Simulation of bridge approach slabs under differential settlement and soil washout

*Yung-Tsang Chen\(^1\) and Shengqian Fan\(^2\)

\(^1\), \(^2\) Department of Civil Engineering, The University of Nottingham, Ningbo 315100, China

\(^1\) ytchen@gmail.com

**ABSTRACT**

The bump at the end of the bridge is a common yet complex problem in bridge engineering community, and bridge approach slabs are often used in mitigating the bump. Despite the extensive use of approach slabs, unsatisfactory performance of them was reported. Among the factors contributing to the bump problem, differential settlement between the approach slab and road way pavement and soil washout underneath the slab near the abutment joint are considered two important factors affecting the overall performance of the approach slab. In this paper, a simplified bridge approach slab model based on beam-on-elastic Winkler’s foundation but capable of simulating the two main factors affecting the slab performance, is proposed. An approach slab design adopted by Zhejiang province is used as an example to verify the feasibility of the proposed model. The results demonstrate that the proposed model is able to give various responses (deformation, bending moment, shear force, soil pressure) of the slab under assumed soil conditions when subjected to design traffic loads. The proposed mathematical model is therefore deemed applicable for simulation of the behavior of approach slabs under design truck loading, as well as quick assessment of the slabs’ responses to decide if remedial measures need to be taken.

**1. INTRODUCTION**

The bump at the end of the bridge is a common yet complex problem in bridge engineering community. Bridge approach slabs are commonly used in mitigating this bump problem, in hopes that a gradual transition between roadway pavement and bridge deck can be provided. Despite the extensive use of approach slabs, unsatisfactory performance of approach slabs has been reported in the US. The bump problem affects about 150000 bridges in the US, with an estimated maintenance cost of at least 100 million US dollars per year (Briaud et al. 1997). In China, similar problems are also reported, e.g. Sha (1999) reported that 9.826 million RMB were spent on settlement treatment of approach slabs in the first 6 years after the opening of

\(^1\) Assistant Professor
\(^2\) Undergraduate Student
Shanghai–Jiading Expressway. This worldwide problem affects comfort of the travelling public; it also leads to high maintenance cost and disruption to traffic.

Factors contributing to the bump problem may include (Chen and Chai 2010): (i) time-dependent consolidation of the natural soil under the embankment and/or fill material due to inadequate compaction, (ii) poor drainage behind the bridge abutment, which results in erosion of fill material and void formation under the approach slab, and (iii) longitudinal and vertical translation as well as rotation of the abutment causing localized damage at the connection of the approach slab. Among the factors contributing to the bump problem, differential settlement between the approach slab and road way pavement due to consolidation of embankment soil/fill and soil washout underneath the slab near the abutment joint due to deteriorated joint material and poor drainage are considered two important observed phenomenon at site affecting the overall performance of the approach slab. Differential settlement changes the transition slope from pavement to bridge deck thus affects riders’ comfort, while soil washout affects bearing capacity of embankment and may result in surface cracks and uneven deflection of the bridge approach, both of which should be taken into account when evaluating the current performance/health condition of the approach slab.

In analyzing the performance and health status of approach slabs, finite element methods are often used. Nassif et al. (2002) developed finite element modeling of bridge approach and transition slabs used in New Jersey. Cai et al. (2005) used finite element method to investigate the interaction between bridge approach slabs and embankment settlement. Ding and Wang (2004) also used finite element method to simulate the ground reaction, internal force and deflection of slabs under loading. In the literature, most research relies on finite-element approach to model the approach slab and to analyze the performance of it under traffic loadings. Finite-element method, if applied, would undoubtedly result in excellent precision, however, such an approach can be expensive in terms of computational time and resources in order to model the boundary and traffic conditions properly. Furthermore, the design of bridge approach slabs are not universal, thus new finite-element model needs to be created when the slab design changes. All these limitations hinder practical bridge engineers from adopting finite element approach in analyzing approach slabs’ performance/current health status; they rather use simple indices, like the amount of settlement, to decide if remedial measures are required, despite the different slab designs do exist in various states.

