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Dynamic Response of Free-Span Submarine Pipelines: Integral Transform Solution

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ABSTRACT

Large numbers of submarine pipelines are laid as the world now is attaching great importance to offshore oil exploitation. Free spanning of submarine pipelines may be caused by seabed unevenness, change of topology, artificial supports, etc. By combining Iwan's wake oscillator model with the differential equation which describes the vibration behaviour of free-span submarine pipelines, the pipe-fluid coupling equation is developed and solved in order to study the effect of both internal and external fluid on the vibration behaviour of free-span submarine pipelines. Through generalised integral transform technique (GITT), the governing equation indicating the transverse displacement is transformed into a system of second-order ordinary differential equations (ODEs) in temporal variable, eliminating the spatial variable. The MATHEMATICA built-in function is then used to numerically solve the transformed ODE system. The good convergence of the eigenfunction expansions proved that this method is applicable for predicting the dynamic response of free-span pipelines subjected to both internal flow and external current.

1. INTRODUCTION

The crucial works of a small number of disparate researchers in the late 19th and early 20th centuries marked the beginnings of a concerted attempt to understand the phenomenon of vortex shedding which continues to this day. A great amount of work

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has been done to study the vortex-induced vibration (VIV) behaviour of underwater structures, such as cable arrays, drilling risers, offshore platforms and pile supported structures.

During the early days, the effect of the internal flow was often ignored. Iwan (1981) proposed a vortex-induced oscillation model that can be used to solve problems that involve non-uniform structures and flow profiles. Xu et al. (1999) developed the fatigue damage models for multi-span pipelines detailed both in time and frequency domain approaches. Pantazopoulos et al. (1993) put forward a Fourier Transformation based methodology to study the VIV of free-span submarine pipelines. Bryndum and Smed (1998) carried experiments in the VIV of submarine free spans under different boundary conditions. Furnes (2003) formulated time domain model a free span pipeline subjected to ocean currents in which the in-line and cross-flow deflections are coupled.

Recently, a significant number of achievements have been gained in understanding the dynamic characteristics of submarine risers and pipelines conveying internal fluid. Shen and Zhao (1996) studied the impact of internal fluid on the fatigue life of submarine pipelines under vortex-induced vibration while simplifying the action of the external flow on the pipe as a type of load, and ignoring the coupling effect of the two. Guo et al. (2004) and Lou (2005) studied the coupled effect of internal and external fluid on the response of VIV of marine risers by the method of Finite Element Method (FEM).

In addition, an increasing amount of interest is paid to the phenomenon of VIV that concerns the influence of soil. Xing et al. (2005) developed a VIV model for the span segment of buried submarine pipelines. In an experimental study conducted by Yang et al. (2008), the cross-line VIV of a submarine pipeline near an erodible sandy seabed under the influence of ocean currents was investigated. By using Visual Basic tools, Xie et al. (2011) developed a VIV fatigue analysis program for submarine pipeline span based on pipe-soil coupling non-linear modal. Wang et al. (2014) proposed a prediction model for the VIV of deepwater steel catenary risers considering the riser-seafloor interaction.

Finite Difference Method (FDM), Finite Element Method (FEM) and some other methods have been taken for the numerical solution of coupled nonlinear oscillator models. However, there exist no previous researches taking the generalised integral transform technique (GITT) approach to solve such coupled fluid and structural equations. GITT is still in its starting stage in the area of structure mechanics. Ma et al. (2006) applied GITT to solve a transverse vibration problem of an axial moving string and the convergence behaviour of integral transform solution was examined. An and Su (2011, 2014) employed GITT to obtain a hybrid analytical-numerical solution for dynamic response of clamped axially moving beams, and later the axially moving Timoshenko beams. Recently, Gu et al. (2012, 2013) used GITT to prove that variation of mean axial tension induced by elongation should not be neglected in the numerical simulation of VIV of a long flexible cylinder, and in addition, predicted the dynamic response of a clamped-clamped pipe conveying fluid, where the convergence behaviour was thoroughly examined.

