A sensitivity-based system identification method of vehicle-track coupling system

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\textbf{ABSTRACT}

Most current techniques used for system identification of structures and vehicles coupling system are static-test-based methods. Methodologies using dynamic responses are highly desirable and under development. This paper presents a method for system identification of the vehicle-track coupling system based on their coupled vibration sensitivity respected to the parameters of the coupling system. The track in the coupling system is considered as the beams with finite length. The explicit motion equation of this coupling system is assembled through interaction forces between the wheels and the track. The elemental flexural rigidity of the track is chosen as a parameter for track identification. Both the elemental flexural rigidity of the track and the vehicular parameters are identified by a dynamic response sensitivity-based finite element model updating approach. By comparing the theoretical measurement responses of the measurement points in two different states, the flexural rigidities of all elements and the vehicular parameters are updated iteratively. For it is able to identify the parameters of moving vehicles, the methodology can be applied to improve the current weigh-in-motion techniques. A numerical three-span continuous beam with multiple local damages is employed to validate the proposed method. The effects of measurement noise have been investigated in the numerical simulation and the identified results demonstrate that the proposed method is able to conduct the system identification of track and vehicle very accurately. A laboratory test is further carried out to verify the proposed method.

1. INTRODUCTION

In the literature many vehicle-track interaction models have been proposed to study the dynamic properties of the coupling system and identifying the vehicle parameters or structural damages (Zhan et al. 2011). In order to study the track condition due to a
vehicle passing throw it, a vehicle-track coupling model is required. Two kinds of model are usually used to study the vehicle-track dynamic interaction, a wheel moving on the track, and a moving irregularity between the vehicle and the track (Knothe et al. 1993). Early studies forced on two typical theories (Ono et al. 1989; Clark et al. 1982; Grassie et al. 1982; Jenkins et al. 1974), continuous support model and discrete support model, which are simple, but are inadequate in understanding vibration characteristics of train and track since the coupling behavior between the vehicle and track is not considered. Later, other detailed models (Ishide et al. 1998; Nielsen et al. 1993; Kisilowski et al. 1991; Zhai et al. 1994; Lei et al. 1997; Ishida et al. 1997; Ishida et al. 2000) were developed, with the coupling behavior is considered.

However, the vibration of the vehicle-track coupling system with random irregularity of track profile is rarely considered (Snyder et al. 1977). The vehicle/track interaction models can be analyzed either in time domain (Newton et al. 1979; Wu et al. 2002) or in the frequency domain (Thompson et al. 1993; Nordborg et al. 1998). A thorough understanding of this complex coupling between vehicles and the track is critical for track design, rail condition assessment, vehicle speed control and overloaded vehicle control.

In the recent years, some great effect has been made for vehicle parameters identification from the dynamic responses of the vehicle-structure system. Different methods for identifying vehicle parameters have been proposed in the literature (Kyongsu et al. 1995; Derradjji et al. 1996) and (Au et al. 2004; Jiang et al. 2004) used a genetic algorithm to identify the parameters of vehicles traveling on a continuous bridge by minimizing the residuals between the measured accelerations and the reconstructed accelerations from the identified parameters. In their study the vehicle was modeled using either a four-parameter model with one DOF or five-parameter model with two DOFs. For the track model, the modified beam functions proposed by (Zheng et al. 1998) were used. (Law et al. 2006) presented a parameter identification method based on the dynamic response sensitivity analysis.

There are a lot of methods for damage identification. (Doebling et al. 1998) provided a comprehensive review on the damage detection methods by examining changes in the dynamic properties of a structure. (Housner et al. 1997) presented an extensive summary on the state-of-the-art in control and health monitoring in civil engineering structures.

Damage detection methods using structural dynamic responses have been widely studied in the past few decades. (Cattarius et al. 1997) used the time histories of vibration response of vibration response of the structure to identify damage in smart structures. (Majumder et al. 2003) proposed a time domain approach for damage identification in beam structures using vibration data. (Koh et al. 2000) studied the structural stiffness parameters of a multi-storey framework in a time domain system identification method. (Lu et al. 2007) proposed a response sensitivity based method to identify the local damages in the beam structures. (Bu et al. 2006) presented a method to identify the local damages of the bridge deck subject to moving vehicle. All this work based on assuming that the parameters of the vehicle are known in the calculation of the dynamic response of the coupling system.

In this paper, a detailed time-domain model is developed for the vehicle-track coupling system. A three-span continuous beam Euler beam is employed to represent
the rail. The train is modeled as a single degree-of-freedom system with three parameters. The track vertical profile irregularity is considered as the stationary ergodic Gaussian random processes and is included in this coupling model. Firstly, the dynamic responses for the coupling system are calculated by the Newmark direct integration method. Secondly, the response sensitivities with respect to both the vehicular parameters and the flexural rigidity of the element are calculated to establish the sensitivity matrix. Using the error between the measured response and the computed one as a minimization criterion, the system identification is performed by the least-squares method. A numerical vehicle-track system and an experimental example are employed to validate the accuracy of the proposed method. In the numerical analysis, the influence of measurement noise on the analysis results is discussed. The results show that the proposed method is computational stable and efficient to identify both the vehicular parameters and the elemental flexural rigidity.

2. Model for vehicle-track coupling system

2.1 Dynamic equation of vehicle model

For the purpose of simplicity of presentation, a three-parameter vehicle model of one degree-of-freedom (DOF) shown in Fig. 1 consisting of a mass, spring, and damper is used to establish the vehicle-track coupling equations.