This paper provides a simple solution to the bump problem. A simplified bridge approach slab model based on Winkler’s foundation but capable of simulating the two main factors affecting the slab performance, is proposed. An approach slab design adopted by Zhejiang province is used as an example to verify the feasibility of the proposed model. The results demonstrate that the proposed model is able to give various responses (deformation, bending moment, shear force, soil pressure) of the slab under assumed soil conditions when subjected to design traffic loads. The proposed mathematical model is therefore deemed applicable for simulation of the behavior of approach slabs under design truck loading, as well as quick assessment of the slabs’ responses to decide if remedial measures need to be taken.

2. CURRENT APPROACH SLAB DETAILS
Similar to the approach slab design in the U.S., there is no unified design of approach slab in China, although general design guidance does exist. For example, The General Code for Design of Highway Bridges and Culverts of China (2015) suggests to construct approach slabs on expressway, level I and level II roadways, and the approach slabs should be designed no thinner than 0.25 m and no shorter than 5 m. However, no specific dimensions and rebar layout are given by the design code. In Ningbo, a city of Zhejiang province, an approach slab with 8 m long, 7.5 m wide, and 0.4 m in thickness is commonly adopted, and it will be used as the demonstration example in this paper to investigate the performance of the approach slab under traffic loading. Fig. 1 shows the details of Ningbo approach slab with a seat-type abutment. The reinforcement area ratio for this approach slab in the longitudinal and transverse directions are 0.93% and 0.51%, respectively. As shown in Fig. 1 (b), in the longitudinal direction, the slab is reinforced by #16 (16 mm in diameter) bars at 0.16 m c/c at the top and #22 (22 mm) bars at 0.16 m c/c at the bottom. The top reinforcement area accounts for 34.5% of the longitudinal reinforcement area. In the transverse direction, #16 top bars at 0.2 m c/c and #16 bottom bars at 0.2 m c/c are used. The slab is designed to accommodate two traffic lanes, 3.75 m each, thus the flexural moment capacity for one traffic lane can be calculated as 872 kN.m.

Fig. 1 Details of the approach slabs used in Ningbo, Zhejiang

3. MATHEMTICAL SIMULATION OF APPROACH SLABS

3.1 Mathematical Modelling and Formulation

Recognizing that differential settlement affects the transition slope from pavement to bridge deck and soil washout affects bearing capacity of embankment and may result in surface cracks and uneven deflection of the approach slab, both of which
should be taken into account when evaluating the current performance/health condition of the slab. This paper proposes to simulate the two main factors affecting the slab performance using a simplified bridge approach slab model based on beam-on-elastic Winkler’s foundation. General mathematical formulation of an approach slab under the effects of differential settlement and soil washout when subjected to traffic loading, as shown in Fig. 2, can be expressed as:

\[ EI y''''(x) + kB u(x - d)[y(x) - y_s(x)] = (1 + IM)[P_1 \delta(x - a) + P_2 \delta(x - a - s_1) + P_3 \delta(x - a - s_1 - s_2)] + \omega_o \]  

where \( EI \) represents flexural rigidity of the unit slab, usually taken as one traffic lane width (= 3.5 m); \( y(x) \) is the deflection of the slab; \( x \) is the distance measured from the abutment; \( k \) represents the coefficient of subgrade reaction; \( B \) is the width of the slab; \( u(x-d) \) is the unit step function representing the soil washout underneath the slab near the abutment end, and \( d \) is the soil washout distance; \( y_s(x) \) is the differential settlement function describing the uneven embankment settlement between the two ends of the slab; \( IM \) is the dynamic amplification factor; \( P_1, P_2, \) and \( P_3 \) are the front, middle, and rear axle loads of a sample three-axle design vehicle, respectively; \( \delta(\bullet) \) is the Dirac delta function; \( a \) is the distance from the abutment to the front axle of the design vehicle; \( s_1 \) and \( s_2 \) are the distances of front-to-middle axle and middle-to-rear axle, respectively; \( \omega_o \) is the design tandem/lane loading. It should be noted that design of approach slabs and design truck loading may vary from country to country, however, via proper modification of the proposed mathematical formulation, responses of the approach slab under combined differential settlement and soil washout effects can be obtained.