It is against this backdrop that the research presented in this article was undertaken. To this end, the remainder of this paper is organised as follows. In Section 2, the mathematical model of the coupled structure and wake oscillator model is put forward. In Section 3, the hybrid numerical-analytical solution is obtained through integral transform. Section 4 presents the numerical results and a parametric study, where the convergence behaviour of the present approach is assessed. Finally, Section 5 concludes this article.

2. DESCRIPTION OF MATHEMATICAL MODEL

As is shown in Fig. 1, a Cartesian coordinate system is adopted to depict the vibration behaviour of a submarine free span under the influence of both the internal and external fluid. The *x*-axis is the initial axis of the pipe; the *y*-axis is in the same direction as the current, horizontally orthogonal to the *x*-axis; and the *z*-axis is in the opposite direction of gravity. Consider a free-span pipeline that is horizontally pinned at $x=0$ and $x=L$. The pipeline is cylindrical with a constant outer diameter of *D* and inner diameter of D_i the axial tension is *T* and the pressure is *P*. Assume that:
(1) The internal fluid flows at a constant velocity of *V.*
(2) The effect of waves is ignored and the current is at a constant speed of *U*.
(3) The property of the pipe is linear and the pipe is elastic.

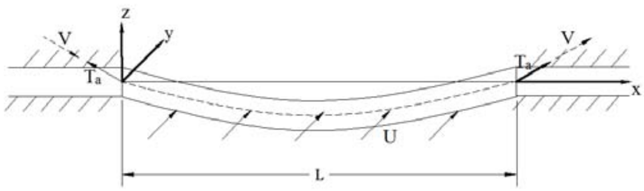

Fig. 1 Free Span of a Submarine Pipeline

Considering the movement of the free spanning pipeline with pinned-pinned boundary conditions in the xoz plane subject to internal and external fluid, tension and the pressure from the internal fluid, according to Guo et al. (2004), Lou et al. (2005) and Faccinetti et al. (2004), the coupled structure and wake oscillator model of the pipe vibration can be described as:

$$\begin{cases} EI\dfrac{\partial^4 z}{\partial x^4}+(m_iV^2+PA_i-T_a)\dfrac{\partial^2 z}{\partial x^2}+2m_iV\dfrac{\partial^2 z}{\partial x\partial t}+(r_s+r_f)\dfrac{\partial z}{\partial t}+m\dfrac{\partial^2 z}{\partial t^2}=\dfrac{\rho_e U^2 DC_L}{2} \\ \dfrac{\partial^2 q}{\partial t^2}+\varepsilon w_f(q^2-1)\dfrac{\partial q}{\partial t}+w_f{}^2 q=\dfrac{\alpha}{D}\dfrac{\partial^2 z}{\partial t^2} \end{cases} \tag{1}$$

where,

EI -- flexural stiffness; P -- internal pressure; Ai -- area of the inner cross section of the pipeline; T_a -- axial tension; ρ_e -- density of external fluid; D -- outside diameter of the pipeline; C_L -- coefficient of instantaneous sectional lift on the pipe; r_s -- structural damping; r_f -- fluid added damping, equaling to $\Upsilon w_f \rho_e D^2$, of which Υ is a coefficient related to the mean sectional drag efficient of the pipe - C_D, and Υ equals to $C_D/(4\pi St)$, (here St is the Strouhal number); and for unit length of the pipeline, $m = m_i + m_p + m_e$, with m_i being the internal fluid mass, m_p being the mass of the pipeline, m_e being the added mass due to external fluid, and $m_e = C_M \rho_e D^2/4$, in which C_M is the added mass coefficient; q is the reduced fluctuating lift coefficient, and $q(x,t) = 2C_L(x,t)/C_{L0}$, and C_L is the lift coefficient, C_{L0} the reference lift coefficient which can be obtained from observation of a fixed structure subject to vortex shedding; $w_f = 2\pi StU/D$ denotes the vortex-shedding frequency; parameters α and ε can be derived from experimental results by Faccinetti et al. (2004), with the former being 12 and the latter 0.3.

