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Combinatorial continuous non-stationary critical excitation in M.D.O.F structures using multi-peak envelope functions

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ABSTRACT

The main objective of the critical excitations methods is to generate the worst possible responses of a structure considering the uncertainty of earthquakes which subjected to the applicable constraints, such as earthquake power and intensity. The conventional critical excitation methods attempted to find the critical earthquake that caused the most severe responses. However, the obtained critical excitations were not physically similar to the any recorded earthquakes' excitations. Since considering this problem as an optimization point of the view, it leads to the discontinuous power spectral density (PSD). The main contribution of this paper is to provide a critical excitation method that it has a proper continuity in their frequency domain same as the real earthquake. The main trigger behind the proposed the continuous excitation refers to the linear combination of two continuous functions. Furthermore, in order to provide a non-stationary model, this research also attempts to present an appropriate envelope function that unlike previous envelope functions covers natural earthquake accelerograms at different peaks. Finally, the proposed method was extended into the M.D.O.F structures and the results were verified with other methods.

Keyword: Random Vibrations, Continuous Critical Excitation, Envelope function, Power Spectral Density.

1. INTRODUCTION

Based on the previous knowledge of earthquakes, natural earthquake excitations do not follow any known or distinctive rule for either frequency or time. This explains why an earthquake has random and uncertain characteristics, and why future earthquakes are not perfectly predictable. However the static and modal seismic designs of structures are still based on the design spectrums generated based on previous earthquakes.

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In order to estimate the worst possible critical response of the structure critical excitation methods, recently, has been developed. It is believed that by considering the some properties of earthquake, such as power and intensity, the maximum response of the structure could be evaluated.

The first researcher who introduced the concept of critical excitation was Papoulis (1967) he used this concept in electrical engineering field. Then, Drenick (1970) applied the critical excitation method for structures in a given time period. In Drenick*'s* method, the most destructive excitation was found based on the maximum response of the system. In same year, Shinozuka (1970) expanded this idea into the frequency domain and offered a narrower upper bound of the maximum response.

In fact, the critical excitation method is an optimization problem to maximize structural response as an objective function subject to the constraints. Until now, many people have used different constraints and objective functions. Iyengar (1972), Manohar and Sarkar (1995, 1998) and Takewaki (2001a, 2001b) deliberately extended the method to stochastic problems to consider the uncertainty of earthquakes. Also, Ben-Haim and Elishakoff (1990), and Pantelides and Tzan (1996) presented several interesting convex models. Ashtari (2006, 2004) used wavelet theory to generate artificial earthquake excitations and introduced more excitations by the linear combination of resultant artificial earthquakes to obtain critical excitation.

Furthermore, recently, Takewaki (2007, 2012) developed a new optimization problem for frequency to find the critical excitation by considering the response resonance of higher modes of structures with input excitation. His proposed constraints were the power limit (area under power spectral density (PSD) function) and the intensity limit (magnitude of PSD function).

In 2006, Ashtari introduced "Optimum Line" as a geometric technique for solving stationary optimization problems. Then, an exact method for SDOF systems and a simple numerical technique were proposed to find stationary critical excitation for MDOF systems. Then, Ashtari proposed a more realistic expression of critical excitation (Ashtari (2004, 2006)). Following those researches, an exact method for SDOF systems and a simple numerical technique were proposed to determine the stationary critical excitation for MDOF systems. Finally, Ashtari and Ghasemi proposed a more realistic expression of critical excitation (Ashtari and Ghasemi (2010a, 2010b, 2013)).

Furthermore, by using critical excitation concept Moustafa and Takewaki (2012) introduced a method to improve the earthquake resilience of buildings.

Then, regards to the damage indices, Moustafa et al. (2010) introduced a critical excitation modeling for an inelastic structure. Afterward, Moustafa and Takewaki (2009) with respect to the probability measurements and entropy idea, they presented a method to obtain resonance records. Nevertheless, it is worth stating that concerning to the Fujita et al. (2010) research, by finding optimal placement of viscoelastic dampers the objective drifts function of a linear structure can be more minimized. Finally, according to the drift and input energy demands, Takewaki (2011) scaled ground motions for designing tall buildings. However, with respect to the previously proposed critical excitations, it is clear that those obtained critical domains are not able to fully cover the earthquake's probable frequency range of power spectral density, and they only have been concentrated on the main frequency domain of the structures. Based on the maximization problem, this issue seems to be acceptable; however, the achieved power spectral density function is not like the power spectral density of a real earthquake. Hence, this research is designed to find a more realistic critical input excitation. Consequently, it will propose a continuous critical excitation, which is more similar to natural earthquakes than previous methods.

In this approach, we first discuss on the theory of random vibrations and then compare the structural response with different critical excitation methods. This paper will consider the frequency domain due to the maximization the objective function of the mean square of the story drifts.

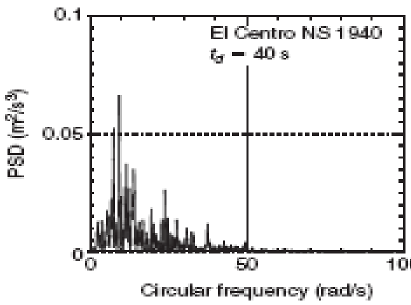

Fig. 1-a El-centro PSD function (Takewaki)2007))

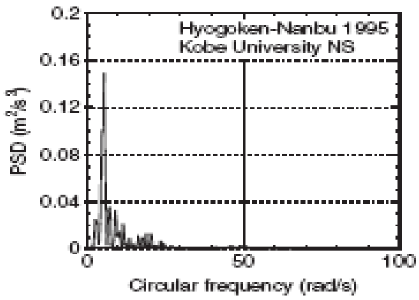

Fig. 1-b Hyogoken-Nanbu PSD function (Takewaki, 2007)