![Fig.1. A single-degree-of-freedom vehicle system.](image)

The equation of motion of the vehicle can be expressed as Eq. (1):

$$[M_v][\dot{Z}_v]+[C_v][Z_v]+[K_v][Z] = \{F_v\} + \{F_g\}$$  \hspace{1cm} (1)

where $[M_v]$, $[C_v]$, and $[K_v]$ are mass, damping, and stiffness matrices for the vehicle, respectively. $\{Z_v\}$, $\{F_v\}$ are the nodal displacement vector and the vehicle-track interaction force for the vehicle, respectively. $\{F_g\}$ is the gravity force vector.

The vehicle-track interaction force $\{F_v\}$ is expressed as Eq. (2):

$$\{F_v\} = M_v\ddot{x} + C_v\dot{x} + K_v(x - y(\ddot{x})) = M_v\ddot{x} - M_vZ_v$$  \hspace{1cm} (2)
where $g$ is the acceleration of gravity. $y\left(\tilde{x}(t)\right)$ represents the vertical deflection of the contact point of the deck at a distance $\tilde{x}(t)$ from the left support. $r\left(\tilde{x}(t)\right)$ is the road surface roughness at a distance $\tilde{x}(t)$ from the left support.

2.2 Dynamic equation of track

A n-span continuous track is shown in Fig. 2. It is modeled as a continuous Euler-Bernoulli beam with $N$ elements in the finite element model. The equation of motion for the track can be written as Eq. (3):

$$
\begin{bmatrix}
M_r
\end{bmatrix}\{Z_r\} + \begin{bmatrix}
C_r
\end{bmatrix}\{\dot{Z}_r\} + \begin{bmatrix}
K_r
\end{bmatrix}\{\ddot{Z}_r\} = \{F_{rv}\}
$$

(3)

where $\begin{bmatrix} M_r \end{bmatrix}$, $\begin{bmatrix} C_r \end{bmatrix}$, and $\begin{bmatrix} K_r \end{bmatrix}$ are mass, damping, and stiffness matrices for the track, respectively. $\{Z_r\}$ is the displacement vector for all DOFs of the track. $z_r$ and $\dot{z}_r$ are the first and second derivative of $\{Z_r\}$ with respect to time, respectively. $\{F_{rv}\}$ is a vector containing all external forces acting on the track.

The damping matrix in Eq. (3) is the Rayleigh damping expressed by the linear combination of mass matrix and stiffness matrix, can be obtained by Eq. (4), Eq. (5) and Eq. (6):

$$
\begin{bmatrix}
C_r
\end{bmatrix} = \alpha \begin{bmatrix}
M_r
\end{bmatrix} + \beta \begin{bmatrix}
K_r
\end{bmatrix}
$$

(4)
\[ \alpha = 4\pi \frac{\xi_1 f_1 f_2^2 - \xi_2 f_2 f_1^2}{f_2^2 - f_1^2} \] (5)
\[ \beta = \frac{1}{\pi} \frac{\xi_2 f_1 - \xi_1 f_2}{f_2^2 - f_1^2} \] (6)

where \( f_1 \) and \( f_2 \) are the first- and second-order frequencies, and \( \xi_1 \) and \( \xi_2 \) are the first- and second-order damping ratios of the track, respectively.

3. Numerical simulation of the random irregularity of track vertical profile

Turbulence of the rail-wheel system is the reason to dynamic vibration of the vehicle-rail coupling system. Generally, there are two classes of the turbulence of the rail-wheel system, which are the deterministic turbulence and the uncertain turbulence. The random irregularity of track vertical profile is the typical uncertain turbulence.

Generally, the irregularity of the track vertical profile can be regarded as the stationary ergodic Gaussian random processes except the area with turnout, road crossing and the rail line with track deterioration (Roberts at el. 1990). In this paper, considering a stationary stochastic process \( \eta(t) \) with expectation of zero and power density function of \( S_x(\omega) \), the sample function of the stochastic process \( \eta(t) \) is simulated by the trigonometry series as (Shinozuka at el. 1971), shown as Eq. (7):
\[ \eta^d(t) = \sum_{v=1}^{NN} a_v \sin(\omega_v t + \phi_v) \] (7)

where \( a_v \) is a Gaussian random variable with expectation zero and variance \( \sigma_v \), and is independent for \( v=1,2,\ldots,NN \), \( \phi_v \) is a random variable with uniformity distribution in \( 0\sim2\pi \), and is independent for \( v=1,2,\ldots,NN \) as well. \( a_v \) and \( \phi_v \) are independent of each other and can be generated by computer by multiplicative method, Monte Carlo method or another algorithm of generating pseudo-random variable.

In order to obtain the variance \( \sigma_v \), we define a frequency band \( \Delta \omega \) as Eq. (8):
\[ \Delta \omega = (\omega_u - \omega_l) / NN \] (8)

where \( \omega_l \) and \( \omega_u \) are the lower and upper limit frequencies in the frequency domain of power spectral density function and \( NN \) is a sufficient large division number.

Defining Eq. (9):
\[ \omega_v = \omega_l + \left( v - \frac{1}{2} \right) \Delta \omega \quad \text{for} \quad v = 1,2,\ldots,NN \] (9)

We have Eq. (10):
\[ \sigma_v^2 = 4S_x(\omega_v) \Delta \omega \quad \text{for} \quad v = 1,2,\ldots,NN \] (10)
In the above computation, the effective power spectral density $S_x(\omega_v)$ is assumed to be in the range of $\omega_l$ to $\omega_u$, and beyond this scope $S_x(\omega_v)$ is taken as zero.

