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Dynamic optimization of snapping through procedure of compressed bistable mechanisms

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ABSTRACT

The fabrication induced strain affects greatly on the snapping through procedure of MEMS bistable devices. However, due to the strong nonlinear coupling effects between the structure parameters and the bistable mechanical properties, it is really difficult to control the snapping through procedure by just modifying the loading cases and structure parameters once the bistable configuration and boundary constraints are fixed. Actually, the snapping mode (symmetric and asymmetric) is deeply related with the structure configuration, which will directly affect the bifurcation frequency. In this paper, through the accurate mechanical analysis of the MEMS bistable mechanisms, a novel optimization based method for adjusting the dynamic snapping through procedure is proposed by modifying multi local segments' geometry. Accordingly, the compressed bistable compliant beam is reconstructed for achieving desired snapping forces and bifurcation frequency. Meanwhile, the influence of the exciting frequency and amplitude of the external dynamic load on the dynamic snapping characteristic is analyzed. The sharp changes on the amplitude - frequency characteristics can be divided into two groups, one is up bifurcation and the other is down bifurcation. The optimization formula was established to achieve a specific bistable parameter, and to minimize the influence of the strain on the bistability. For experimental study, a compressed beam with reinforced local segments is manufactured. The good agreement between the numerical simulation and experimental results validates the effectiveness of the dynamic optimization method for controlling the snapping through procedure, thus expanding the potential application range of compressed compliant bistable mechanisms.

1. INTRODUCTION

Distinguished by attractive properties such as snap-through, accurate positioning and state holding capability, bistable mechanisms have great applications in industrial electronics, such as energy harvesting systems [1], memories [2, 3], valves and relays [4-8], switches and actuators [9-12], and so on.

In recent years, a series of bistable mechanisms have been developed using different structure combinations. Howell [13] and Yu[14] introduced pseudo-rigid-body-model in the design of compliant mechanism with four links and translational joints. Pucheta[15] used the precision-position and rigid-body replacement method to design bistable compliant mechanisms, which can determine the initial and final equilibrium positions of partially compliant mechanisms. Hao [16] presented an in-plane comprehensive static analysis of a translational bistable mechanism using nonlinear finite element analysis, and furthermore, Hao gave a simple analytical (empirical) equation for estimating the negative stiffness. Chen [17] developed a tristable mechanism configuration employing orthogonal connected compliant mechanisms. Actually, the bistable characteristics including the snapping force, the maintaining force, stable positions, and the travelling stroke are of great importance for different potential applications. In Fig.1, some bistable terms are introduced to depict the nonlinear mechanical property, including the snapping force F_{cr1}, the maintaining force F_{cr2}, stable positions, the travelling stroke (displacement). To further explore the bistable mechanics, the bistable index parameter γ defined as the raito of the snapping force to the maintaining force was introduced to indicate the bistability.

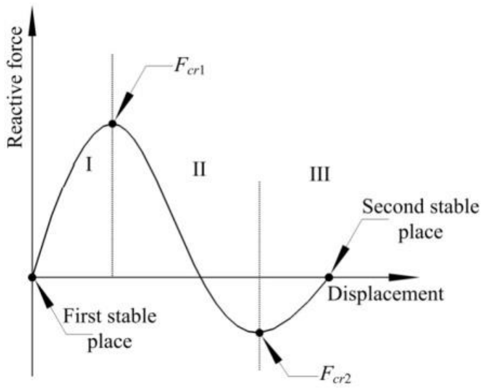

Fig.1 Satic mechanics of the bistable mechanism

Due to the strong nonlinearity, the bistable characteristics are highly coupled to each other, which means that the specific bistable feature e.g. snapping force F_{cr1} can not be designed or adjusted freely by just changing the geometric size once the structure configuration is fixed. For example, changing the span length of the cosine-shaped beam can modify the bistable index parameter, the stable position, and the snapping force as well, thus, resulting in a repeat design of all the other bistable parameters. Therefore, a new bistability design method is really needed to decouple the design process and further to expand the application space for compliant bistable mechanisms.