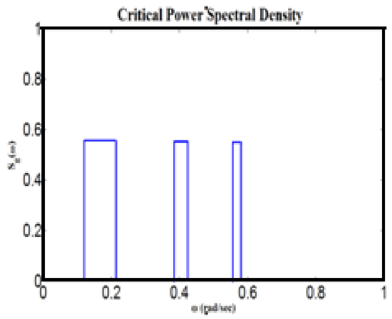

Fig. 1-c PSD function of Takewaki's method

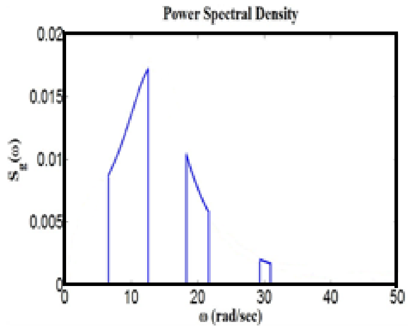

Fig. 1-d PSD function of Ashtari's method

Fig. 1 shows the power spectral density of El-Centro earthquake (Fig. 1-a) and

Kobe earthquake (1-b). As it can be seen those natural earthquake ground motions continuously cover a specific frequency range. Nevertheless, pervious proposed critical excitations like Takewaki (Fig. 1-c) and Ashtari (Fig.1-d) methods suffer discontinuities in their power spectral density; therefore, these artificial critical excitations are not likely probable.

This paper intends to introduce a new method to attain more realistic critical excitations. Hence, the power spectral density (PSD) of the excitation is assumed to be a linear combination of two functions known as Kanai-Tajimi (1957, 1960) function and the square of the dynamic response function ($F(\omega)$) . The Kanai-Tajimi function is a spectral description of ground motion, and $F(\omega)$ depends on the dynamic characteristics of the structure. Both of these functions are continuous. Therefore, the combinatorial PSD of them leads to a continuous critical excitation. The optimization problem will be solved using the Lagrangian method. Moreover, for considering the non-stationary modeling of earthquakes several envelope functions will be introduced, which unlike the previous envelope function, will be able to follow the earthquakes accelerograms at various times in a good shapes. Since, the previous envelope functions are not accurate enough to describe severe earthquake's movements occurring at the final moments of the earthquakes. The proposed envelope functions in this paper do not have this limitation and can better represent the earthquake movment. The method is simple and simulates natural earthquakes using mathematical expressions.

This technique is applied to many multi-degree-of-freedom models that have different natural periods. Finally, the results are compared with the previous methods separately proposed by Ashtari [13] and Takewaki (2007).

2. NON-STATIONARY CRITICAL EXCITATION THEORY

According to the theory of random vibration, the sum of the mean square of story-drifts can be defined as

$$f(t) = \sum_{k=1}^{n} \sigma_{Dk}(t)^2 = \int_{-\infty}^{\infty} H_M(t;\omega) S_w(\omega) d\omega \tag{1}$$

where $S_w(\omega)$ denote the PSD function of a stationary Gaussian process with zero mean $(w(t))$, and $H_M(t;\omega)$ can be expressed by:

$$H_M(t;\omega) \sum_{k=1}^{n} \left[\left\{ \sum_{j=1}^{n} \Gamma_j \left(\varphi_k{}^{(j)} - \varphi_{k-1}{}^{(j)} \right) A_{Cj}(t;\omega) \right\}^2 + \left\{ \sum_{j=1}^{n} \Gamma_j \left(\varphi_k{}^{(j)} - \varphi_{k-1}{}^{(j)} \right) A_{Sj}(t;\omega) \right\}^2 \right] \tag{2}$$

where Γ_j is the j^{th} participation factor, $\varphi_k{}^{(j)}$ is the k th component in the j^{th} eigenvector $\varphi^{(j)}$, and A_{Cj}, and A_{Sj} are respectively equal to:

$$\Gamma_j = \frac{\sum_{i=1}^{n} m_i \varphi_i{}^{(j)}}{\sum_{i=1}^{n} m_i {\varphi_i{}^{(j)}}^2} \tag{3}$$

$$A_{Cj}(t;\omega) = \int_0^t e(\tau) g_j(t-\tau) cos\omega\tau d\tau \tag{4}$$

$$A_{Sj}(t;\omega) = \int_0^t e(\tau) g_j(t-\tau) sin\omega\tau d\tau \tag{5}$$

which $e(t)$ is given deterministic envelope function and $g_j(t) = H_e(t) \left(1/\omega_{dj} \right) e^{-\xi\omega_j t} \sin(\omega_{dj} t)$ ($H_e(t)$ is the Heaviside step function, which is $\omega_{dj} = \sqrt{1-\xi^2}\omega_j$ in Equation (4) and Equation (5))

2.1 Critical excitation optimization problem

Considering these equations, problem of non-stationary critical excitation consists of double maximization procedures that can be described mathematically by:

$$Max_{S_w(\omega)} Max_t \{ f(t; S_w(\omega)) \} \tag{6}$$

subjected to

$$\int_{-\infty}^{\infty} S_w(\omega) d\omega \le \bar{S}_w \qquad (\bar{S}_w: given\ power\ limit) \tag{7}$$

and,

$$\sup \mathrm{S_w}(\omega) \le \bar{s}_w \qquad (\bar{s}_w: given\ PSD\ amplitude\ limit) \tag{8}$$

The first maximization of Equation (6), which is performed with respect to time for the given PSD function, which it can be solved by utilizing the critical excitation method as a stationary input. The second maximization procedure can be achieved with respect to time.

