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Performance of a new enriched 2D solid finite element

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ABSTRACT

In this paper, we introduce a new enriched scheme for the 2D 4-node solid finite element, which is free from the linear dependence problem. The linear dependence problem is resolved by applying the piecewise linear shape functions as the partition of unity function. The enriched finite element method can improve the accuracy of solutions without remeshing or introducing additional nodes, and has an important advantage that it can be selectively applied to the area of interest, where the gradient of solutions is high. The stability and effectiveness of the new enriched 2D solid finite element method are demonstrated through static and dynamic analyses of various problems.

1. INTRODUCTION

The finite element method is being used widely in structure engineering field, but there are still cases that inaccurate solutions are obtained when a high stress gradient is present (Bathe, 1996, Kim and Bathe, 2013). Various studies have been conducted to obtained reliable solutions efficiently.

The enriched finite element method (Babuška and Melenk, 1997, Dolbow et al, 1999, Duarte et al, 2000, Ham and Bathe, 2012, Yoon et al, 2012, Jeon and Lee, 2014, Kim and Bathe, 2014) can improve solution without remeshing by incorporating enrichment functions at each node. In addition, enrichment functions can be used to only to local area where solutions are needed to be improved. However, depending on enrichment functions and elements, there is the linear dependence problem that needs to be resolved.

The linear dependence problem of the 2D 3-node triangular and 3D 4-node tetrahedron elements can be resolved by suppressing not only the standard degrees of freedom but also the enriched degrees of freedom when imposing the boundary

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conditions (Tian et al, 2006, An et al, 2011). However, for the 2D 4-node quadrilateral element, the linear dependence problem cannot be resolved with such treatment.

In this paper, we introduce a new enriched 4-node 2D solid finite element (Kim and Lee) free from the linear dependence problem, and demonstrate its stability and effectiveness via several numerical examples.

2. A NEW ENRICHED 4-NODE 2D SOLID FINITE ELEMENT

The enriched finite element approximates the solution variable by the product of the partition of unity function and the enrichment functions defined on each cover (Babuška and Melenk, 1997, Kim and Bathe, 2013). The cover is union of elements attached to each node shown in Fig. 1.

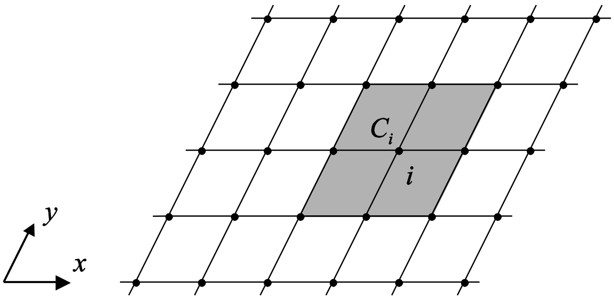

Fig. 1 Cover region corresponding to node i (Kim and Lee).

In this paper, we consider linear and quadratic polynomials as the enrichment functions, and the piecewise linear shape function (Kim and Lee) that is defined on each sub triangular domain is used as the partition of unity function, see Fig. 2. The piecewise linear shape function is

$$h_i^m(r,s) = 0.25(1 + b_i^m r + c_i^m s) \quad \text{with} \tag{1}$$
$$b_i^m = (-1)^{i+1} \times |m-3| + (-1)^i \times 2 \times (\delta_{i3} + \delta_{i4}),$$
$$c_i^m = (-1)^{i+1} \times |m-2| + (-1)^i \times 2 \times (\delta_{i2} + \delta_{i3}),$$

in which m and i indicate the number of sub-domain and node in 4-node quadrilateral element.

The geometry and displacement interpolations of the new enriched 4-node 2D solid element with linear enrichment function (Kim and Lee) are given by

$$\mathbf{x} = \sum_{i=1}^{4} h_i \mathbf{x}_i \ , \tag{2}$$

$$\mathbf{u} = \sum_{i=1}^{4} h_i \mathbf{u}_i + \sum_{i=1}^{4} \mathbf{H}_i^{en} \mathbf{u}_i^{en} \quad \text{with} \tag{3}$$

$$\mathbf{H}_i^{en} = h_i \begin{bmatrix} \xi_i & \eta_i & 0 & 0 \\ 0 & 0 & \xi_i & \eta_i \end{bmatrix}, \quad \mathbf{u}_i^{en} = \begin{bmatrix} u_i^{\xi} & u_i^{\eta} & v_i^{\xi} & v_i^{\eta} \end{bmatrix}^{\mathrm{T}},$$

where $\xi_i = (x - x_i)/\lambda_i$ and $\eta_i = (y - y_i)/\lambda_i$, in which λ_i is the largest diagonal length of element in cover i. Note that only the displacement field is enriched and the geometry interpolation of enriched finite element is identical to the standard finite element.

