

## **Moving-vehicle-excited bridge impact test and related structural identification theory**

Liming Zhou<sup>1)</sup> and \*Jian Zhang<sup>2)</sup>

<sup>1), 2)</sup> School of Civil Engineering, Southeast University, 210096,  
<sup>2)</sup> [jian@seu.edu.cn](mailto:jian@seu.edu.cn)

### **ABSTRACT**

A new mobile impact testing method using a moving vehicle as the excitation and its corresponding flexibility identification theory is proposed for rapid bridge tests. While maintaining the advantage of an impact test, i.e. both the impact force and structural reactions are measurable, the proposed moving impact can be more efficient than traditional point impacts such as the sledge hammer impact and the drop-weight impact. But a subsequent challenge lies in the structural identification theory because the proposed impact test produces a spatially continuous excitation force while classical impact forces are spatially discrete. Therefore, a novel data processing strategy is proposed to make use of the continuous excitation force to identify the basic modal parameters as well as the structural flexibility. Finally, laboratory and numerical experiments are performed and identification results successfully validated that the proposed impact test is suitable for rapid dynamic tests for bridges.

### **1. INTRODUCTION**

Structural Health Monitoring (SHM) has been developed over a long time and SHM systems have been installed on many long-span bridges (Adewuyi and Wu 2015; Kim et al. 2007; Deng and Cai. 2007; Brownjohn et al. 2010; Catbas et al. 2013; Fujino et al.2015; Lynch 2004;Lynch 2007; Liu et al. 2017; Rahbari et al. 2015; Soyoz and Feng 2010; Sabato et al.2017). However, due to limited budgets for short-span bridges, especially for those in rural areas, an economical, efficient and effective testing method is necessary for evaluation of their safety. Some attempts have been made to develop testing methods using mobile vehicles.

Pioneering work by Yang and Lin (2005) led to the identification of basic modal parameters from a moving vehicle's acceleration. Guo et al. (2009) evaluated bridge performance by establishing the relationship between a passing vehicle's reactions and structural damage. A modified vehicle capable of producing an impact upon a bridge whilst simultaneously scanning for structural damage based on its reactions was developed by Xiang et al.(2005). Though these methods were efficient, they were limited to the identification of basic modal parameters and preliminary damage.

As a kind of forced vibration test, the impact test, by measuring both input forces and

output responses, has the merit to extract the Frequency Response Function (FRF) consistent with analytic solutions, including its shape and amplitude, while ambient vibration testing data is restricted to the shape of the structural FRF (i.e. it does not include amplitude). As a result, the impact test can be used to successfully identify not only basic modal parameters (frequencies, damping ratios and mode shapes) but also deep-level parameters, such as the scaling factor and flexibility matrix (Zhang et al. 2014).

Although impact tests have long been studied, they have not been widely used in engineering practices for the following reasons. First, it is not easy to excite a bridge because the energy generated by the available impacting equipment is limited. For example, a sledge hammer cannot fully excite the dynamic characteristics of a bridge because of its low impact force (less than 20 kN), and despite De Vitis et al. (2013) and Zhang and Moon (2012) developing a drop-weight exciter which could generate a 100kN impact force with a wide frequency band (0–200Hz), even this excitation device was inefficient because it needed to be stopped when exciting the bridge. Second, the lack of a structural identification theory to impact testing discourages use of the method; such a theory must be based upon impact test data, and this work is still in progress. Brownjohn et al. (2007) performed bridge vibration tests using an exciter, but the impact force was not used in the data processing stage. Catbas et al. (2004) and Brown and Witter (2011) developed a flexibility identification theory by using impacting forces and structural responses, from which the structural deflection under any static load can be accurately predicted, and Zhang and Moon (2012) applied it to several short-/middle-span bridges and proved that predicted deflections from the impact testing data were comparable with those from static truck load tests. A drawback to the traditional impact testing method is that it requires a large number of sensors deployed across the entire structure. To overcome this problem, the idea of subdividing the structure to be analyzed into smaller sub-structures was proposed: the test data from all sub-structures could then be integrated to allow the flexibility matrix of the entire structure to be identified. These proposals led to a series of algorithms including the multiple reference method (Zhang and Moon 2012), the single reference method (Zhang et al. 2014) and the reference-free method (Guo et al. 2018). However, these test methods were not convenient because they all required an impact test to be performed on each sub-structure independently.

In order to overcome limitations of the impact tests described above, we have developed a bridge testing method using continuous wheel forces excitation, and we put forward a corresponding structural flexibility identification theory. Our method utilizes a moving vehicle to continuously excite a bridge under conditions in which the wheel forces are measurable. By continuously instead of intermittently exciting the bridge, the proposed test is more efficient than a sledge-hammer test or a drop-weight test. History of the impacting device development is shown in Fig. 1. However, current test data processing methods are no longer applicable because the vehicle–bridge interaction forces act on the continuous bridge space, whereas the traditional impacting forces from a hammer or a drop-weight exciter act on discrete bridge nodes. Thus, it is necessary to develop a new structural identification theory for our proposed test.