2.2 Continuous critical excitation

It is obvious that if the PSD function is the Dirac's delta function at the peak of $H_M(t;\omega)$ without any amplitude limit, the structural response will be maximized. However, because of amplitude limitation the excitation will generate the rectangle(s) as high as $\bar{s}_w$, and the power will be determined the frequency range $\Delta\omega$.

To solve this problem and achieve a continuous spectral density excitation, a linear combination of $F(\omega)$ and the Kanai-Tajimi function will consider providing a continuous critical excitation.

$$S_g(\omega) = \propto F(\omega) + \beta S_{K.T.}(\omega) \tag{9}$$

where $F(\omega) = |H(\omega)|^2$ indicates the dynamic properties of structure and $S_{K.T.}(\omega)$ represents the power spectral density of the ground motion of the previous natural earthquakes:

$$S_{K.T.}(\omega) = S_0 \frac{{\omega_g}^4 + 4{\xi_g}^2{\omega_g}^2\omega^2}{\left(\omega^2 - {\omega_g}^2\right)^2 + 4{\xi_g}^2{\omega_g}^2\omega^2} \tag{10}$$

where ω_g is the fundamental frequency of the ground's motion, ξ_g is the ground damping which represents the sharpness of the power spectral density function $S_{K.T.}(\omega)$, and S_0 is a white noise constant parameter. By considering β coefficient, it is not necessary to obtain S_0 parameter precisely.

At *ω=0* the Kanai-Tajimi function goes to S_0. Therefore, the equation (11) must be filtered to achieve $S = 0$ at $\omega = 0$ to get similar to the natural earthquake. To do so, here is going to just use the Lai's filter.

$$S_{K.T-L}(\omega) = S_0 \frac{{\omega_g}^4 + 4{\xi_g}^2{\omega_g}^2\omega^2}{\left(\omega^2 - {\omega_g}^2\right)^2 + 4{\xi_g}^2{\omega_g}^2\omega^2} \left(\frac{\omega^2}{\omega^2 + {\omega_c}^2}\right) \tag{11}$$

where ω_c is a variable that determines the low-frequency content of ground motion.

In this study, equation (12) has been presented to find the continuous critical excitation, which respect to the constraints of the earthquake such as intensity, and power α and β will be achieved by solving two-unknown, two-equation system.

$$\begin{cases} \bar{S} = \int_{-\infty}^{+\infty} \propto F(\omega)d\omega + \int_{-\infty}^{+\infty} \beta(S_{K.T.}(\omega))d\omega \\ \bar{s} = Max[\propto F(\omega) + \beta S_{K.T.}(\omega)] \end{cases} \tag{12}$$

It should be noted that in the second equation above, $\bar{s}$ will be obtained at $\omega = \omega_n$ or around this area, where ω_n is equal to the fundamental natural frequency of the structure.

It is clear that if the ω_g of ground motion is closed to ω_n , a stronger critical excitation will be generated; however, it is rare for ω_g to be equal to ω_n. Therefore, these excitations are denoted the very rare events which is called as a rare continuous critical excitations. Thus, substituting $\omega_g = \omega_n$ in the Kanai-Tajimi equation, Equation (11) becomes (13) to describe the rare continuous critical excitation. The equation (12) would be a solution for assumed critical excitation in form equation (9), which this solution just presented as mathematical interpretation without any mathematically optimization effort, therefore we called the equation (12) as simplified solution.

$$S_{K.T-L}^{Rare}(\omega) = S_0 \frac{{\omega_n}^4 + 4{\xi_n}^2{\omega_n}^2\omega^2}{(\omega^2 - {\omega_n}^2)^2 + 4{\xi_n}^2{\omega_n}^2\omega^2} \left(\frac{\omega^2}{\omega^2 + {\omega_c}^2}\right) \tag{13}$$

$S_{K.T-L}^{Rare}(\omega)$ is the PSD of the rare critical continuous excitation using Lai's filtering. In the proposed method, the fluctuations of earthquake excitation in frequency domain can be simulated by multiplying the absolute value of the sine function by the second part of equation (9):

$$S_g(\omega) = \propto F(\omega) + \beta(S^n_{K.T-L}(\omega))|\sin(\omega)| \qquad (14)$$

This sine function generates an artificial critical earthquake which more similar to the critical earthquakes' excitation.

2.3 Solving the continuous critical excitation problem using the lagrangian method

In this section, by using the Lagrangian optimization method the continuous critical excitation problem is solved and the results are compared with the outcome of the equation (12). The given conditions of this problem for multi-degree-of-freedom structures are:

$$Max:\ f = \sum_{i=1}^{n} {\delta_{Di}}^2 = \int_{-\infty}^{\infty} F(\omega) S_g(\omega) d\omega \qquad (15)$$

subjected to

$$\int_{-\infty}^{\infty} S_g(\omega) d\omega \leq \bar{S} \qquad (16)$$

$$S_g(\omega) \leq \bar{s} \qquad (17)$$

Choosing the following functions for the Lagrangian method leads to:

$$f = \sum_{i=1}^{n} {\delta_{Di}}^2 = \int_{-\infty}^{\infty} F(\omega) S_g(\omega) d\omega \qquad (18)$$

$$g = \int_{-\infty}^{\infty} S_g(\omega) d\omega - \bar{S} \qquad (19)$$

$$h = S_g(\omega) - \bar{s} \qquad (20)$$

By substituting Equation (9) into the three above equations can be written as:

$$f = \sum_{i=1}^{n} {\sigma_{Di}}^2 = \int_{-\infty}^{\infty} F(\omega)(S^n_{K.T-L}(\omega))\, d\omega \qquad (21)$$

$$g = \int_{-\infty}^{\infty} \{\propto F(\omega) + \beta\left(S^n_{K.T-L}(\omega)\right)\} d\omega - \bar{S} \qquad (22)$$

$$h = \{\propto F(\omega) + \beta\left(S^n_{K.T-L}(\omega)\right)\}_{Max} - \bar{s} \qquad (23)$$