The power spectral density $S_x(\omega_v)$ of the railway track for the line grades of one to six (line grade one is the worst line and six is the best line) from America Railway Standard is used as input excitation, which has Eq. (11):

$$S(\omega) = \frac{\kappa A_v \omega^2}{(\omega^2 + \omega_c^2)\omega}$$

(11)

<table>
<thead>
<tr>
<th>Line grade</th>
<th>$A_v$ (cm² rad/m)</th>
<th>$\omega_c$ (rad/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.2107</td>
<td>0.8245</td>
</tr>
<tr>
<td>2</td>
<td>1.0181</td>
<td>0.8245</td>
</tr>
<tr>
<td>3</td>
<td>0.6816</td>
<td>0.8245</td>
</tr>
<tr>
<td>4</td>
<td>0.5376</td>
<td>0.8245</td>
</tr>
<tr>
<td>5</td>
<td>0.2095</td>
<td>0.8245</td>
</tr>
<tr>
<td>6</td>
<td>0.0339</td>
<td>0.8245</td>
</tr>
</tbody>
</table>

In table 1, $A_v$ and $\omega_c$ are coefficients associated with line grade, as shown in Table 1, and $\kappa$ is a constant, normally 0.25.

### 4. Method for vehicle-track coupling system

Vehicles travelling on the track are connected to the track rail via contact points. The wheel-rail interaction forces acting on the track $\{F_{rv}\}$ and the interaction forces acting on the vehicles $\{F_{vr}\}$ are actually action and reaction forces existing at the contact points. In terms of the finite element modelling, these interaction forces may not apply right at any element nodes. Therefore, the interaction forces need to be transformed into equivalent nodal forces $\{F_{ne}\}$ in the finite element analysis. According to the virtual work principle and the concept of shape function, the following relationship holds, Eq. (12):

$$\{F_n\} = [N_i] \times F$$

(12)

where $\{F_{ne}\}$ is the vector with the dimension equal to the total number of DOFs of the track. $[N_i]$ is a $n \times 1$ vector with zero entries except at the DOFs corresponding to the nodal displacements of the beam elements on which the load is acting, $n$ is the total DOFs of the track after introducing the boundary conditions. $[N_i] = [0 \ 0 \ldots N_{ri} \ldots 0]$, $N_{ri}$ is the vector of shape functions in the global coordinates evaluated for the $i$th element supporting the moving vehicle with $(i-1)l \leq x(t) \leq il$, $x(t)$ is the location of the moving vehicle in the global coordinate, $l$ is the length of the finite element. The $N_{ri}$ can be obtained in Eq. (13):
The vehicle is assumed to contact with the track all the time and no jumps occur between the vehicle’s wheels and the track. Combining Eq. (1) and Eq. (3), the equation of motion of the vehicle-track system is written as Eq. (14):

\[
N_i = \begin{bmatrix}
1 - 3 \left( \frac{x(t) - (i-1)l}{l} \right)^2 + 2 \left( \frac{x(t) - (i-1)l}{l} \right)^3
\end{bmatrix}
\]

The vehicle is assumed to contact with the track all the time and no jumps occur between the vehicle’s wheels and the track. Combining Eq. (1) and Eq. (3), the equation of motion of the vehicle-track system is written as Eq. (14):

\[
[K_r] \ddot{Z}_r + \begin{bmatrix}
M_r & N_r M_r \\
0 & M_v
\end{bmatrix} \begin{bmatrix}
\dot{Z}_r \\
\dot{Z}_x
\end{bmatrix} + \begin{bmatrix}
K_r & 0 \\
0 & K_v
\end{bmatrix} \begin{bmatrix}
Z_r \\
Z_x
\end{bmatrix} = \begin{bmatrix}
N_r M_r g \\
K_r \dot{x}(t)
\end{bmatrix}
\]

Based on the above methodology, a Matlab program is developed to assemble the motion equations of the vehicle-bridge coupling system and to solve the coupling equations. Eq. (14) can be solved by the Newmark integration method in time domain.

5. Dynamic response sensitivity analysis with respect to both the track and the vehicular parameters

5.1 Sensitivity of response with respect to track parameter

In this paper, the elemental flexural rigidity of the track is chosen as a parameter for track identification. Differentiating both sides of Eq. (24) with respect to the elemental flexural rigidity, of the \(j\)th element, the following differentiation equation can be obtained, named Eq. (15):

\[
\frac{\partial}{\partial EI} \begin{bmatrix}
M_r & N_r M_r \\
0 & M_v
\end{bmatrix} \begin{bmatrix}
\dot{Z}_r \\
\dot{Z}_x
\end{bmatrix} + \frac{\partial}{\partial EI} \begin{bmatrix}
K_r & 0 \\
0 & K_v
\end{bmatrix} \begin{bmatrix}
Z_r \\
Z_x
\end{bmatrix} + \frac{\partial}{\partial EI} \begin{bmatrix}
M_r & N_r M_r \\
0 & M_v
\end{bmatrix} \begin{bmatrix}
\ddot{Z}_r \\
\ddot{Z}_x
\end{bmatrix} = \begin{bmatrix}
\frac{\partial}{\partial EI} \dot{Z}_r \\
\frac{\partial}{\partial EI} \dot{Z}_x
\end{bmatrix}
\]
\[ + \begin{bmatrix} C_r & 0 & -N_r^T C_r & -N_r^T C_r \end{bmatrix} \begin{bmatrix} \frac{\partial Z_r}{\partial (EI')} \\ \frac{\partial Z_r}{\partial (EI')} \\ \frac{\partial Z_r}{\partial (EI')} \\ \frac{\partial Z_r}{\partial (EI')} \end{bmatrix} + \begin{bmatrix} K_r & 0 & -N_r^T K_r & -N_r^T K_r \end{bmatrix} \begin{bmatrix} \frac{\partial Z_r}{\partial (EI')} \\ \frac{\partial Z_r}{\partial (EI')} \\ \frac{\partial Z_r}{\partial (EI')} \\ \frac{\partial Z_r}{\partial (EI')} \end{bmatrix} = \frac{\partial}{\partial EI} \begin{bmatrix} N, M, g \\ K_r (\ddot{x}(t)) \end{bmatrix} \] (15)