As the free span considered in this article is pinned-pinned, the boundary conditions is as follows:

$$z(0,t)=0\,,\ \ \frac{\partial^2 z(0,t)}{\partial x^2}=0\,,\ \ z(L,t)=0\,,\ \ \frac{\partial^2 z(L,t)}{\partial x^2}=0 \tag{2a}$$

$$q(0,t)=0\,,\ \ \frac{\partial^2 q(0,t)}{\partial x^2}=0\,,\ \ q(L,t)=0\,,\ \ \frac{\partial^2 q(L,t)}{\partial x^2}=0 \tag{2b}$$

Then the following dimensionless variables are introduced:

$$x^*=\frac{x}{L}\,,\ \ z^*=\frac{z}{D}\,,\ \ t^*=\frac{t}{L^2}\sqrt{\frac{EI}{m_p}}\,,\ \ V^*=VL\sqrt{\frac{m_p}{EI}}\,,\ \ w_f{}^*=w_f L^2\sqrt{\frac{m_p}{EI}}\,,\ \ \beta=\frac{\rho_e U^2 C_{L0} L^4}{4EI} \tag{3}$$

By combining Eq. (3) into Eq. (1), two dimensionless equations are obtained (leaving out the asterisks for simplicity)

$$\begin{cases} \dfrac{\partial^4 z}{\partial x^4} + (\dfrac{m_i V^2}{m_p} + \dfrac{PA_i L^2}{EI} - \dfrac{T_a L^2}{EI})\dfrac{\partial^2 z}{\partial x^2} + \dfrac{2m_i V}{m_p}\dfrac{\partial^2 z}{\partial x \partial t} + (r_s + r_f)\dfrac{L^2}{\sqrt{EIm_p}}\dfrac{\partial z}{\partial t} + \dfrac{m}{m_p}\dfrac{\partial^2 z}{\partial t^2} = \beta q \\ \dfrac{\partial^2 q}{\partial t^2} + \varepsilon w_f (q^2 - 1)\dfrac{\partial q}{\partial t} + w_f{}^2 q = \alpha \dfrac{\partial^2 z}{\partial t^2} \end{cases} \tag{4}$$

along with the boundary conditions

$$z(0,t) = 0\,,\ \ \frac{\partial^2 z(0,t)}{\partial x^2} = 0\,,\ \ z(1,t) = 0\,,\ \ \frac{\partial^2 z(1,t)}{\partial x^2} = 0 \tag{5a}$$

and

$$q(0,t) = 0\,,\ \ \frac{\partial^2 q(0,t)}{\partial x^2} = 0\,,\ \ q(1,t) = 0\,,\ \ \frac{\partial^2 q(1,t)}{\partial x^2} = 0\,. \tag{5b}$$

The initial conditions is defined as:

$$z(x,0) = 0\,,\ \ \frac{\partial z(x,0)}{\partial t} = 0\,,\ \ q(x,0) = 1\,,\ \ \frac{\partial q(x,0)}{\partial t} = 0\,. \tag{6}$$

3. INTEGRAL TRANSFORM SOLUTION

According to the idea of GITT method, the next step is to select the auxiliary eigenvalue problem and propose the eigenfunction expansion for Eq. (4) under the boundary conditions (5). For the transverse displacement of a free span, the eigenvalue problem is chosen as:

$$\frac{\mathrm{d}^4 X_i(x)}{\mathrm{d}x^4} = \phi_i^4 X_i(x),\ \ 0 < x < 1 \tag{7a}$$

with the boundary conditions being

$$X_i(0) = 0,\ \ \frac{\mathrm{d}^2 X_i(0)}{\mathrm{d}x^2} = 0,\ \ X_i(1) = 0,\ \ \frac{\mathrm{d}^2 X_i(0)}{\mathrm{d}x^2} = 0 \tag{7b}$$

where X_i and ϕ_1 are respectively the eigenfunction and the eigenvalue of problem (7), satisfying the following orthogonality,

$$\int_0^1 X_i(x) X_j(x)\,\mathrm{d}x = \delta_{ij} N_i \tag{8}$$

where δ_{ij} is the Kronecker delta, and for $i \neq j$, $\delta_{ij} = 0$; for $i = j$, $\delta_{ij} = 1$.