Equation (21-23) can be reduced to the following forms.

$$f = \propto \mathrm{c}_1 + \beta \mathrm{c}_2 \qquad (24)$$

$$g = \propto \mathrm{c}_3 + \beta \mathrm{c}_4 - \bar{S}$$

(25) $h = \propto \mathrm{c}_5 + \beta \mathrm{c}_6 - \bar{s}$ (26)

where c_i constitutes constant values. Using the Lagrange derivation lead to

$$\begin{cases} \frac{\partial f}{\partial \alpha} - (\lambda \frac{\partial g}{\partial \alpha} + \gamma \frac{\partial h}{\partial \alpha}) = 0 \\ \frac{\partial f}{\partial \beta} - (\lambda \frac{\partial g}{\partial \beta} + \gamma \frac{\partial h}{\partial \beta}) = 0 \end{cases} \qquad (27)$$

By substituting c_i's the above equations can be simply expressed as

$$\begin{cases} \mathrm{c}_1 - \mathrm{c}_3\lambda - \mathrm{c}_5\gamma = 0 \\ \mathrm{c}_2 - \mathrm{c}_4\lambda - \mathrm{c}_6\gamma = 0 \end{cases} \qquad (28)$$

By solving the two-unknown, two-equation system above, λ and γ can be found. These two equations are independent of $\propto$ and β. The coefficients $\propto$ and β are only calculated based on the boundary conditions and problem constrains, exactly like the simplified method proposed in the equation (12).

$$\begin{cases} \bar{S} = \int_{-\infty}^{+\infty} \propto F(\omega) d\omega + \int_{-\infty}^{+\infty} \beta S^n_{K.T-L}(\omega) d\omega \\ \bar{s} = \{\propto F(\omega) + \beta(S^n_{K.T-L}(\omega))\}_{Max} \end{cases} \qquad (29)$$

This Equation (29) shows that the Lagrangian optimization problem is independent of Lagrangian coefficients. Therefore, it means we still can use the equation (12) to find the continuous critical excitation.

3. SEVERAL ENVELOPE FUNCTIONS PROPOSED FOR NON-STATIONARY MODELING OF AN EARTHQUAKE

Envelope functions represent non-stationary behavior of the earthquake. BY selecting a simple mathematical an envelope function leads to a better simulation of the earthquake phenomena. In this section, three envelope functions will be presented which they are have ability to cover more than one peaks of the accelerogram of any earthquake. Indeed, ditch effort of this section is that to present new multi-peaks envelope functions, in order to provide more realistic behavior of actual earthquakes in term of general shape of earthquakes' accelerograms.

3.1 The first proposed envelope function

From the basic concept of functional shape investigation, it is clear that following term ($\frac{c_i}{(t^2 - t_{Max}{}^2)^2}$) can cause a peck at t_{Max}. Additionally base on the power series definition, the summation of many functions which are positive can consider all maximum points in one formula. It is pretty obvious that any accelerograms consists of summation several pecks at different moments. Hence, with power series believes it would be possible to model any earthquake's accelerogram. As matter of fact the similarity of the frequency response function (if the damping term is ignored) and this selected generator function inspires us to propose such envelope function.

$$e(t) = \sum_{i=1}^{n} \frac{c_i}{(t^2 - t_{iMax}{}^2)^2} \tag{30}$$

where n is the number of maximum points in the natural earthquake, t_{iMax} is the time of the desired peak point, and c_i's are constant values depending on the accelerogram pattern. Obviously, if the number of maximum points increases, the overall shape of the envelope function in equation (15) becomes more similar to the accelerogram of a natural earthquake. One of the advantages of this shape is that controlling on desired accuracy. The other advantages are that it specifies accurately the main points in the accelerogram of a natural earthquake, and that the non-stationary nature of the earthquake can be precisely displayed with respect to vibration amplitude and the duration of severe vibrations. Also, unlike the most famous envelope functions such as Bolotin's envelope function, which can merely display one maximum point, the presented envelope function has the ability to show several peak points at any desired time. Therefore, this new model, which proposed here, can be regarded as decent multi-peak envelope function.

3.2 The second proposed envelope function

As mentioned in last section, most of conventional envelope functions such as the Bolotin's envelope function could not cover severe excitations of the accelerogram at the final moment of ground motion. In order to overcome on this issue another new envelope function is presented which established based on a single harmonic wave. For this purpose, the severe excitations are cover by the Bolotin's function, however, since the decreasing rate of exponential function is very dramatic; thanks have sine wave we can consider other severe excitation at very end of the motion.

$$e(t) = \begin{cases} (e_o.e^{-at} - e_1.e^{-bt}), & 0 \leq t < t_o \\ e_2.\sin\left(\frac{2\pi t}{c.T}\right) & , \ t_o \leq t < T \end{cases} \tag{31}$$

where T is the total duration of a natural earthquake, t_o is the desired time depending on the natural earthquake (it can be achieve by trial and error effort to find best adjustment), e_o and e_1 and e_2 are constant coefficients representing the shape of the envelope function, and *a* and *b* and *c* are constant values representing the position of the maximum points in earthquake excitation.

Two important properties of this envelope function, which the Bolotin's function suffers from lack of those, are as follows: the first issue is related to the initial moment modeling and the second one is referred to its ability for modeling the final movements of the accelerogram's motions. Therefore, this new type of proposed multi-peak envelope function can be utilized as more exact function comparison with the Bolotin's function, thus, for more certification of these advantages this method is investigated in the next section.