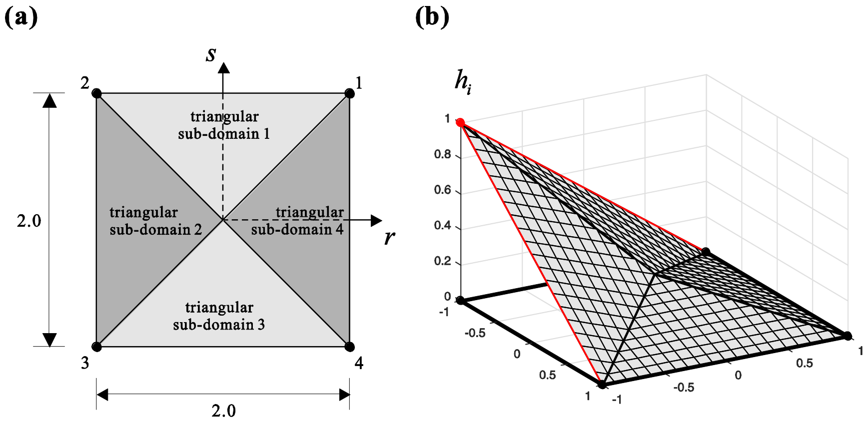

Fig. 2 Piecewise linear shape function: (a) triangular subdivision of quadrilateral element and (b) piecewise linear shape function corresponding to node i (Kim and Lee).

3. LINEAR DEPENDENCE PROBLEM

In this section, we investigate the linear dependence problem. The rank deficiencies of the global stiffness matrices are calculated by counting zero eigenvalues. Fig. 3 shows regular and distorted meshes are used to construct the global stiffness matrices, and the calculated rank deficiencies are presented in Table. 1.

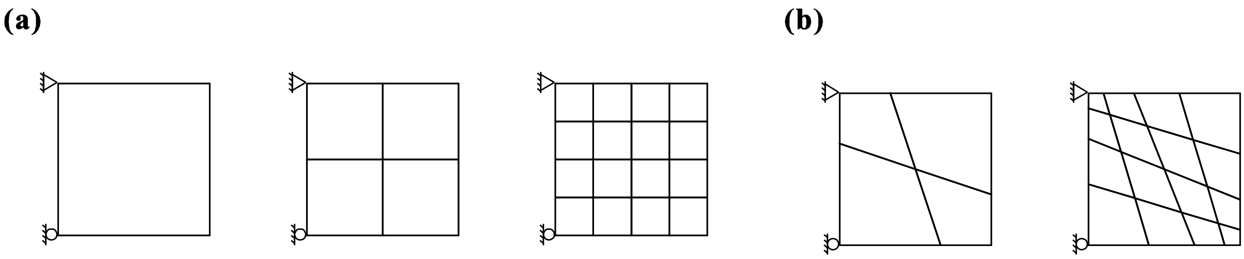

Fig. 3 Quadrilateral meshes: (a) square meshes and (b) distorted meshes (Kim and Lee).

As mentioned in the previous section, by applying the piecewise linear shape function, rank deficiency is not observed regardless mesh topology and enrichment function. Note that the $\mathbf{u}_i^{en}$ is suppressed at the fixed boundary, and this condition is applied in numerical examples in the next section.

Table 1. The rank deficiencies of the global stiffness matrices. Meshes shown in Fig. 3 are used.

Mesh type	Number of elements	Rank deficiency / Total degrees of freedom	
		Linear enrichment	Quadratic enrichment
Square	1	0/13	0/25
	4	0/43	0/85
	16	0/139	0/277
Distorted	4	0/43	0/85
	16	0/139	0/277

4. NUMERICAL EXAMPLES

In this section, we consider a wrench problem where the static analysis is performed and Cook's skew beam problem where the transient response is calculated.