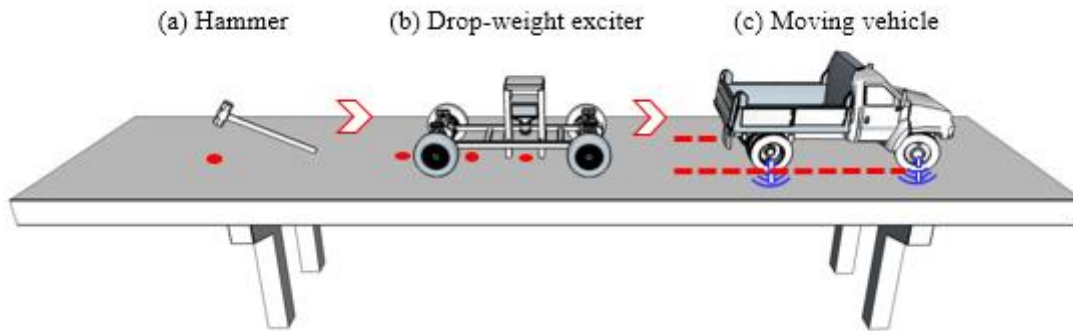


Fig. 1 History of the impacting device development

The article is set out as follows. In Section 2, we present the framework of the continuous wheel forces excited bridge dynamic test. In Section 3, we derive the structural flexibility identification theory, which includes equivalent load distribution, the eFRFs construction and modal scaling factor identification. In Sections 4, we validate the effectiveness of our proposed method by presenting experimental. Finally, in Section 5 we present our conclusions.

## 2. THE FRAMEWORK OF STRUCTURAL IDENTIFICATION THEORY

The cable element, derived by using the concept of an equivalent modulus of elasticity and assuming the deflection curve of a cable as catenary function, is proposed to model the cables...In order to obtain basic modal parameters (frequencies, shapes and damping ratios) as well as the flexibility, the framework of a strategy processing the proposed rapid test data is illustrated in Fig. 2.

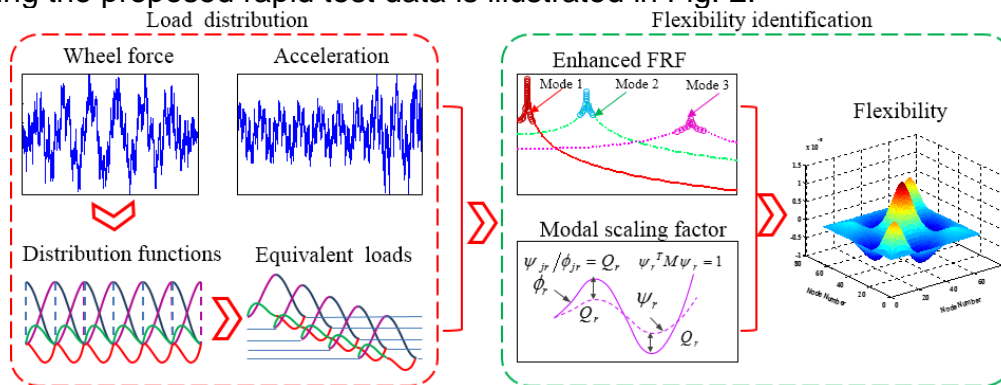


Fig. 2 Framework of the developed structural identification theory

Step 1. A moving-vehicle impact test is performed, during which bridge accelerations are measured by accelerometers as well as the wheel force (e.g. by monitoring the tire pressure).

Step 2. The equivalent load distribution function, which transforms a wheel force acting on infinite DOFs into nodal forces acting on finite DOFs, is proposed to help estimate the FRF together with nodal accelerations.

Step 3. Basic modal parameters including frequencies, damping ratios and mode shapes are identified through the estimated FRF and the enhanced FRFs (eFRFs). Then, modal scaling factors are calculated by the polynomial fitting algorithm. Finally, the displacement flexibility matrix is identified.