Fig. 2 Modelling of approach slabs under differential settlement and soil washout
3.2 Design Truck Load of Chinese Standard

This paper uses an approach slab design in Ningbo, a city of Zhejiang province in China, thus Chinese design truck loading should be investigated and adopted in the proposed model. General Code for Design of Highway Bridges and Culverts states that the truck load can be classified as load levels 1 and 2 (China Communications Construction Company, 2015). Table 1 shows the highway classification and corresponding design truck load that should be used. It is generally recognized that load level 1 will cause more severe effects than load level 2, thus, on account of the most severe case, truck load level 1 will be used as the input loading in the mathematical formulation. Fig. 3 shows the axle and tandem load distribution of Chinese level 1 design truck.

<table>
<thead>
<tr>
<th>Highway classification</th>
<th>Expressway</th>
<th>First-class highway</th>
<th>Second-class highway</th>
<th>Third-class highway</th>
<th>Fourth-class highway</th>
</tr>
</thead>
<tbody>
<tr>
<td>Truck load level</td>
<td>Level 1</td>
<td>Level 1</td>
<td>Level 1</td>
<td>Level 2</td>
<td>Level 2</td>
</tr>
</tbody>
</table>

![Moving direction]

30kN 120kN 120kN 140kN 140kN

$q_k=10.5\text{kN/m}$

![Axle and tandem load distribution of load level 1 design truck.](image)

Based on the Chinese level 1 design truck, the governing equation in Eq. (1) can be modified accordingly and be written below.

\[
EIy''''(x) + kBu(x - d)[y(x) - y_s(x)] = (1 + IM)[P_1\delta(x - a) + P_2\delta(x - a - s_1) + P_3\delta(x - a - s_1 - s_2) + P_4\delta(x - a - s_1 - s_2 - s_3) + P_5\delta(x - a - s_1 - s_2 - s_3 - s_4)] + \omega_o
\]  

(2)

where the axle loads $P_1$, $P_2$, $P_3$, $P_4$, and $P_5$ are 30, 120, 120, 140, and 140 kN, respectively; the spacing between axles $s_1$, $s_2$, $s_3$, and $s_4$ are 3.0, 1.4, 7.0, and 1.4 m, respectively; design tandem load $\omega_o$ becomes 10.5 kN/m.
For the dynamic amplification factor of bridges used in China, the General Code for Design of Highway Bridges and Culverts (2015) suggests to use the following formula to obtain the factor:

\[
\begin{align*}
    f < 1.5 \text{ Hz}, & \quad \mu = 0.05 \\
    1.5 \text{ Hz} \leq f \leq 14 \text{ Hz}, & \quad \mu = 0.1767 \ln f - 0.0157 \\
    f > 14 \text{ Hz}, & \quad \mu = 0.45
\end{align*}
\]  

(3)

where \( f \) stands for the nature frequency of the pavement or bridge and \( \mu \) is the impact factor/dynamic amplification factor. Therefore, to investigate the most severe case, an impact factor (\( IM \)) of 0.45 may be used.

### 3.3 Boundary Conditions and Slab Properties

To obtain the response of the approach slab, proper boundary conditions should be set. In this paper, four boundary conditions are employed to obtain the numerical solution of the proposed slab model:

- \( y(0) = 0 \), represents no vertical displacement at the abutment end,
- \( y''(0) = 0 \), represents no bending moment of the slab at the abutment end,
- \( y''(l) = 0 \), represents no bending moment of the slab at the pavement end,
- \( y(l) = 0 \), represents no deflection at pavement end. Note that this assumption is based on the fact that dowel bars are used to connect the approach slab and roadway pavement to constrain the movement of the slab at the pavement end. If no dowel bars are present, meaning no shear force of the slab at the pavement end, then \( y'''(l) = 0 \) should be used to replace \( y(l) = 0 \).