4.1 WRENCH PROBLEM

The wrench problem shown in Fig. 4(a) is solved. Wrench is subjected to a uniform pressure load on line AA. We first perform the linear static analysis using 143 4-node standard finite elements, and then we selectively enrich nodes that are in high stress gradient area as shown in Fig. 4(b). The number of degrees of freedom in each case are 360 and 666. Calculated effective stress along line BB are presented in Fig. 5. By applying enrichment functions adaptively, the accuracy of solution is improved effectively.

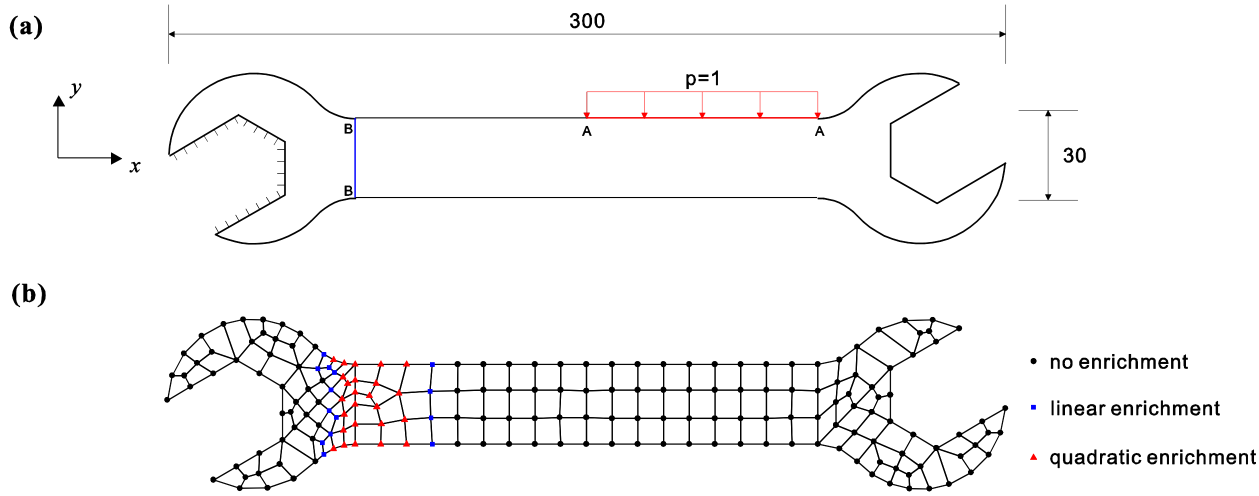

Fig. 4 Wrench problem: (a) problem description and (b) mesh used ($E = 1.0 \times 10^7$ and $\nu = 0.3$).

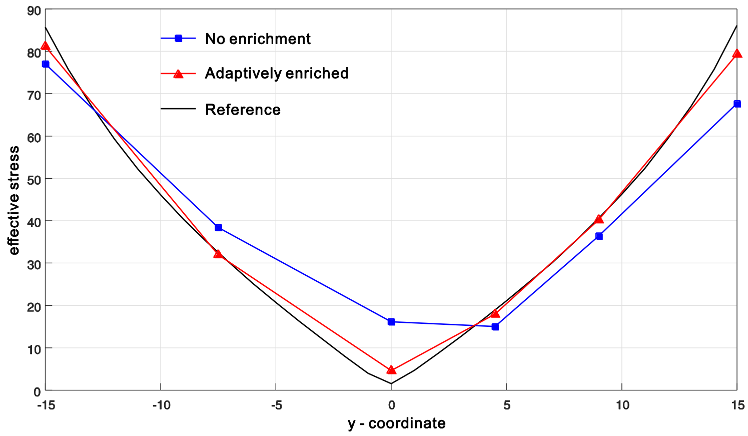

Fig. 5 Effective stress distribution along line BB shown in Fig. 4(a).

4.2 COOK'S SKEW BEAM PROBLEM

Transient response of the cook's skew beam shown in Fig. 6 is calculated. Newmark time integration method with time step $\Delta t = 0.001$ is used, and the initial condition is ${}^0\mathbf{U} = {}^0\dot{\mathbf{U}} = {}^0\ddot{\mathbf{U}} = \mathbf{0}$. We perform numerical simulations using

- 32x32 standard 4-node elements (2112 DOFs),
- 16x16 standard 9-node elements (2112 DOFs),
- 16x16 enriched 4-node elements with linear enrichment (1632 DOFs),
- 8x8 enriched 4-node elements with quadratic enrichment (864 DOFs),

and the results are presented in Fig. 7.