In this paper, a new optimization-based design method was proposed for controlling the snapping mode to achieve desired bistable mechanics, with which, the target bistable character can be re-designed without affecting the other bistable properties. With the goal of adjusting the bistable index parameter γ to 1.0 from the original value of 0.598 and reducing the snapping force as well, a novel bistable cosine-shaped beam with fifteen reinforced local segments was obtained by using the multi-step iteration method. Comparing with the original uniform structure in the same overall size, the snapping force is decreased by 47.4% as well as the maintaining force reduced by 69.5%, thus achieving a bistable index parameter of 1.021 that is nearly 2.0% deviated from the required value of 1.0. In addition, the bistable index parameter can also be adjusted to 1.2 from the original value of 0.598. Hence, the effectiveness and practicability of the proposed optimization method was validated.

2. Optimization model of the specific bistable mechanics

Cosine-shaped beam with both ends fixed has been frequently used as bistable structures in MEMS actuators and sensors. In Fig.2, the beam was divided into *n* elements depending on finite element method. And the bistable index parameter γ, $\gamma=|F_{cr1}/F_{cr2}|$, was introduced to describe the relationship between the two switching forces.

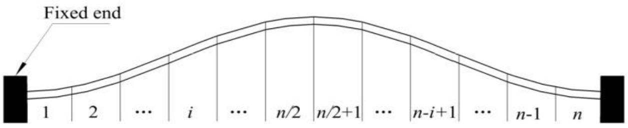

Fig.2 Bistable cosine-shaped beam finite element model

The updated Lagrange method was used to analyze the snap-through process of the cosine-shaped beam with both ends clamped. By taking the configuration at time *t* as the reference value, the virtual displacement principle equivalent to the equilibrium condition and force boundary condition of the configuration at time $t+\Delta t$ can be expressed as follows[18].

$$({}_t^tK_L + {}_t^tK_{NL})u = {}^{t+\Delta t}Q - {}_t^tF \tag{1}$$

where, u is the increment vector of node displacement, ${}^{t+\Delta t}Q$ loading vector, and ${}_t^tK_L$, ${}_t^tK_{NL}$ and ${}_t^tF$ integral items expressed as follows.

$$\begin{cases} {}_t^tK_L = \sum_e \int_{{}^tV_e} {}_t^tB_L^T \, {}_tD \, {}_t^tB_L \, {}^tdV \\ {}_t^tK_{NL} = \sum_e \int_{{}^tV_e} {}_t^tB_{NL}^T \, {}^t\tau \, {}_t^tB_{NL} \, {}^tdV \\ {}_t^tF = \sum_e \int_{{}^tV_e} {}_t^tB_L^T \, {}^t\hat{\tau} \, {}^tdV \end{cases} \tag{2}$$

Where ${}_t^tB_L^T$ and ${}_t^tB_{NL}^T$ represent the transformational matrixes of linear and nonlinear strains, ${}_tD$ the constitutive matrix of material, ${}^t\tau$ and ${}^t\hat{\tau}$ are the *Cauchy* stress matrix and the vector.

From numerical simulation, it can be found that the geometric configuration of local segment along the beam will greatly influence the snap-through mode and the two threshold forces, as shown in Fig.3. By modifying the local segment, the bistable index parameter can be adjusted to be 3.31 without affecting the travelling stroke of the bistable mechanism. Meanwhile, the snapping mode can be changed from symmetric to asymmetric.

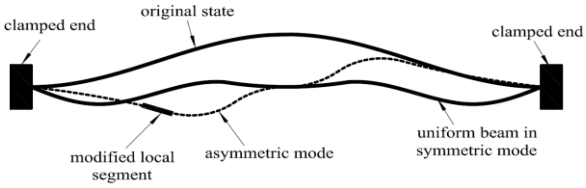

Fig. 3 Asymmetric and symmetric modes during the snap-through procedure

To optimize the bistable mechanics, five design variables involving the length, the width, the thickness, element number and their positions should be considered in the design model. Then, the bistable beam is divided into 2n parts, and each element length was set to be $L/2n$. Additionally, the widths of all the elements were assumed to be the same with the original structure. Here, the position and the number of the multiple reinforced segments are involved in the element labeling information from 1 to n. Therefore, the optimization problem of multi-variables can be converted to find the element labeling and the corresponding thickness.