Properties of the current Ningbo approach slab includes:

- The length of the approach slab is 8 m, and the width of slab is 3.75 m, representing one traffic lane width.
- The flexural rigidity for one traffic lane (3.75 m in width), \( EI \), is 258300 kNm²,
- The coefficient of subgrade reaction, \( k \), is assumed to be 49.7 MN/m³,

By inputting all the required parameters and accompany boundary conditions, the solution to Eq. (2) can be obtained numerically. After obtaining the slab deflection \( y(x) \), important slab responses, such as the bending moment of this slab, \( Ely''(x) \), and shear force of the slab, \( Ely'''(x) \) can all be calculated by differentiating the slab deflection \( y(x) \) with respect to \( x \) and then multiplying by the flexural rigidity \( EI \).

### 4. CASE STUDIES

#### 4.1 Effect of Soil Support

The general approach of designing an approach slab is to treat the slab as a simply-supported beam subjected to design traffic loads; therefore, as can be seen in Fig. 1, the rebar layout of the approach slab in Ningbo is mainly to resist positive bending moment, without considering the effect of soil support. To investigate the effect of soil support, the slab is first treated as a simply-supported slab under truck load without the soil under the slab. Fig. 4 shows the bending moment envelopes of the
Ningbo approach slab without considering the soil support. It can be seen from Fig. 4 that maximum positive moment of 535.85 kN.m occurs at the first and second 140 kN axle loads placed at 3.65 m and 5.05 m from the abutment, respectively, and the resulting maximum moment is 61.5% of the full flexural capacity (= 872 kN.m) of the slab. Fig. 5 shows the bending moment envelopes of the Ningbo approach considering the soil support. It can be seen from Fig. 5 that the maximum bending moment is significantly reduced to 56.18 kN.m, which is less than 10% of the full flexural capacity of the slab. It can therefore be concluded that the embankment soil under the approach slab contributes enormous in reducing the moment demand caused by the traffic loads. However, it should be noted that the resulting negative bending moment, which is not shown in Fig. 4, may be the cause of surface cracking on the slab, which should be considered in design of the approach slab.

Fig. 4 Bending moment envelopes without soil spring
4.2 Effect of Soil Washout

Void formation under the approach slab may be caused by differential settlement between the abutment and the pavement and washout of embankment soil near the abutment, therefore, effects of differential settlement and soil washout on slab’s overall performance should be investigated. In the proposed formulation shown in Eq. (1), unit step function \( u(x-d) \) is used to simulate the soil washout, where parameter \( d \) represents the washout length, meaning that there will be no soil support thus no soil spring under the approach slab from 0 to \( d \) meters measured from the abutment. In the simulated case, \( d \) is varied from 0, 0.5, 1, 2, and 3 meters.

Fig. 6 (a)-(d) shows the bending moment envelopes for Ningbo slab under increasing soil washout length. It can be seen from Fig. 6 (a) and (b) that the case of 0.5 m washout and 1 m washout are nearly identical, just the bending moment increased slightly from 56.01 to 61.93 kN.m. However, when the washout distance increases to 2 m and 3 m, maximum bending moment increases significantly, as shown in Fig. 6 (c) and (d). For 2 m washout, the maximum bending moment is 98.53 kN.m, which is nearly twice the case of 0.5 m washout. For 3 m washout, the maximum bending moment further increases to 145 kN.m. Although it’s less likely that the 3 m washout can exist without any remedial measures being taken, it is nonetheless an indication that if large washout takes place, an increase in bending moment and deflection may cause riders’ discomfort and this may be an issue for the bridge authority. It can also be seen from Fig. 6 (d) that maximum negative bending moment has reached -44.9 kN.m. The increased negative moment, with increasing washout length, is the main cause of surface cracks. Therefore, in considering the slab
maintenance, soil washout length is a contributing factor and should be taken into account in deciding if remedial measures need to be taken.