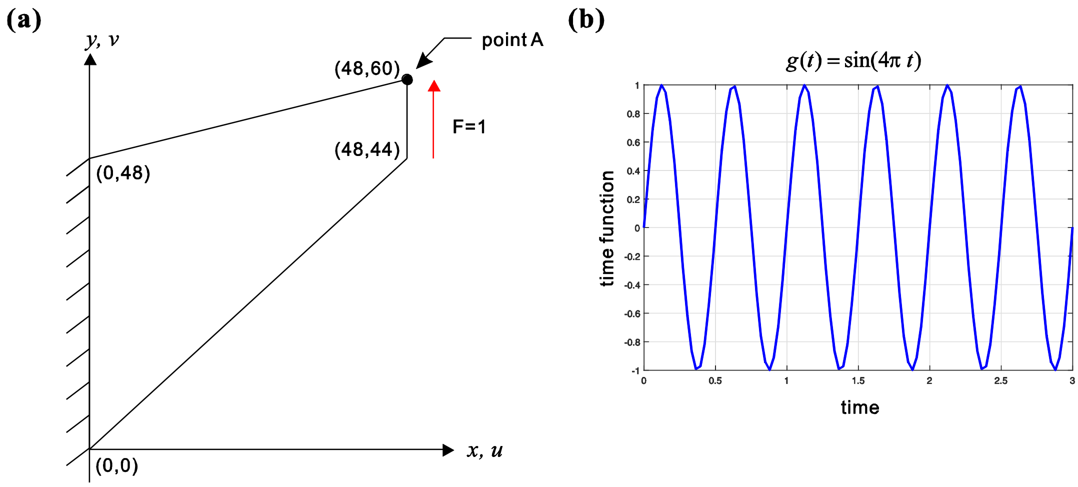

Fig. 6 Cook's skew beam problem: (a) problem description and (b) time function of applied load ($E = 1.0$, $\nu = 1/3$, and $\rho = 0.3 \times 10^{-4}$).

Except standard 4-node elements, results of the other elements are in good agreement with reference solution. The result shows that the new enriched 4-node finite element can provide more accurate solutions even with a smaller degree of freedoms than the standard finite elements.

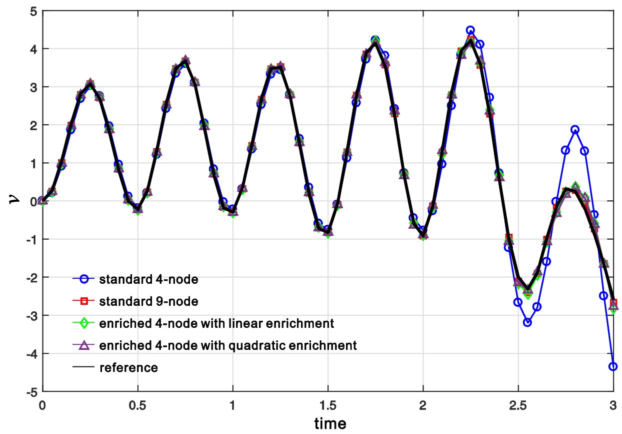

Fig. 7 Displacements response at point A shown in Fig. 6(a).

5. CONCLUSIONS

In this paper, a new enriched 4-node 2D solid finite element was introduced. The geometry and displacement interpolations of the new enriched 4-node 2D solid finite element are based on the piecewise linear shape function, and this element is free from the linear dependence problem. In the wrench problem, the effectiveness of the adaptive enrichment scheme was demonstrated, where the solution is improved without mesh refinement. Transient response of the cook's skew beam was also effectively obtained using enriched elements in terms of the number of degrees of freedoms. The development of enriched 3D solid finite elements and shell elements free from the linear dependence problem can be the future research (Lee and Bathe, 2004, Lee et al, 2012, Jeon et al, 2014, Lee et al, 2014, Jeon et al, 2015, Lee et al, 2015, Ko et al, 2017).

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