3.3The third proposed envelope function

The previous two functions are sufficiently flexible for creating several maximum points. In point of view for showing better distribution of earthquake fluctuations, and illustrating the non-stationary nature of an earthquake this third envelope function is introduced. Nevertheless, they are rather complicated than last two ones. It is worth mentioning the inspiring function for such envelope function stem from the similarity between the hearts beat in cardiograph and functional behavior of beat function. Therefore, the third proposed envelope function was based on beat function.

$$e(t) = \left| e_o.\sin(\left(\frac{2\pi}{b.T} + c\right).t) + e_1.\sin\left(\frac{2\pi.t}{b.T}\right) + d \right| \tag{32}$$

One of the magnificent of this function is that ability to model several peaks of earthquake's accelerogram. However, it has an issue which the amplitude of beat function is constant during in test sampling, therefore, to solve this problem of same heightens of peaks, partition the range of function would be recommended, it would be possible regarding to the amount of desired number of peaks and changing the values of constant parameters of e_o and e_1. Thus, it can be given as one of the most flexible multi-peak envelope function.

4. COMPARISON OF THE RESULTS OF THE COMBINATORIAL CONTINUOUS CRITICAL EXCITATION WITH PREVIOUS DISCONTINUOUS CRITICAL EXCITATION

This section examines on the results of non-stationary continuous critical excitation compared with discontinuous critical excitations found in the previous studies of Takewaki (2007) and Ashtari (2004). The effects of different envelope functions are investigated to find continuous critical excitation.

For this purpose, a two-degree-of-freedom structure with the following mass, and stiffness values with a damping ratio of 0.02 was subjected to the El-Centro earthquake.

$k_1 = 1.2 \times 10^6\ KN, \qquad k_2 = 6 \times 10^5\ KN.$

$m_1 = 6 \times 10^3\ Ton, \qquad m_2 = 3 \times 10^3\ Ton.$

Fig. 2 shows the real accelerogram of the El-Centro earthquake whose peak acceleration (PGA) was scaled to $0.3g$.

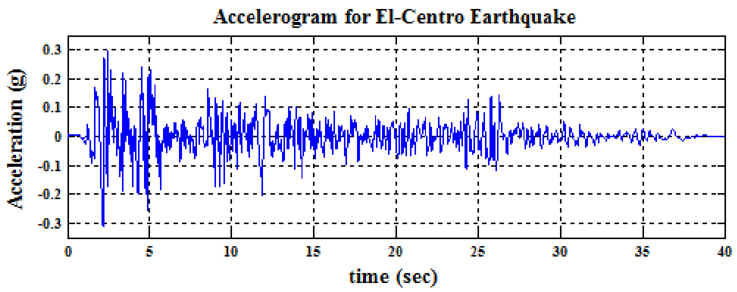

Fig. 2 Scaled accelerogram of El-Centro earthquake

Fig. 2 shows the Bolotin envelope function whose equation can be written as:

$$e(t) = e^{-0.14t} - e^{-0.33t} \tag{33}$$

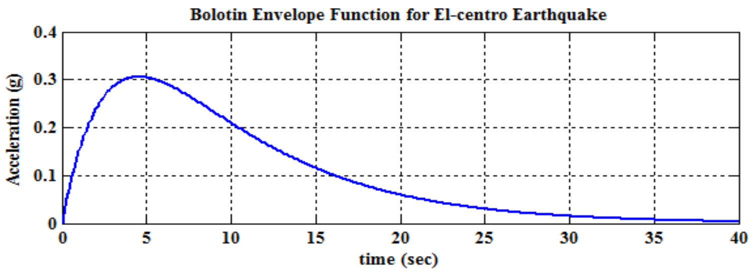

Fig. 3 Bolotin's envelope function

As shown in Fig. 2 and Fig. 3, Bolotin's function does not follow the final peaks like the El-Centro earthquake after $t = 20$ seconds. It should also be noted that the function is conservative during the initial moments. Fig. 3 shows the second proposed envelope function based on Equation (34).

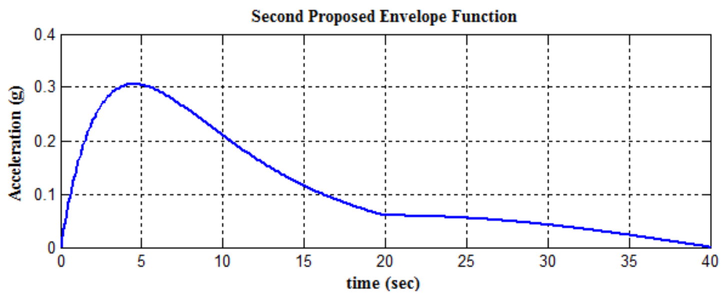

Fig. 4 The second proposed envelope function of scaled El-Centro earthquake

$$e(t) = \begin{cases} (e^{-0.14t} - e^{-0.33t}) & , \ 0 \le t < 20 \\ 0.0594 \sin\left(\frac{2\pi.t}{80}\right) & , \ 20 \le t < 40 \end{cases} \tag{34}$$

where e_2 was achieved at $t = 20$ second.

$$a \sin\left(\frac{2\pi * 20}{80}\right) = (e^{-0.14*20} - e^{-0.33*20}) \ \Rightarrow \ a = 0.0594 \tag{35}$$