The normalization integral is evaluated as

$$N_i = \int_0^1 X_i^2(x)\,\mathrm{d}x \tag{9}$$

Problem (7) is now readily solved analytically to yield

$$X_i(x) = \sin(\phi_i x) \tag{10}$$

where the eigenvalue is obtained by:

$$\phi_i = i\pi \quad , \quad i = 1, 2, 3, \ldots \tag{11}$$

and the normalization integral is evaluated as

$$N_i = \frac{1}{2}, \quad i = 1, 2, 3, \ldots \tag{12}$$

Therefore, in this case, the normalised eigenfunction coincides with the original eigenfunction itself, i.e.

$$\tilde{X}_i(x) = \frac{X_i(x)}{N_i^{1/2}} \tag{13}$$

This solution proceeds by putting forward the integral transform pair – the integral transformation itself and the inversion formula. Through integral transform, the spatial coordinate x is eliminated.

For the transverse displacement:

$$\bar{z}_i(t) = \int_0^1 \tilde{X}_i(x) z(x,t)\,\mathrm{d}x, \text{ transform} \tag{14a}$$

$$z(x,t) = \sum_{i=1}^{\infty} \tilde{X}_i(x)\bar{z}_i(t), \text{ inversion} \tag{14b}$$

Similarly, the eigenvalue problem is chosen for $q(x,t)$ is

$$\frac{\mathrm{d}^4 Y_k(x)}{\mathrm{d}x^4} = \varphi_k{}^4 Y_k(x), \quad 0 < x < 1 \tag{15a}$$

with the boundary conditions being:

$$Y_k(0) = 0, \quad \frac{\mathrm{d}^2 Y_k(0)}{\mathrm{d}x^2} = 0, \quad Y_k(1) = 0, \quad \frac{\mathrm{d}^2 Y_k(1)}{\mathrm{d}x^2} = 0 \tag{15b}$$

where Y_k is the eigenfunction of problem (15) and φ_k the corresponding eigenvalue. And similarly,

$$\int_0^1 Y_k(x) Y_l(x)\,\mathrm{d}x = \delta_{kl} N_k, \tag{16}$$

The same mathematical manipulation is carried here as Eqs.(8-13), and the eigenvalue problem (15) defines the integral transform pair for the wake variable as follows:

$$\bar{q}_k(t) = \int_0^1 \tilde{Y}_k(x) q(x,t)\,\mathrm{d}x, \text{ transform} \tag{17a}$$

$$q(x,t)=\sum_{k=1}^{\infty}\tilde{Y}_k(x)\bar{q}_k(t)\text{, inversion} \tag{17b}$$

To perform GITT, the dimensionless equation system (4) is multiplied by operators $\int_0^1 \tilde{X}_i(x)\,\mathrm{d}x$ and $\int_0^1 \tilde{Y}_k(x)\,\mathrm{d}x$, and also the inverse formula (14) and (17) are applied, resulting in a set of ordinary differential equations:

