5. Modal analysis of the bridge

Fig.6. Compacted chart of the modal analysis result

4. CONCLUSIONS

In this study, we found that nowadays it is possible to use neural network and ARX model to track the behavior of a structure instruments with a SHM in quasi real time. This was an issue in the past because of computational time. The simple eigenvalue decomposition of a large matrix can be performed on a regular laptop. Linking the different part of the method: the ARX model, which is written in Matlab, the FEM model, which is parameterized in Opensees or APDL, and the neural network algorithm, which is also written in Matlab or C++, is rather simple.

While the AR model and ARX model are well developed coding the algorithm and reaching good performance proved to be a challenge. The tests were conducted using script language and greater performances are expected with binary code. Parallel programming is also expected to improve the overall performance, such as CUDA C on GPGPU. The FEM model and solver proved to be also an important bottleneck in the system ID because the eigenvalue analysis is performed on a large matrix due to the level of detail requires in the FEM model. The singular value decomposition on large matrices still take time. Neural network proved to be the most straightforward part of the process once the parameters that had to be taken into account were identified.

In this study the data used are from: a remote sensing vibrometer pointed at a short span bridge on campus, and a cable stayed bridge outside campus with a low number of sensor. In the future, a large scale deployment of SHM will be used as test-bed. Good scalability is expected but it must be proved to validate the entire model proposed.

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