It can be seen from Eq. (1), Eq. (3) and Eq. (4) that \(K_r\) and \(C_r\) are dependent of \(EI_j\), and \(M_v, C_v, K_v, N_r, r(x(t))\) are independent of \(EI_j\). In consequence, \(EI_j\) is related to \(K_r\) as Eq. (16):

\[ \frac{\partial C_r}{\partial EI'} = \beta \frac{\partial K_r}{\partial EI'} \] (16)

Eq. (15) can be simplified as Eq. (17):

\[ \begin{bmatrix} M_r & N_r M_r \\ 0 & M_r \end{bmatrix} \begin{bmatrix} \frac{\partial Z_r}{\partial (EI')} \\ \frac{\partial Z_r}{\partial (EI')} \end{bmatrix} + \begin{bmatrix} C_r & 0 & -N_r^T C_r & -N_r^T C_r \end{bmatrix} \begin{bmatrix} \frac{\partial Z_r}{\partial (EI')} \\ \frac{\partial Z_r}{\partial (EI')} \\ \frac{\partial Z_r}{\partial (EI')} \\ \frac{\partial Z_r}{\partial (EI')} \end{bmatrix} + \begin{bmatrix} K_r & 0 & -N_r^T K_r & -N_r^T K_r \end{bmatrix} \begin{bmatrix} \frac{\partial Z_r}{\partial (EI')} \\ \frac{\partial Z_r}{\partial (EI')} \\ \frac{\partial Z_r}{\partial (EI')} \\ \frac{\partial Z_r}{\partial (EI')} \end{bmatrix} = \begin{bmatrix} -\frac{\partial K_r}{\partial (EI')} Z_r - \beta \frac{\partial K_r}{\partial (EI')} Z_r \\ 0 \end{bmatrix} (j = 1, 2, \ldots, N) \] (17)

It can be observed that the displacement response \(Z_r\) and velocity response \(\dot{Z}_r\), obtained from Eq. (14), is the input data for Eq. (17). As a forward problem, the response sensitivities of the nodes can also be obtained from Eq. (17) by the Newmark integration method.

5.2 Sensitivity of response with respect to vehicular parameters

Differentiating both sides of Eq. (14) with respect to the mass, spring and damper parameters of the vehicle, respectively, the following differentiation equation can be obtained as Eq. (18) to Eq. (20):

\[ \begin{bmatrix} M_r & N_r M_r \\ 0 & M_r \end{bmatrix} \begin{bmatrix} \frac{\partial Z_r}{\partial (M')} \\ \frac{\partial Z_r}{\partial (M')} \end{bmatrix} + \begin{bmatrix} C_r & 0 & -N_r^T C_r & -N_r^T C_r \end{bmatrix} \begin{bmatrix} \frac{\partial Z_r}{\partial (M')} \\ \frac{\partial Z_r}{\partial (M')} \\ \frac{\partial Z_r}{\partial (M')} \\ \frac{\partial Z_r}{\partial (M')} \end{bmatrix} + \begin{bmatrix} K_r & 0 & -N_r^T K_r & -N_r^T K_r \end{bmatrix} \begin{bmatrix} \frac{\partial Z_r}{\partial (M')} \\ \frac{\partial Z_r}{\partial (M')} \\ \frac{\partial Z_r}{\partial (M')} \\ \frac{\partial Z_r}{\partial (M')} \end{bmatrix} = \begin{bmatrix} \frac{\partial Z_r}{\partial (M')} \end{bmatrix} \]
Similarly, the response sensitivity with respect to the parameters of the three-parameter vehicle can be obtained from Eqs. (18)-(20).

6. Identification of both the track and vehicular parameters from measured dynamic response

The identification problem is to find the parameters of the track and the vehicle, i.e. \( A = [E_1, E_2, ..., E_n, M_v, K_v, C_v]^T = [A_{E_1}, A_{E_2}]^T \), based on the calculated responses \( R_c \), which best match the measured responses \( R_m \), written as Eq. (21):

\[
\{R_m\} = [Q]\{R_c\} \tag{21}
\]

where the selection matrix \([Q]\) is a constant matrix with elements of zeros or ones, which matches the DOFs corresponding to the measured response components. The inverse problem is to minimize the error between the calculated and measured responses as Eq. (22):

\[
\delta R = \{R_m\} - [Q]\{R_c\} \tag{22}
\]

The identification problem can be solved in the following two steps.

6.1 Sensitivity of response with respect to vehicular parameters
Using the penalty function method (Friswell et al. 1995), the sensitivity equation for parameter identification can be expressed as Eq. (23):

$$[S_v] \times \{ \delta A_v \} = \{ \delta R \} \quad (23)$$

where $\{ \delta A_v \}$ is the perturbation in the vehicular parameters, $[S_v]$ is the time varying response sensitivity matrix, which is the sensitivity of dynamic response with respect to the vehicular parameters. Substituting Eq. (22) to Eq. (23) and rewriting it in full, we have Eq. (24):

$$
\begin{bmatrix}
R_v(1) \\
R_v(2) \\
\vdots \\
R_v(NT)
\end{bmatrix} - [\mathcal{Q}]
\begin{bmatrix}
R_v(1) \\
R_v(2) \\
\vdots \\
R_v(NT)
\end{bmatrix}
= 
\begin{bmatrix}
\frac{\partial R_v(1)}{\partial M} & \frac{\partial R_v(1)}{\partial K} & \frac{\partial R_v(1)}{\partial C} \\
\frac{\partial R_v(2)}{\partial M} & \frac{\partial R_v(2)}{\partial K} & \frac{\partial R_v(2)}{\partial C} \\
\vdots & \vdots & \vdots \\
\frac{\partial R_v(NT)}{\partial M} & \frac{\partial R_v(NT)}{\partial K} & \frac{\partial R_v(NT)}{\partial C}
\end{bmatrix}
\begin{bmatrix}
\delta M \\
\delta K \\
\delta C
\end{bmatrix}
\quad (24)
$$