Then, the target of minimum snapping force and the specific bistable parameter of 1.0 can be represented by minimizing the objective function of $f_i(D_j)=|\gamma-\alpha|+F_{cr1}/F_{ocr1}+F_{cr2}/F_{ocr2}$, where F_{ocr1} and F_{ocr2} represent the snapping force and the maintaining force of the original uniform structure. To conclude, the optimization formulation can be obtained, as formulated in the following.

$$
\begin{aligned}
&Find \quad i, D_j \\
&\min \quad f_i(D_j) = |\gamma\text{-}\alpha| + \frac{F_{cr1}}{F_{ocr1}} + \frac{F_{cr2}}{F_{ocr2}} \\
&s.t. \quad \begin{cases} K_{Ti}\left(A_i, i\right)\cdot w = F_i\left(A_i, i\right) \\ \gamma = \left|\dfrac{F_{cr1}}{F_{cr2}}\right| \\ F_{cr1} = \max\left[F_i\left(A_i, i\right)\right], \quad i = 1, 2, ..., n \\ F_{cr2} = \left|\min\left[F_i\left(A_i, i\right)\right]\right|, \quad i = 1, 2, ..., n \end{cases}
\end{aligned}
\tag{3}
$$

In which, $F_{cr1}$and F_{cr2} represent the snapping force and the maintaining force of the modified bistable structure, F_{ocr1} and F_{ocr2} the snapping force and the maintaining force of the original uniform structure, $K_{Ti}(A_i,i)$ the stiffness of strengthened beam with the i^{th} element modified, w the displacement, $F_i(A_i, i)$ the reactive force at loading position when the beam is modified at the i^{th} segment.

Then, with the optimization formulation Eq.(3), the flow chart for designing specific bistable feature can be obtained, as shown in Fig.4, in which, the multi-step optimization method was adopted.

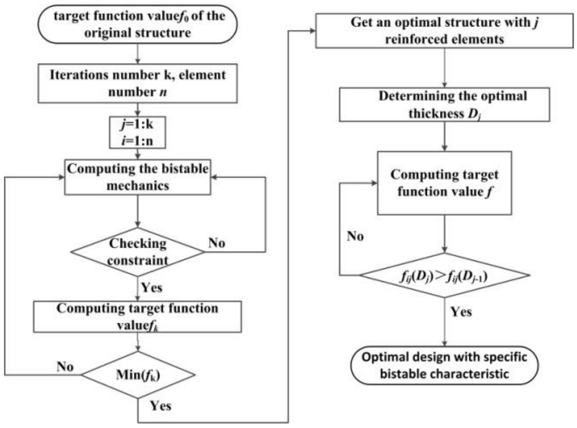

Fig. 4 The flow chart of the optimization procedure

From figure 4, the iteration number increases with the element number. Assuming that the total number of elements is 2*n* and the reinforced segments of *m*, the total iteration number can be determined P_n^m for symmetric arrangement and P_{2n}^m for the asymmetric case.

3. Numerical examples

The multi-step iteration method was introduced to obtain the optimal structure with predefined bistable mechanics. Through 29 times overall iterations, a cosine-shaped beam with 15 reinforced elements was finally obtained, as shown in Fig.5. The snapping threshold force F_{cr1} decreases to 10.3mN, and the maintaining force to F_{cr2} to 10.1mN, thus the specific bistable index parameter becomes to 1.021, which is only 2% deviated from the goal value of 1.0. More important, the travelling stroke of the bistable mechanism has not been changed during the optimization process.

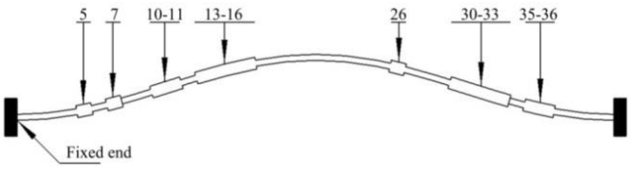

Fig.5 The configuration of the optimal structure with predefined bistable mechanics

Furthermore, the relationship between the thickness of the reinforced element and the bistable characteristics was numerically analyzed, as listed in Table I. In the range from 40.0μm to 100.0μm, the optimal value is 50.0 μm with the snapping force F_{cr1} =10.3mN, and the maintaining force F_{cr2}=10.1mN.

Finally, the nonlinear force-displacement curve of the newly obtained bistable structure was shown in Fig. 6. They axle represents the reactive force which is positive when the direction is opposite to the load direction. And the x axle stands for the displacement of the load point. Compared with the mechanics of the original uniform bistable beam, the snapping force decreased from 19.6mN to 10.3μN by 47.4% and the maintaining force decreased from 32.8mN to 10.1mN by 69.5%, which have been significantly decreased, the energy of actuation decreased distinctly for the area is decreased and the bistable parameter, γ=1.021,can reach the design goal within 2.0%. Additionally, the snap-through path (mode) was changed but the stroke during the optimization procedure.