$$\begin{cases} \phi_i^4\bar{z}_i(t)+\left(\dfrac{m_iV^2}{m_p}+\dfrac{PA_iL^2}{EI}-\dfrac{T_aL^2}{EI}\right)\displaystyle\sum_{j=1}^{\infty}A_{ij}\bar{z}_j(t)+\dfrac{2m_iV}{m_p}\displaystyle\sum_{j=1}^{\infty}B_{ij}\dfrac{\mathrm{d}\bar{z}_j(t)}{\mathrm{d}t}+ \\ (r_s+r_f)\dfrac{L^2}{\sqrt{EIm_p}}\dfrac{\mathrm{d}\bar{z}_i(t)}{\mathrm{d}t}+\dfrac{m}{m_p}\dfrac{\mathrm{d}^2\bar{z}_i(t)}{\mathrm{d}t^2}=\beta\displaystyle\sum_{k=1}^{\infty}C_{ik}\bar{q}_k(t) \qquad (18a) \\ \dfrac{\mathrm{d}^2\bar{q}_k(t)}{\mathrm{d}t^2}+\varepsilon w_f\displaystyle\sum_{l=1}^{\infty}\sum_{r=1}^{\infty}\sum_{s=1}^{\infty}D_{klrs}\bar{q}_l(t)\bar{q}_r(t)\dfrac{\mathrm{d}\bar{q}_s(t)}{\mathrm{d}t}-\varepsilon w_f\dfrac{\mathrm{d}\bar{q}_k(t)}{\mathrm{d}t}+w_f^2\bar{q}_k(t)= \\ \alpha\displaystyle\sum_{i=1}^{\infty}E_{ki}\dfrac{\mathrm{d}^2\bar{z}_i(t)}{\mathrm{d}t^2} \qquad (18b) \end{cases}$$

where the coefficients are analytically determined by the following integrals:

$$A_{ij}=\int_0^1\tilde{X}_i(x)\frac{\mathrm{d}^2\tilde{X}_j(x)}{\mathrm{d}x^2}\mathrm{d}x,\quad B_{ij}=\int_0^1\tilde{X}_i(x)\frac{\mathrm{d}\tilde{X}_j(x)}{\mathrm{d}x}\mathrm{d}x,\quad C_{ik}=\int_0^1\tilde{X}_i(x)\tilde{Y}_k(x)\mathrm{d}x,$$

$$D_{klrs}=\int_0^1\tilde{Y}_k(x)\tilde{Y}_l(x)\tilde{Y}_r(x)\tilde{Y}_s(x)\mathrm{d}x,\quad E_{ki}=\int_0^1\tilde{Y}_k(x)\tilde{X}_i(x)\mathrm{d}x \tag{19}$$

In a similar manner, the boundary and the initial conditions are also transformed to omit the spatial variable, yielding

$$\bar{z}_i(0)=0,\quad \frac{\mathrm{d}^2\bar{z}_i(0)}{\mathrm{d}t^2}=0,\quad \bar{q}_k(0)=0,\quad \frac{\mathrm{d}^2\bar{q}_k(0)}{\mathrm{d}t^2}=0,\quad i,k=1,2,3,\ldots \tag{20a}$$

$$\bar{z}_i(0)=0,\quad \frac{\mathrm{d}\bar{z}_i(0)}{\mathrm{d}t}=0,\quad \bar{q}_k(0)=\int_0^1\tilde{Y}_k(x)\mathrm{d}x,\quad \frac{\mathrm{d}\bar{q}_k(0)}{\mathrm{d}t}=0,\quad i,k=1,2,3,\ldots \tag{20b}$$

For computational purposes, the expansions for the transverse displacement $z(x,t)$ and the reduced lift coefficient $q(x,t)$ are truncated to finite *N* order. The equation system (18), in truncated series, are subsequently calculated by the NDSolve routine of *Mathematica*. Once $\bar{z}_i(t)$ and $\bar{q}_k(t)$ are numerically evaluated, the inversion formulas

Eqs.(14) and (17) are than applied to obtain the explicit analytical expressions for the dimensionless $z(x,t)$ and $q(x,t)$.

4. RESULTS AND DISCUSSIONS

In this Section, the numerical results of the transverse displacement $z(x,t)$ a free-span submarine pipeline subject to both internal and external fluid under pinned-pinned boundary condition is presented.