where $NT$ is the total number of time steps, and to make sure the equation is over-determined, $NT$ should be larger than the number of unknown parameters. Eq. (24) can be solved by least-squares method as follow Eq. (25):

$$\{ \delta A_v \} = ([S_v]^T [S_v])^{-1} [S_v]^T \{ \delta R \} \quad (25)$$

Like many other inverse problems, Eq. (25) is an ill-conditioned system of equations and the solution is unstable. In order to provide bounds to the solution, the damped least-squares method [37] and singular value decomposition technology are used in the pseudo-inverse calculation. Eq. (25) can be rewritten as Eq. (26):

$$\{ \delta A_v \} = ([S_v]^T [S_v] + \lambda I)^{-1} [S_v]^T \{ \delta R \} \quad (26)$$

where $\lambda$ is the non-negative damping coefficient governing the participation of least-squares error in the solution. The solution of Eq. (26) is equivalent to minimizing the function Eq. (27):

$$J(\{ \delta A_v \}, \lambda) = \|S_v \delta A_v - \delta R\|^2 + \lambda \|\delta A_v\|^2$$

(27)
where second term in Eq. (27) gives bounds to the solution. When the parameter \( \lambda \) approaches zero, the estimated vector \( \{\delta A_v\} \) approaches to the solution obtained from the simple least-squares method.

Many methods have been developed to get the damping coefficient \( \lambda \). In this paper, the well-known L-curve method (Tikhonov at el. 1963) is used to determine the optimal regularization parameter \( \lambda \).

Once the increment in the vehicular parameter vector is obtained from Eq. (26), the updated parameters can be expressed as Eq. (28):

\[
\{A_v\} = \{A_v\}_0 + \{\delta A_v\}
\]  

(28)

### 6.2 Identification of track parameter

As long as the vehicular parameters have been obtained from the above, we can conduct structure elemental flexural rigidity identification. Using the penalty function method, the sensitivity equation for damage identification can be express as Eq. (29):

\[
[S_{ei}]{\times}\{\delta A_{ei}\} = \{\delta R\}
\]  

(29)

Similarly, \( \{\delta A_{ei}\} \) is the perturbation in the elemental flexural rigidity vector, \( [S_{ei}] \) is the time varying response sensitivity matrix, which contains the partial derivatives of the dynamic response with respect to the elemental flexural rigidity. For example, at time \( t=t_i \), the sensitivity matrix can be expressed as Eq. (30):

\[
S_{ei}(t = t_i) = \begin{bmatrix}
\frac{\partial R_{11}(t_i)}{\partial EI^1} & \frac{\partial R_{12}(t_i)}{\partial EI^2} & \cdots & \frac{\partial R_{1n}(t_i)}{\partial EI^n} \\
\frac{\partial R_{21}(t_i)}{\partial EI^1} & \frac{\partial R_{22}(t_i)}{\partial EI^2} & \cdots & \frac{\partial R_{2n}(t_i)}{\partial EI^n} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial R_{n1}(t_i)}{\partial EI^1} & \frac{\partial R_{n2}(t_i)}{\partial EI^2} & \cdots & \frac{\partial R_{nn}(t_i)}{\partial EI^n} \\
\frac{\partial R_{NM1}(t_i)}{\partial EI^1} & \frac{\partial R_{NM2}(t_i)}{\partial EI^2} & \cdots & \frac{\partial R_{NMn}(t_i)}{\partial EI^n}
\end{bmatrix}
\]  

(30)

where \( NM \) is the total number of measured points.

Eq. (29) can be solved by the damped least-squares method as follows, expressed as Eq. (31):

\[
\{\delta A_{ei}\} = \left([S_{ei}]^T [S_{ei}] + \lambda I\right)^{-1} [S_{ei}]^T \{\delta R\}
\]  

(30)

The solution of Eq. (30) is equivalent to minimizing the function Eq. (31):

\[
J\left(\{\delta A_{ei}\}, \lambda\right) = \left\| [S_{ei}]\delta A_{ei} - \delta R \right\|^2 + \lambda \left\| \delta A_{ei} \right\|^2
\]  

(31)
The updated element flexural rigidity can be expressed as Eq. (32):

\[
\{ A_{EI} \} = \{ A_{EI} \}_0 + \{ \delta A_{EI} \}
\]  

(31)

6.3 Iteration scheme

In the beginning, the unknown vehicular parameter vector \( \{ A_v \}_0 \) and the set of physical parameter \( \{ (A_{EI})^0 \} \) from the intact model of the track should be assumed.

Step 1: Update both the reference finite element model of the track with the elemental flexural rigidity vector \( \{ (A_{EI})^k \} \) and the vehicular parameter vector \( \{ A_v \}_k \) of the \( k \)th iteration step, to get the dynamic response from Eq. (14) and the response of differentiation equations from Eq. (17) and Eqs. (18)-(20). The error vector \( \{ \delta R \} \) is computed from Eq. (22).

Step 2: Calculate the sensitivity matrix \( [S_v] \) of the response with respect to the different vehicular parameters and sensitivity matrix \( [S_{EI}] \) of the response with respect to the different physical parameters of the rail of the \( k \)th iteration step, to get the vectors \( \{ \delta A_v \} \) and \( \{ \delta A_{EI} \} \).