Fig. 7 shows the deflection envelopes for Ningbo slab under increasing soil washout length. It can be seen from the figure that at short washout length, e.g. 0.5 and 1 m, the maximum deflection of the slab is very small, less than 0.5 mm. With increasing washout length, the maximum slab deflection also increases, but even at maximum washout length of 3 m, the slab’s maximum deflection is slightly larger than 1 mm, which is considered insignificant and should not cause discomfort to the travelling public.

Fig. 6 Bending moment envelopes for Ningbo slab under increasing soil washout length
4.3 Effect of Differential Settlement

Apart from soil washout, differential settlement is also a main reason causing the void formation. On account of strong impact on bump problem by differential settlement, researches were carried out to evaluate the influence of differential settlement. In U.S., a visual survey was conducted by Long et al. (1998) on 1181 bridge approaches in Illinois, and a rating system of the bump problem using differential settlement proposed by them is shown in Table 2. It is reported that about 58% of approach slab can be classified as having slight bump, which means the differential settlement is no more than 25 mm. China Communications Construction Company (2015) stated that the slope of differential settlement (the differential settlement divided by the slab length) should be no larger than 0.2%. In Japan, 0.2% slope is also considered the threshold for bridge maintenance (Qiao 2011), and 0.4% slope is used in some countries such as Sweden and France (0.4-0.6%) (Qiao, 2011). To investigate the effect of differential settlement on the overall performance of bridge approaches, in this paper, the differential settlement gradient of 0.1%, 0.2%, 0.3% and 0.4% will be used. For a 8 m long slab, the settlements are respectively 0.008 m, 0.016 m, 0.024 m and 0.032 m.

To simulate the differential settlement, \( y_s(x) \) shown in Eq. (1) is used to represent the settlement of embankment. The reaction force provided by soil spring is thus equal to the coefficient of subgrade reaction \( k \) times slab width \( B \) and the relative deformation of soil spring, i.e. \( y(x) - y_s(x) \). Note that the embankment settlement function, \( y_e(x) \), can be any possible shape of the settlement. In this paper, uniform settlement is used, thus settlement function \( y_e(x) \) is a constant. For solving Eq. (1), abutment is assumed to pin-ended and have zero settlement, i.e. \( y(0) = 0 \) and \( y''(0) = 0 \), and pavement end is assumed dowel-connected to the pavement slab and settles the same amount as the embankment settlement, i.e. \( y''(l) = 0 \) and \( y(l) = \text{embankment settlement.} \)
Table 2 Rating system of approach slabs proposed by Long et al. (1998)

<table>
<thead>
<tr>
<th>Qualitative Visual Rating</th>
<th>Approach Interface Description</th>
<th>Approximate Differential Movement</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>No bump</td>
<td>~0mm</td>
</tr>
<tr>
<td>1</td>
<td>Slight bump</td>
<td>~25mm</td>
</tr>
<tr>
<td>2</td>
<td>Moderate bump</td>
<td>~50mm</td>
</tr>
<tr>
<td>3</td>
<td>Significant bump</td>
<td>~75mm</td>
</tr>
<tr>
<td>4</td>
<td>Large bump</td>
<td>&gt;75mm</td>
</tr>
</tbody>
</table>

Fig. 8 shows the bending moment envelopes under given uniform embankment settlement. It can be seen from Fig. 8 that the bending moment increases significantly with increasing embankment settlement. At 8 mm settlement (0.1% slope), the maximum bending moment has increased to 274 kN.m, which is almost five times the moment at no soil settlement (= 56 kN.m). This moment is also larger than the maximum moment at 3 m washout case (= 145 kN.m). It is therefore worth noting that the differential settlement, although could be of small amplitude, has significant impact on the moment effects. At 16 mm settlement (0.2% slope), as can be seen from Fig. 8, the moment further increases to 370 kN.m. At 32 mm settlement (0.4% slope), the maximum bending moment has reached 502 kN.m, about 60% of the full flexural capacity of the slab. This moment is considered significant as only 32 mm uniform embankment settlement can cause this dramatic increase in moment demand.