The main parameters used in the present work is set as follows:

Table 1 Main Parameters of the Pipeline and the Fluid

L (m)	D (m)	D_i (m)	ρ_p (kg/m^3)	ρ_e (kg/m^3)	ρ_i (kg/m^3)	EI (Nm^2)	C_M	C_D	C_{L0}	St
80	0.324	0.292	8200	1025	908.2	3.68×10^7	1	1.2	0.3	0.2

Assume *P = 0*, T_a *= 50 kN*. The dimensionless transverse deflection $z(x,t)$ is calculated with two different values of the dimensionless constant internal fluid velocity, i.e. *V = 2, 4*, and two different values of constant current velocity, i.e. *U = 0.05 m/s, 0.1 m/s*. The convergence behaviour of the integral transform solution is examined for an increasing truncation terms *N = 4, 8* at *t = 5, 20, 50*. The results of (i) *V = 2*, *U = 0.05 m/s*, (ii) *V = 4*, *U = 0.05 m/s*, (iii) *V = 2*, *U= 0.1 m/s*, and (iv) *V = 4*, *U = 0.1 m/s*, are displayed in Tables 2 and 3.

The parametric study indicates that either along with the increasing velocity of the current or the internal flow, the amplitude and the vibration frequency of the system will rise (see Tables 2-3). If the frequency of the vibration of free span reaches its natural frequency, resonance will occur, resulting in not only a severe damage to the pipeline but also environmental pollution.

Table 2 Convergence Behaviour of $z(x,t)$ for *V = 2, 4* and *U = 0.05 m/s*

V = 2; U = 0.05 m/s

x	*N=4*	*N=8*	*N=16*
t=5			
0.1	*0.000246477*	*0.000246178*	*0.000245858*
0.3	*0.000629393*	*0.000626839*	*0.000626067*
0.5	*0.000753193*	*0.000750787*	*0.000749853*
0.7	*0.000594333*	*0.000591919*	*0.000591149*
0.9	*0.000224056*	*0.000223493*	*0.00022318*
t=20			
0.1	*-0.000482822*	*-0.000487265*	*-0.000488488*
0.3	*-0.00124594*	*-0.00125394*	*-0.00125687*
0.5	*-0.00152536*	*-0.0015363*	*-0.00153979*
0.7	*-0.00124337*	*-0.00125136*	*-0.00125428*
0.9	*-0.00048113*	*-0.000485571*	*-0.000486792*
t=50			
0.1	*-0.000810771*	*-0.000827957*	*-0.000833269*
0.3	*-0.00209535*	*-0.00213259*	*-0.00214511*
0.5	*-0.00256874*	*-0.00261509*	*-0.00262998*
0.7	*-0.00209411*	*-0.00213133*	*-0.00214384*
0.9	*-0.000809954*	*-0.000827118*	*-0.000832425*

V = 4; U = 0.05 m/s

x	*N=4*	*N=8*	*N=16*
t=5			
0.1	*0.000608827*	*0.000608508*	*0.000607994*
0.3	*0.00160222*	*0.00159775*	*0.00159652*
0.5	*0.00201252*	*0.0020085*	*0.00200704*
0.7	*0.00167448*	*0.00167035*	*0.00166912*
0.9	*0.000655365*	*0.000654412*	*0.000653885*
t=20			
0.1	*-0.000720719*	*-0.000727059*	*-0.000728882*
0.3	*-0.00186633*	*-0.00187904*	*-0.00188352*
0.5	*-0.00228878*	*-0.00230545*	*-0.00231084*
0.7	*-0.00185927*	*-0.00187193*	*-0.00187639*
0.9	*-0.000716109*	*-0.000722382*	*-0.000724192*
t=50			
0.1	*-0.00122897*	*-0.00125392*	*-0.00126177*
0.3	*-0.0031872*	*-0.00324447*	*-0.00326355*
0.5	*-0.00391482*	*-0.00398584*	*-0.00400879*
0.7	*-0.00318246*	*-0.00323964*	*-0.0032587*
0.9	*-0.00122586*	*-0.00125074*	*-0.00125857*