The third proposed envelope function can be obtained as follows (see Fig. 5),

$$f(t) = \begin{cases} e(t) = 0.15\left(\sin\left(\left(\frac{2\pi}{1}+1\right).t\right) + \sin\left(\frac{2\pi.t}{1}\right)\right), & 0 < t < 15 \\ e(t) = 0.05\left(\sin\left(\left(\frac{2\pi}{1}+1\right).t\right) + \sin\left(\frac{2\pi.t}{1}\right)\right), & 15 < t < 23 \\ e(t) = 0.075\left(\sin\left(\left(\frac{2\pi}{1}+1\right).t\right) + \sin\left(\frac{2\pi.t}{1}\right)\right), & 23 < t < 30 \\ e(t) = 0.01\left(\sin\left(\left(\frac{2\pi}{1}+1\right).t\right) + \sin\left(\frac{2\pi.t}{1}\right)\right), & 30 < t < 40 \end{cases} \tag{36}$$

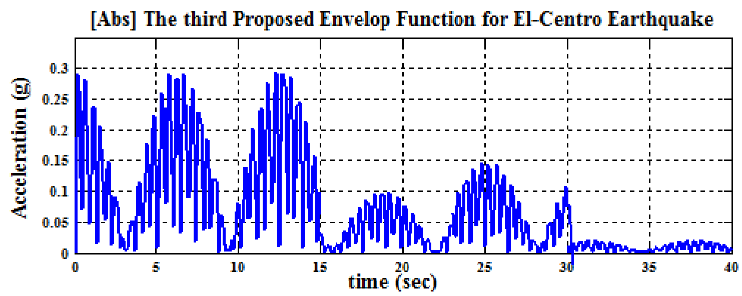

Fig. 5 Aabsolute value of envelope function of Equation (36)

Fig. 6 The frequency response function of the structures based on the different type of envelope functions.

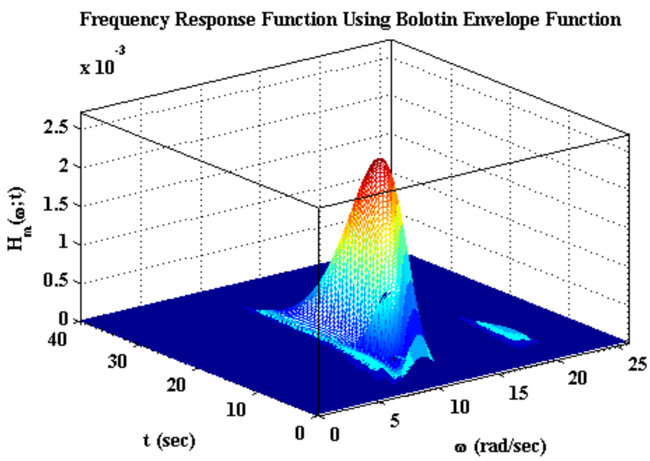

Fig. 6-a The frequency response function of the structure using the Bolotin's envelope function

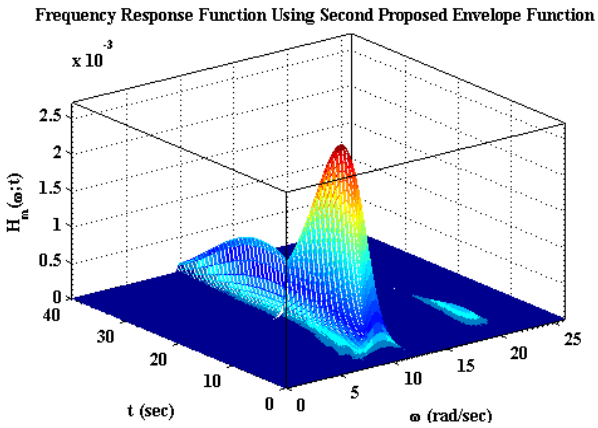

Fig. 6-b The frequency response function using envelope function of Equation (34)

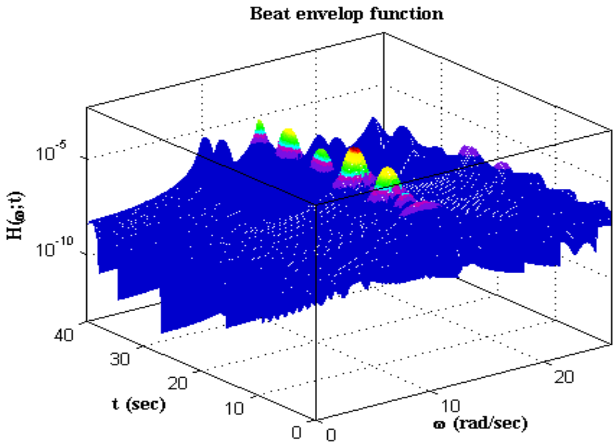

Fig. 6-c The frequency response function of the envelope function of Equation (36)

Fig. 6 The frequency response function using the Bolotin's envelope function and the proposed envelope functions

Due to the envelope functions proposed in this article more reliable response can be obtained, especially at the final moments, of an earthquake. As can be seen, Fig. 6 shows three different types of frequency response functions. Form just graphical manifestation of those, the advantages and disadvantages of those are recognizable. As matter of fact, Fig. 6-a shows the frequency response of the structure based on the Bolotin's envelope function. However, due to observe the effects of envelope function on response function($H_M(t;\omega)$), the proposed envelope functions were examined; the second proposed envelope function covers the motion at the end of ground movements and it decently includes two main pecks, the third proposed envelope function (Fig. 6-c) by utilizing new multi-peak presents more real shape of frequency response function however it cannot be exactly categorized as envelope function, because of some fluctuations. Therefore, the second one (Fig. 6-b) could be considered as most flexible and coverable envelope function.

Table (1) compares the results of conventional work with the results obtained by presented combinatorial continuous non-stationary critical excitation using the proposed envelope functions.