Fig. 9 shows the deflection envelopes of the slab under uniform embankment settlement. It can be seen from Fig. 9 that with the increase in embankment settlement, the slab’s deflection also increases, and the maximum slab deflection at each embankment settlement is very close to the given settlement length. Fig. 10 shows the soil pressure under given uniform settlement. From Fig. 10, it can be seen that at a small settlement of 0.008 m, a more than 3 m long void underneath the slab near the abutment end has been created, as zero soil pressure is observed. At 0.016 m settlement (0.2% slope), the void further increases to about 4.5 m. At 0.032 m settlement (0.4% slope), the void length has reached almost 6 m, considering the slab is only 8 m long, the void is deemed significant, and remedial measures should be taken. It can also be seen from the figure that soil pressure decreases with increasing settlement length; this may imply that the slab itself is taking more loading effects by bending of the slab, and the behavior of the slab under loading is moving toward a simply-supported beam.
Fig. 8 Bending moment envelopes under uniform settlement

Fig. 9 Slab deflection envelopes under uniform settlement
In this paper, a simplified bridge approach slab model based on beam-on-elastic Winkler’s foundation but capable of simulating the two main factors affecting the slab’s performance, namely differential settlement between the abutment and embankment and soil washout underneath the slab near the abutment end, is proposed. An approach slab design adopted by Zhejiang province is used as an example to verify the feasibility of the proposed model, and Chinese design truck is used as the loading model. Four different slope changes (0.1%, 0.2%, 0.3%, and 0.4%) between abutment and pavement and four different washout scenarios (0.5 m, 1 m, 2 m, and 3 m) are used in the case studies. Results from case studies demonstrate that the proposed model is able to give various responses (e.g. deformation, bending moment, shear force) of the slab under assumed soil conditions when subjected to design traffic loads. Results also show that a given embankment settlement, although could be fairly small, will produce significant effects on the moment demand of the slab and void formation underneath the slab. On the contrary, when the void is formed due to washout of embankment soil near the abutment end, the impact on slab’s overall performance is insignificant, even for a 3 m washout length. The proposed mathematical model is capable of simulating differential settlement and soil washout, and is therefore deemed applicable for simulation of the behavior of approach slabs under design truck loading, as well as quick assessment of the slabs’ responses to decide if remedial measures need to be taken.

Fig. 10 Soil pressure envelopes under uniform settlement

5. CONCLUSIONS

In this paper, a simplified bridge approach slab model based on beam-on-elastic Winkler’s foundation but capable of simulating the two main factors affecting the slab’s performance, namely differential settlement between the abutment and embankment and soil washout underneath the slab near the abutment end, is proposed. An approach slab design adopted by Zhejiang province is used as an example to verify the feasibility of the proposed model, and Chinese design truck is used as the loading model. Four different slope changes (0.1%, 0.2%, 0.3%, and 0.4%) between abutment and pavement and four different washout scenarios (0.5 m, 1 m, 2 m, and 3 m) are used in the case studies. Results from case studies demonstrate that the proposed model is able to give various responses (e.g. deformation, bending moment, shear force) of the slab under assumed soil conditions when subjected to design traffic loads. Results also show that a given embankment settlement, although could be fairly small, will produce significant effects on the moment demand of the slab and void formation underneath the slab. On the contrary, when the void is formed due to washout of embankment soil near the abutment end, the impact on slab’s overall performance is insignificant, even for a 3 m washout length. The proposed mathematical model is capable of simulating differential settlement and soil washout, and is therefore deemed applicable for simulation of the behavior of approach slabs under design truck loading, as well as quick assessment of the slabs’ responses to decide if remedial measures need to be taken.
ACKNOWLEDGEMENTS

We gratefully acknowledge the financial support from Zhejiang Qianjiang Talent Scheme (QJD1402009) and Ningbo Natural Science Foundation (2014A610172).

REFERENCES


Qiao, H. (2010), Researches into the design approach and technical parameters of approach slabs, Master Thesis, Chang’an University, Xian, China.