Table 3 Convergence Behaviour of $z(x,t)$ for *V = 2, 4* and *U = 0.1 m/s*

V = 2; U = 0.1 m/s

x	N=4	N=8	N=16
t=5			
0.1	-0.00106358	-0.00107869	-0.0010834
0.3	-0.00275696	-0.0027884	-0.00279983
0.5	-0.00340531	-0.00344649	-0.00346019
0.7	-0.00280937	-0.00284057	-0.002852
0.9	-0.00109743	-0.00111297	-0.00111768
t=20			
0.1	-0.000699271	-0.000720542	-0.000726849
0.3	-0.00184928	-0.00190262	-0.00191804
0.5	-0.00230868	-0.00237113	-0.00238983
0.7	-0.00187465	-0.00192839	-0.00194393
0.9	-0.000715982	-0.000737644	-0.000744038
t=50			
0.1	0.00153519	0.00158746	0.00161046
0.3	0.00389293	0.00401326	0.00406872
0.5	0.00470787	0.0048684	0.00493452
0.7	0.00387069	0.00399057	0.00404595
0.9	0.00152056	0.00157242	0.00159537

V = 4; U = 0.1 m/s

x	N=4	N=8	N=16
t=5			
0.1	-0.00240079	-0.00242261	-0.0024299
0.3	-0.0062373	-0.00628876	-0.00630683
0.5	-0.00765249	-0.00771742	-0.00773931
0.7	-0.00617993	-0.00623138	-0.0062495
0.9	-0.00236394	-0.00238703	-0.00239437
t=20			
0.1	-0.00190223	-0.00194586	-0.00195777
0.3	-0.00502415	-0.00513528	-0.00516521
0.5	-0.00627603	-0.00640922	-0.00644599
0.7	-0.00511722	-0.00522983	-0.00526024
0.9	-0.00196312	-0.00200814	-0.00202037
t=50			
0.1	0.00044856	0.000480821	0.000514123
0.3	0.000999697	0.00107406	0.00115676
0.5	0.00106964	0.00118013	0.00127924
0.7	0.000905482	0.000976879	0.00105917
0.9	0.000386975	0.000416935	0.000449964

Fig. 2-5 are the GITT solution for the dimensionless $z(x,t)$ under different internal and external fluid velocities in 3-D diagrams.

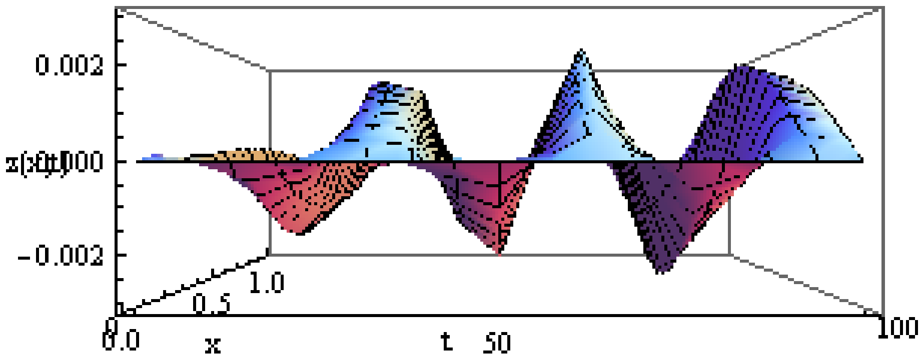

Fig. 2 GITT Solution for the dimensionless $z(x,t)$ with *the V = 2, U = 0.05 m/s*