Table I The influences of thickness *D* on the bistable parameters

D/μm	F_{cr1}/mN	F_{cr2}/mN	*γ*	*f*
40	9.4	6.8	1.384	1.069
50	**10.3**	**10.1**	**1.021**	**0.852**
60	10.7	10.2	1.057	0.915
70	11.0	10.3	1.073	0.946
80	11.2	10.4	1.078	0.961
90	11.3	10.4	1.085	0.973
100	11.3	10.6	1.072	0.969

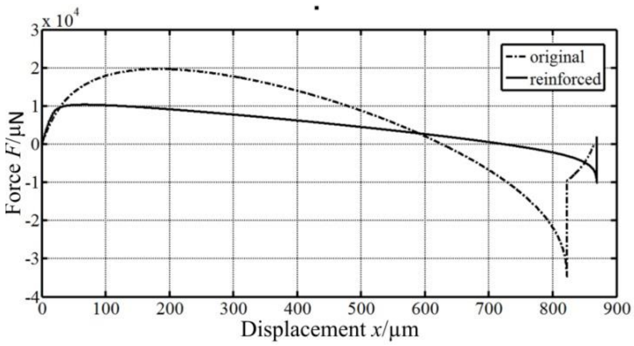

Fig.6 The mechanics difference between original structure and the newly proposed structure

3. Discussion

The sharp change on the displacement-force curve shown in Fig.6 is mainly induced by the mode switching in the snap-through procedure between two stable states. Actually, the mode switching occurs twice when the buckling mode transfers from one energy state to the other, e.g. from the lower order mode to the higher order mode and inversely switching back to lower mode, as shown in Fig.7. Also the bistable beam accumulates more energy at higher order buckling mode than that of lower mode. So the two mode switching procedures represent the energy accumulation and release processes. During the accumulation process at the beginning, the structure deforms gradually into the asymmetric mode known as the unstable bifurcation buckling mode while the displacement load applies to the central point of the beam. Then, at the energy release process, the deformed beam snaps quickly to the low energy state (the second stable state) as the high energy state is unstable. Therefore, the mode transfer is much faster from the high energy mode to the low energy status, thus resulting in a sharp change on the displacement-force curve. Additionally, the simulation results of the bistable state switching from the second state to the first state, the sharp change also occurs when the beam snaps from the higher energy state to the low energy state (the original stable state).

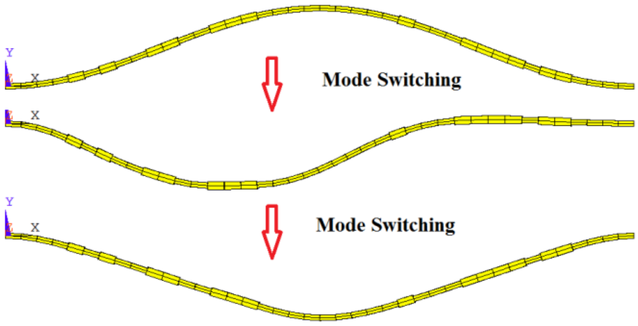

Fig.7 the mode switching during the snap-through procedure

Most important, adding properly arranged reinforced segments to the bistable mechanism can effectively control the higher order buckling mode to be symmetric or asymmetric during the snapping procedure.

4. Conclusion

Due to the complex coupling effects among bistable characters, it is difficult to adjust one target bistable character without affecting other bistable characters by just changing the overall structure parameter such as size and configuration. Hence, an optimization method was proposed to modify the bistability through the integrated design of the cross section parameters of multiple local segments and their arrangements. The optimization formula was established to achieve the specific bistable parameter, and to minimize the snapping force without changing its stroke. The numerical results show that the snapping force can be significantly decreased and the bistable index parameter can achieve the design goal of 1.0 with only 2.0% of deviation. Also, with the proposed model, the bistable index parameter can also be adjusted to 1.2. Therefore, the proposed model can be used to design any specific bistable character without influencing other related bistable features, thus effectively expanding the potential applications of such compliant bistable mechanisms.

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