Step 3: Compute \( \{ A_v \}^{k+1} \) and \( \{ A_{EI} \}^{k+1} \) from Eq. (28) and Eq. (33), respectively.

Step 4: Repeat steps 1 to 3 to get the final value of the set of vehicular parameters vector \( \{ A_v \} \) and the damage index vector \( \{ A_{EI} \} \). The following convergence criterions are used, written as Eq. (34) and Eq. (35):

\[
\left\| \{ (A_v)_{k+1} \} - \{ (A_v)_k \} \right\| \leq \varepsilon_v \tag{34}
\]

\[
\left\| \{ (A_{EI})_{k+1} \} - \{ (A_{EI})_k \} \right\| \leq \varepsilon_{\gamma} \tag{35}
\]

where \( \| \cdot \| \) means the norm of a vector, the tolerance limits \( \varepsilon_v \) and \( \varepsilon_{\gamma} \) are \( 1.0 \times 10^{-4} \) and \( 1.0 \times 10^{-6} \), respectively.

7. Numerical example

7.1 Model layout

A three-span continuous beam with spans of 5m+5m+5m is studied to illustrate the feasibility and efficiency of the proposed system identification method. The finite element model of the continuous beam is shown in Fig. 3. The continuous beam is numerically modelled by 15 elements with each 1m long. The beam has 16 nodes and 43 DOFs in total. The material constants of the beam elements are chosen as: Young’s modulus \( E = 210 \text{Gpa} \), cross sectional area \( A = 0.6 \times 1 \times 1 \text{m} \), density \( \rho = 3500 \text{kg/m}^3 \), and Poisson’s ratio=0.15. The parameters of three-parameter vehicle model are: \( M_v = 3.0 \times 103 \text{kg} \), \( C_v = 1.0 \times 103 \text{Ns/m} \), \( K_v = 6.0 \times 105 \text{N/m} \). In the identification, the speed of the vehicle is 10m/s and the time step in the response calculation is 0.001s. The lower
and upper limit frequencies in Eq. (8), \( \omega_l \) and \( \omega_u \), are \( 2\pi(0.02-2) \text{rad/m} \) which correspond to the irregularity of the track with unevenness wavelength 0.5-50m and \( NN \) takes 2500. The track random irregularity chosen in this paper is shown in Fig. 4.

Fig. 3. Element division of the track.

Fig. 4. The track random irregularity adopted in this paper

7.2 Identification of the elemental flexural rigidity of the track and vehicular parameters

In this paper, the elemental flexural rigidity of the track is chosen as a parameter for track identification. Assuming the bending rigidities of 2ed, 7th, and 11th element have a reduction of 15\%, 20\%, and 30\%, respectively. The initial value of the mass of the vehicle in the identification is taken as \( m^0=2000 \text{kg} \), the initial value of the spring and the damping coefficients are taken as \( k^0=3.5\times10^5 \text{N/m} \) and \( c^0=800 \text{Ns/m} \). All the response data of the railway while the vehicle passes through the track are used in the identification. Three acceleration measurements point 1-ponit 3 as shown in Fig. 3 are used in the identification, where locate at 2ed, 8th, and 13th element, respectively. The true values and calculation values of the vehicular parameters are shown in Table 4, respectively. It can be found that the identified results calculated by the proposed method are close to the real values. The study case indicates the correctness and effectiveness of the proposed method. To examine the accuracy of the identified parameters of the railway, the relative \( EI \) reductions have been calculated. Fig. 5 shows the relative \( EI \) reductions between the identified \( EI \) values and the real \( EI \) values. The main calculated relative \( EI \) reductions is 0.13\%, 0.85\%, and 0.78\% of 2ed, 7th, and 11th element, respectively. This study case illustrates the correctness and effectiveness of the proposed method.
Table 2. Identified results for different cases

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Noise level (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>$M_i$/kg</td>
<td>TV</td>
</tr>
<tr>
<td>3000</td>
<td>2991</td>
</tr>
<tr>
<td>$K_i$/1e5N/m</td>
<td>6.5</td>
</tr>
<tr>
<td>$C_i$/Ns/m</td>
<td>1000</td>
</tr>
</tbody>
</table>

a True value of the parameter. b Identified value of the parameter using the proposed method.

Fig. 5. Identification of multiple damages in the rail (noise free)

7.3 Effect of different noise levels

In the simulations discussed above, extremely accurate results were achieved in ideal situations. Since measurement noise always exists in real tests, the effect of measurement noise was investigated here, and the noise-polluted responses were used to identify the parameters.

The normally distributed random noise is added to the measured response of the track to simulate the measurement noise (Lu et al. 2007), written as Eq. (36):

$$y_m = y_c + e_n N_0 \sigma(y_c)$$

(36)

where $y_m$ and $y_c$ are the polluted response and the calculated one, respectively. $e_n$ is the ratio of the noise amplitude to the response amplitude (between 0 and 1). $N_0$ is the standard normal distribution vector with a mean value of zero and a unit standard
deviation. \( \sigma(y_c) \) is the standard deviation of the calculated response time history, which indicates the deviation of the response from its mean value.

Three different levels of noise, namely 1%, 5% and 10%, were used in the present study. The identification vehicular parameters and their errors are shown in Table 2. From the table it can be seen that the errors for the identified \( M_v \) increase as the noise level increases, and the error reaches a maximum of 1.533% with a noise level of 10%. Similarly, the error for the identified spring stiffness \( K_v \) reaches a maximum of 4.153% with a 10% noise level. The error for the identified damping \( C_v \) reaches a maximum of 4.4% with a 10% noise level. It should be noted that the identified damping is significant even when the noise level is only 1%. It shows the dynamic responses of the rail are not sensitive to the changes in vehicle damping. Fig. 6 shows the relative \( EI \) reduction after 30 iterations between the identified \( EI \) values with different levels of noise and the real \( EI \) values, with a maximum identified error - 0.68%, -1.38%, and -1.92% at element 7 for 1%, 5%, and 10% noise level, respectively. It shows that the proposed method has a good ability of resisting noise.