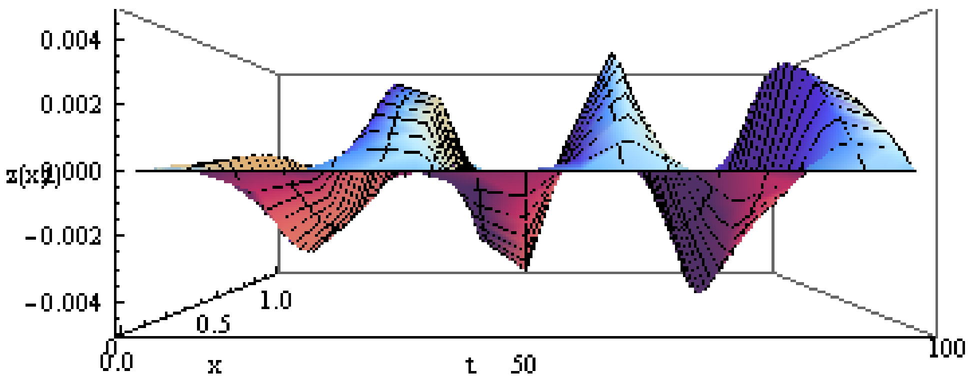

Fig. 3 GITT Solution for the dimensionless $z(x,t)$ with *the V = 4, U = 0.05 m/s*

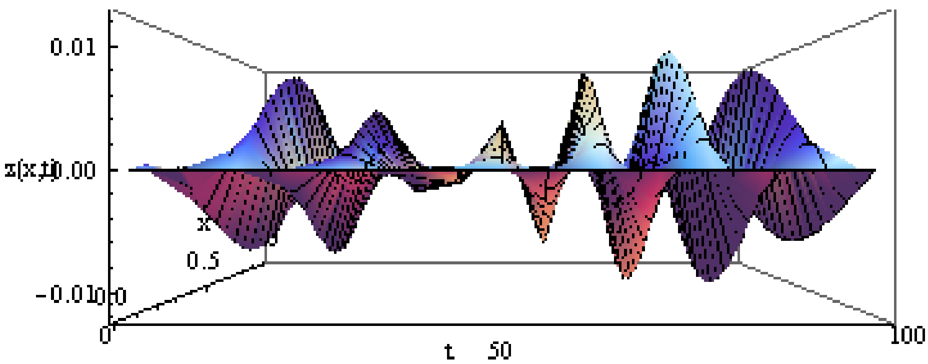

Fig. 4 GITT Solution for the dimensionless $z(x,t)$ with *the V = 2, U = 0.1 m/s*

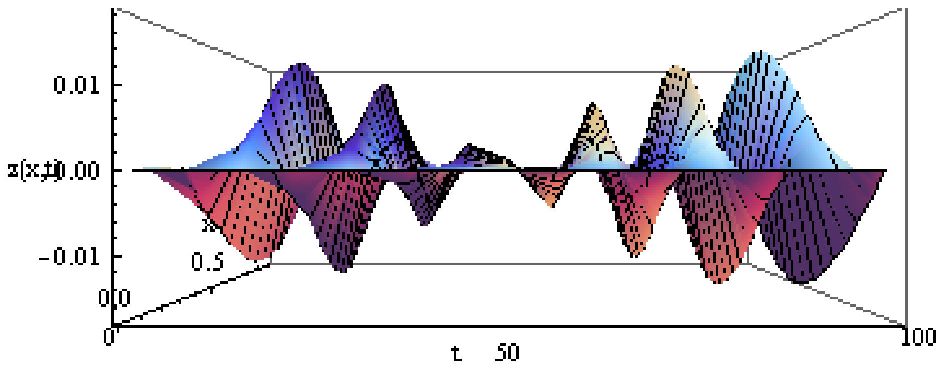

Fig. 5 GITT Solution for the dimensionless $z(x,t)$ with *the V = 4*, *U = 0.1 m/s*

5. CONCLUSION

It is shown in the present studies that GITT is an adequate approach for analyzing the dynamic response of a free-span submarine pipeline under the influence of both the external current and the internal flow. It can be employed for benchmarking purposes, yielding sets of reference results with controlled accuracy, and on the other hand, as an engineering simulation tool with its lower truncation orders and exceptional computational performance.

Due to the limit of time, this article only cover the work presented above. It is suggested that more parameters need to be studied under higher truncation orders, and the results be compared with and verified by other methodologies.

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