Case	Method	$S(\omega)$	Envelope function	t max (sec)	Response (cm)
No. 1	Ashtari	Ashtari Method	Bolotin	8	9.23
No. 2	Rare continuous critical excitation	$S_g(\omega) = \propto F(\omega) + \beta S_{K.T.}(\omega)$	Bolotin	9.5	10.76
No. 3	Rare real continuous critical excitation	$S_g(\omega) = \propto F(\omega) + \beta S_{K.T.}(\omega)\lvert\sin(\omega)\rvert$	Bolotin	9.5	7.85
No. 4	Rare continuous critical excitation	$S_g(\omega) = \propto F(\omega) + \beta S_{K.T.}(\omega)$	Beat: sum(sin)	7.5	9.31
No. 5	Rare real continuous critical excitation	$S_g(\omega) = \propto F(\omega) + \beta S_{K.T.}(\omega)\lvert\sin(\omega)\rvert$	Beat: sum(sin)	7.5	6.82
No. 6	Continuous critical excitation	$S_g(\omega) = \propto F(\omega) + \beta S_{K.T.}(\omega)$	Bolotin	9.5	6.65
No. 7	Real continuous critical excitation	$S_g(\omega) = \propto F(\omega) + \beta S_{K.T.}(\omega)\lvert\sin(\omega)\rvert$	Bolotin	9.5	4.99
No. 8	Continuous critical excitation	$S_g(\omega) = \propto F(\omega) + \beta S_{K.T.}(\omega)$	Beat: sum(sin)	7.5	4.93
No. 9	Real continuous critical excitation	$S_g(\omega) = \propto F(\omega) + \beta S_{K.T.}(\omega)\lvert\sin(\omega)\rvert$	Beat: sum(sin)	7.5	3.74
No.10	Takewaki	Takewaki	Bolotin	8	10.89

Table 1 - Comparison of responses of different critical excitation methods

As it is shown, rare continuous critical excitation methods, apart as Takewaki (2002, 2007) critical excitation method, it can generate more critical responses comparison with the other method. However, other proposed methods have thought-provoking advantage for presenting more reality PSD function. For instance when we utilize $S_g(\omega) = \propto F(\omega) + \beta S_{K.T.}(\omega)\lvert\sin(\omega)\rvert$ as critical PSD we can see one of the distinguish chrematistics of natural fact which is fluctuation trend, but for attaining either critical and more reality PSD which have critical response with near natural PSD method that known as Real Rear Continuous Critical Excitation in this paper might be appropriable one.

Fig. 6 shows PSD of different proposed critical excitation. As shown in Fig. 6, all of the critical power spectral density functions are continuous and more like a natural earthquake. Therefore, the critical responses are more likely to be probable.