![Fig. 6. Comparison on the multiple damages identification in the track with different noise levels.](image)

8. Experimental validation

8.1 Description of experimental model

A steel simply supported side-main-side beam and a four wheels car with front and rear axles were constructed in laboratory to verify the proposed method as shown in Fig. 7. The main beam located in the laboratory is 1000mm long with a 150×9mm² uniform cross-section, and the mass density was measured as 7818.52kg/m³. The initial elastic modulus was considered as 200Gpa for all the members. The leading beam made sure that the car velocity was a constant speed, and the tailing beam was used to decelerate the car velocity to zero after the car passed the main beam. The car axle was fixed to the car body with two steel blocks. The distance between the front
wheel and rear wheel was 11.7cm. The distance of the left and right wheel-sets was 7.3cm. The mass of the car body itself was 2.308kg, and a 2.990kg weight was added to the car. Thus the total weight of the car body was \((2.308+2.990) \times 9.8=51.92\)N.

An electric motor was used to control car velocity in the laboratory tests. The motor model is 51K90K-CRF as shown in Figure 8. A traction rope connected the motor and the car, which was keeping the same horizontal line with the centre of gravity of the car in the entire work hours.

Accelerometer DH187 was used in the laboratory tests to record the acceleration responses of a structure. The SINOCERA LC-04A hammer with a rubber tip was utilized in the laboratory tests to produce excitations for structure modal texts. To control the time points that the car got into the main beam and left the main beam, five strain gauges were installed in the main beam, whose distances from the start point of the main beam were 0, 1/4 \(l\), 1/2 \(l\), 3/4 \(l\), \(l\) (\(l\) represented the length of the main beam). The accelerometers and strain gauges were mounted under the main beam, to make sure the car travelled safely. The position illustrations of the accelerometers and strain gauges are shown in Fig. 9. Beside, the number of accelerometers and strain gauges corresponding to the node points of the finite element is shown in Table 3.

The structure was tested in four states: the undamaged state and three damage scenarios, which are given in Table 4. The car-beam system was tested under all beam conditions, corresponding to ‘Case 0’, ‘Case 1’, ‘Case 2’, ‘Case 3’, successively.

![Fig. 7. The experimental beam specimen.](image)
Fig. 8. Moto used in the experimental tests.

Fig. 9. Layout positions of the sensors mounted on the experimental main beam.

Table 3. Layout of the sensors

<table>
<thead>
<tr>
<th>Measured</th>
<th>Channe</th>
<th>Correspo</th>
<th>Senso</th>
<th>Sensitivi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accelerat</td>
<td>3</td>
<td>3</td>
<td>81443</td>
<td>5.027</td>
</tr>
<tr>
<td>Accelerat</td>
<td>5</td>
<td>5</td>
<td>81445</td>
<td>5.259</td>
</tr>
<tr>
<td>Accelerat</td>
<td>7</td>
<td>7</td>
<td>81442</td>
<td>5.374</td>
</tr>
<tr>
<td>Accelerat</td>
<td>9</td>
<td>9</td>
<td>81444</td>
<td>5.142</td>
</tr>
<tr>
<td>Force</td>
<td>1</td>
<td>4</td>
<td>13076</td>
<td>2.93</td>
</tr>
<tr>
<td>Strain</td>
<td>15</td>
<td>1</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>Strain</td>
<td>16</td>
<td>3.5</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>Strain</td>
<td>13</td>
<td>6</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>Strain</td>
<td>14</td>
<td>8.5</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>Strain</td>
<td>12</td>
<td>11</td>
<td>/</td>
<td>/</td>
</tr>
</tbody>
</table>

Table 4. Configuration of the damage scenarios
8.2 Experimental result

Fig. 10 and Fig. 11 are the measured acceleration time history and the measured strain time history under undamaged case respectively. In Fig. 11, the peak value of the strain curve respects the time point when the car travelled through this situation. The time when the front wheel of the car firstly run onto the main beam is $t_1=0.485\text{s}$. Similarly, the time when the back wheel of the car left the main beam is $t_2=2.205\text{s}$. Then the total coupling time of the car and the main beam is $t=t_2-t_1=1.72\text{s}$. The velocity is $v=(L_b+L_v)/(1+0.117)/1.72=0.65\text{m/s}$. $L_b$ and $L_v$ represent the length of the main beam and the distance of the front and rear wheels, respectively. The random irregularity of track vertical profile is adopted as Fig. 12.

![Fig. 10. Measured acceleration time history (Case 0).](image-url)
8.3 Experimental result analysis

8.3.1 First stage for updating the initial intact model

The two-stage model updating method is utilized in the identification of the laboratory-tested car-beam coupling structure. The beam is first updated using the experimental data under undamaged state in order to minimize the discrepancy between the analytical finite element model and the experimental model. Subsequently,
the second stage of the model updating based on the measured acceleration data under damage state by the proposed method is applied in the updated finite element model for both identification of the elemental flexural rigidity of the railway and vehicular parameters.

In the first model updating stage, the elastic modulus of all ten elements as the updating parameters are modified using the experimental acceleration response data under the undamaged state in the first step. The initial finite element model is shown in Fig. 9. The identified relative EI reductions for all the ten elements are shown in Fig. 14. The identified results of experimental vehicular parameter are shown in Table 6. According to these identified parameters, the updated acceleration response can be obtained. The measured dynamic acceleration response, initial dynamic acceleration response and the updated dynamic acceleration response of Node 4 in vertical direction is shown in Fig. 15, respectively. It is shown that the updated acceleration response is much close to the measured acceleration response, which means the updated elastic modulus is much appropriate to model the real situation.

![Updated parameters of elastic modulus after first stage model updating.](image)

**Table 6 Identified results of experimental vehicular parameter**

<table>
<thead>
<tr>
<th>Identified parameter</th>
<th>TV</th>
<th>IV</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M(kg)</td>
<td>5.298</td>
<td>5.285</td>
<td>0.245</td>
</tr>
<tr>
<td>C(Ns/m)</td>
<td>1e5</td>
<td>8.974e2</td>
<td>0.991</td>
</tr>
<tr>
<td>K(N/m)</td>
<td>5e6</td>
<td>4.316e6</td>
<td>0.137</td>
</tr>
</tbody>
</table>
8.3.2 Second stage for identification of both the elemental flexural rigidity of the track and the vehicular parameters

Based on the updated parameters in the first stage model updating, the identification of both the elemental flexural rigidity of the railway and vehicular parameters are performed by the dynamic response sensitivity-based model updating method. Damage scenarios of the experimental beam have been illustrated in Table 4. The cuts at Location 1 and Location 2 in Table 4 are considered as the local damages in Element 3 and Element 8, respectively. According to the displacement-based finite element method [30], the analytical equivalent stiffness reductions of the damaged element are theoretically as 25 percent, and 50 percent for the cut depth of 25mm and 50mm, respectively. One typical cut is demonstrated in Fig. 18.

The experimental model and finite element model of the car-beam coupling system are shown in Fig. 16 and Fig. 17, respectively. The measured acceleration responses of AY(3), AY(5) and AY(7) under all damage configurations (Case 1, Case 2, and Case 3) are used to identify the damages and the vehicular parameter. The sampling frequency is 200Hz. Typically, the acceleration response of AY(5) is shown in Fig. 19.

The parameters of the three-parameter vehicle model are: \( M_v = 5.298 \text{kg}, \) \( C_v = 8.974 \times 2 \text{Ns/m}, \) \( K_v = 4.316 \times 6 \text{N/m}. \) The initial value of the mass, the spring and the damping coefficients are taken as \( M_v^0 = 5 \text{kg}, \) \( C_v^0 = 800 \text{Ns/m}, \) \( K_v^0 = 4 \times 6 \text{N/m}. \) The identified vehicular mass, spring and damping coefficient and their errors are shown in Table 7, respectively. It is noted that the error corresponding to the identified damping reaches 31.77%. This large error could be attributed to the fact that the track
responses are not sensitive to the change in vehicle damping, which means even a large change in vehicle damping will produce only a slight difference in the track response. The relative EI reduction of the laboratory beam for different damage configurations (Case 1, Case 2, and Case 3) are shown in Fig. 20. The equivalent stiffness reduction of Element 3 is, respectively, calculated as about 24.76 percent (Case 1) and 49.53 percent (Case 2) as the depth of the cut increases gradually with $d=25\text{mm}$ (Case 1) and 50mm (Case 2). For Case 3, Element 3 and Element 8 are believed to be damaged with equivalent stiffness reduction of 47.24 percent and 39.18 percent, respectively. Comparing the calculated relative EI reduction to the analytical equivalent stiffness reductions of the damaged element, it is shown that both the elemental flexural rigidity of the railway and the vehicular parameters can be identified accurately for the laboratory beam by the proposed method.

Fig. 16. Configuration of the car-beam coupling specimen (Unit: mm).

Fig. 17. Finite element model of the laboratory beam specimen.
Figure 18. One typical cut of the main beam specimen.

Figure 19. Measured acceleration response time history for AY(5).

Table 7 Identified results of experimental vehicular parameter

<table>
<thead>
<tr>
<th>Identified parameter</th>
<th>TV</th>
<th>IV</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M(kg)</td>
<td>5.285</td>
<td>5.227</td>
<td>1.097</td>
</tr>
<tr>
<td>C(Ns/m)</td>
<td>8.974e2</td>
<td>612.3</td>
<td>31.77</td>
</tr>
<tr>
<td>K(N/m)</td>
<td>4.316e6</td>
<td>3.896e6</td>
<td>9.731</td>
</tr>
</tbody>
</table>
9. Conclusion

This paper proposes an approach making use of the dynamic responses of the vehicle-track coupling system to identify both the elemental flexural rigidity of railway and the vehicular parameters, which is based on the dynamic response sensitivity-based finite element model updating approach. A numerical simply supported beam with multiple local damages is employed to validate the accuracy of the proposed method. The effects of measurement noise have been carried out and the identified results demonstrate that the proposed method is able to identify both the vehicular parameters and the railway parameter very accurately. Afterward, a laboratory test is further carried out to verify the accuracy of the proposed method. Model updating of the intact finite element model is first conducted in the measured modal data in the undamaged state, and the updated model is subsequently used for the identification of both the railway and the vehicular parameters based on the measured accelerations. Based on the results obtained from the case studies and the field testing, the following conclusion can be drawn:
1) The dynamic responses of the coupling system can be used to successfully identify both the elemental flexural rigidity of the railway and the vehicular parameters.
2) The noise has an insignificant effect on the identified mass and spring stiffness of the vehicle. However, it has a significant impact on the identified damping coefficient for the dynamic responses are not sensitive to the change in the vehicle damping coefficient. The proposed method has a good ability of resisting noise to identify the elemental flexural rigidity of the railway.
3) In the field testing, if the total weight of the vehicle is the only concern, the identification process has a good accuracy.

REFERENCES