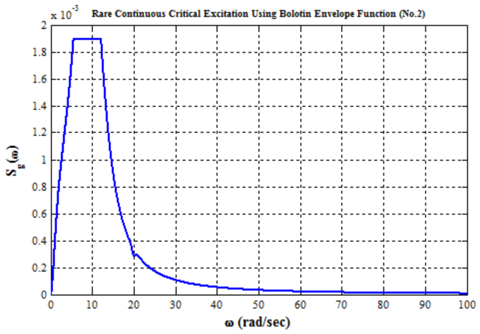

Fig. 7-a Case No. 2

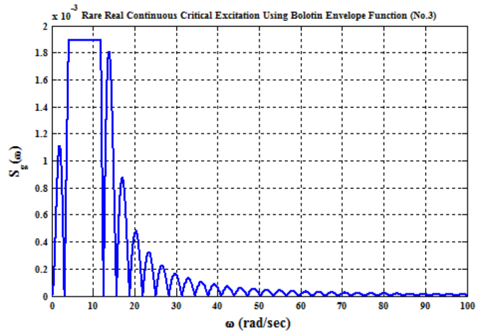

Fig. 7-b Case No.3

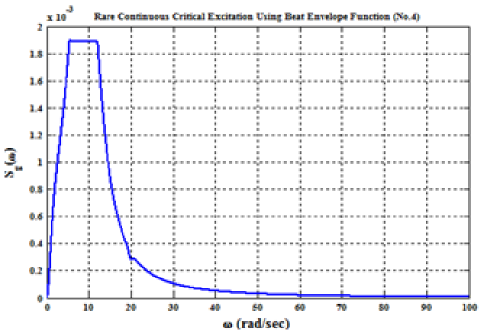

Fig. 7-c Case No.4

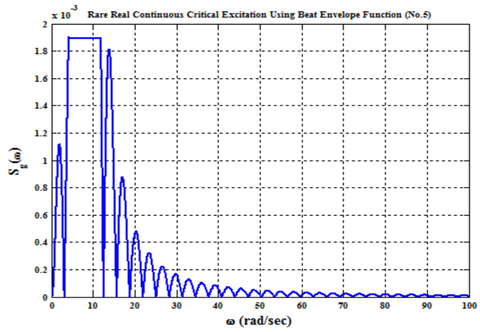

Fig. 7-d Case No.5

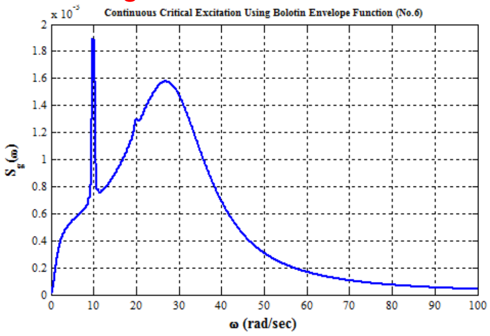

Fig. 7-c Case No.4

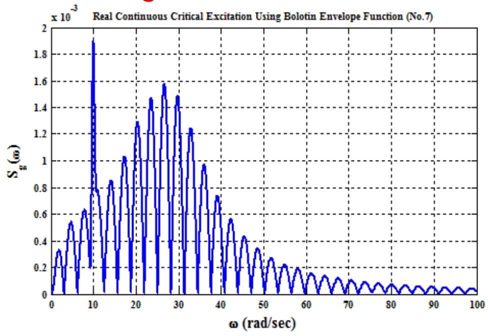

Fig. 7-d Case No.5

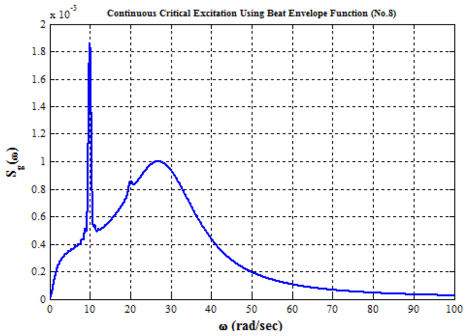

Fig. 7-c Case No.4

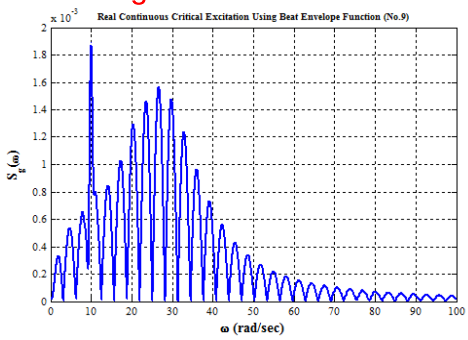

Fig. 7-d Case No.5

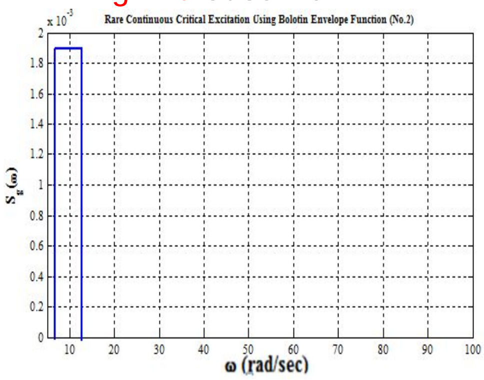

Fig. 7-c Case No.4

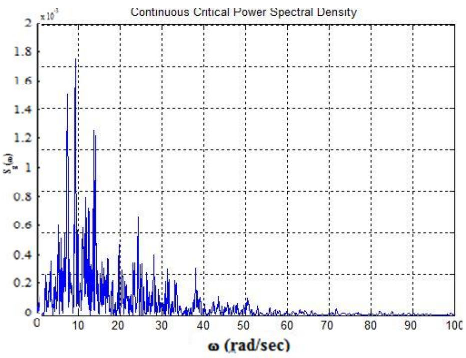

Fig. 7-d Case No.5

Fig. 7 Critical power spectral density of different proposed methods (see Table 1)

Fig. 7 shows the comparison among all methods (see Table 1), rare continuous critical excitation coerces extra congestion near natural frequency of the structure which concludes that the structural inter-story drift dramatically increases However, the most significant advantage of proposed method “combinatorial continuous critical excitation” is that all of the produced PSDs unlike previous critical excitation they obviously are continuous in their frequency domain. At the first glance, the discrete PSD does not reckon as a significant issue especially in scope of critical excitation, nonetheless, it is a clear axiom that the possibility of occurrence of earthquake with discrete spectral density is equal to zero, because there is no recorded earthquake with discrete spectral density function. Therefore, the absence of continuous method is felt.

Another outcome of Fig. 7 can be the tangible fluctuations in PSD, which is related to the behavior of the attendance of harmonic function in proposed continuous critical excitation, (terms sin(x)) $S_g(\omega) = \propto F(\omega) + \beta S_{K.T.}(\omega)|\sin(\omega)|$, thus the shape of PSD tends to the actual PSD.

And final fact which may be concealed in Fig. 7, is that multi-peak envelope functions have little reduction effects on final response of the structure. The reason of this reduction was predictable, because the Bolotin envelop function is doing overestimation of earthquake motion in time domain, Therefore, the maximum response in this especial example tends to time of the maximum acceleration of El-Centro, for instance the maximum response occurs at 9.5 sec for Bolotin envelop function, the maximum response for second proposed function happens at 7.5 sec and the maximum acceleration of El-Centro happened about 2 sec (see Table 1).

5. CONCLUSIONS

The main objective of this paper was an introduction of new generation of critical excitation of earthquakes. The main controversial side of conventional critical excitation is that they are suffering the abruptly discontinuity in their density functions. Hence, this article proposed a combinatorial continuous non-stationary critical excitation for multi-degree-of-freedom structures. As it can be seen, this method was established base on a linear combination of the square frequency of the response function ($F(\omega)$), and the Kanai-Tajimi spectral density. Then for calculating the maximum response of the structure, the structure was subjected to the two pivotal constraints of the earthquake such as intensity and power limits. The main purpose of generating continuous critical excitation stems from the probability of occurrence of the critical earthquake. Therefore, for attaining critical excitation of the earthquake which is more similar to the actual excitation of the critical earthquake the continuous critical excitation was proposed. Then for finding more flexible continuous critical excitation several types of continuous critical excitations were investigated.

From examination of some possible cases of the continuous critical excitation some feasible cases were introduced. For instance to reach more critical responses, rare continuous critical excitation was suggested.

Also, in the non- stationary condition this paper proposed three new envelope functions that could simulate severe motions of any earthquake at the final or any moments. In other words, they help us to consider the several peaks of the earthquake accelerogram at any different moments.

Finally, it was observed that the responses of the continuous critical excitation are almost critical same as the regular critical excitation also they are more probable and in terms of spectral density function appearance they are similar to natural earthquakes.

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