### 3. STRUCTURAL IDENTIFICATION THEORY BASED ON THE CONTINUOUS WHEEL FORCE

#### 3.1 Equivalent load distribution

The moving-vehicle impact test is more rapid to perform than the sledge-hammer test or the drop-weight test, but a challenge is that traditional impact theories only deal with excitation forces acting on discrete nodes, which is unsuitable for the wheel force acting on continuous space. In this part, the continuous wheel force is transformed into nodal forces by the equivalent load distribution. The equivalent load distribution in fact originates from the shape functions in the finite element analysis. The equivalent nodal loads of a bridge having  $n$  nodes are

$$F_E^{bT}(t) = \begin{Bmatrix} F_1(t) \\ M_1(t) \\ F_2(t) \\ M_2(t) \\ \vdots \\ F_{n-1}(t) \\ M_{n-1}(t) \\ F_n(t) \\ M_n(t) \end{Bmatrix} = N_v^b(x)\{f^b(x, t)\}\Delta x \quad (1)$$

and

$$N_v^b(x) = \begin{bmatrix} N_1(x_1^{e1}) & \dots & N_1(x_m^{e1}) & 0 & \dots & 0 & \dots & 0 & \dots & 0 & 0 & \dots & 0 \\ N_2(x_1^{e1}) & \dots & N_2(x_m^{e1}) & 0 & \dots & 0 & \dots & 0 & \dots & 0 & 0 & \dots & 0 \\ N_3(x_1^{e1}) & \dots & N_3(x_m^{e1}) & N_1(x_1^{e2}) & \dots & N_1(x_m^{e2}) & \dots & 0 & \dots & 0 & 0 & \dots & 0 \\ N_4(x_1^{e1}) & \dots & N_4(x_m^{e1}) & N_2(x_1^{e2}) & \dots & N_2(x_m^{e2}) & \dots & 0 & \dots & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \dots & 0 & 0 & \dots & 0 & \dots & N_3(x_1^{e(n-1)}) & \dots & N_3(x_m^{e(n-1)}) & N_1(x_1^{en}) & \dots & N_1(x_m^{en}) \\ 0 & \dots & 0 & 0 & \dots & 0 & \dots & N_4(x_1^{e(n-1)}) & \dots & N_4(x_m^{e(n-1)}) & N_2(x_1^{en}) & \dots & N_2(x_m^{en}) \\ 0 & \dots & 0 & 0 & \dots & 0 & \dots & 0 & \dots & 0 & N_3(x_1^{en}) & \dots & N_3(x_m^{en}) \\ 0 & \dots & 0 & 0 & \dots & 0 & \dots & 0 & \dots & 0 & N_4(x_1^{en}) & \dots & N_4(x_m^{en}) \end{bmatrix} \quad (2)$$

$$\{f^b(x, t)\}\Delta x = \begin{Bmatrix} f(x_1^{e1}, t)\Delta x \\ \vdots \\ f(x_m^{e1}, t)\Delta x \\ f(x_1^{e2}, t)\Delta x \\ \vdots \\ f(x_m^{e2}, t)\Delta x \\ \vdots \\ f(x_1^{e(n-1)}, t)\Delta x \\ \vdots \\ f(x_m^{e(n-1)}, t)\Delta x \\ f(x_1^{en}, t)\Delta x \\ \vdots \\ f(x_m^{en}, t)\Delta x \end{Bmatrix} \quad (3)$$

where  $x_1^{e1} \dots x_m^{e1}, x_1^{e2} \dots x_m^{e2}, \dots, x_1^{e(n-1)} \dots x_m^{e(n-1)}, x_1^{en} \dots x_m^{en}$  are coordinates of the distributed force, the superscript represents the element and the subscript represents a

specific position in the element.  $\Delta x$  is the distance between two continuous coordinates and it is assumed that all coordinates are uniformly distributed though not necessarily.  $N_v^e(x) = [N_1(x) \ N_2(x) \ N_3(x) \ N_4(x)]$ .  $F_E^b(t), N_v^b(x), \{f^b(x, t)\}$  are equivalent nodal loads, the distribution function and the distributed force of the bridge respectively.

### 3.2 Displacement flexibility identification

Once the continuous impacting force is transformed into equivalent nodal loads, FRF can be estimated using equivalent nodal loads and structural accelerations. It should be noted that each node has two inputs (the vertical force and the bending moment) and a single output signal (the acceleration). Namely, the number of inputs  $N_i$  of the bridge is greater than that of the outputs  $N_o$ . However, only the relationship between the vertical force and the acceleration is of interest here. Thus, the concerned parts need to be stripped from the original FRF. By performing the Singular Value Decomposition (SVD) on the original FRF matrices, the left singular matrix, the right singular matrix and the singular value matrix can be obtained. Then, natural frequencies and mode shapes are identified by the Complex Mode Indicator Function (CMIF) method.

Subsequently, eFRFs are constructed to improve the accuracy of the structural identification. The FRF is a superposition of modal parameters in the frequency domain. In the frequency domain, the modal parameter identification of a single FRF will be affected by the coupling between different modes. Even if advanced algorithms can solve the modal coupling problem, a new problem is likely to appear that identified modal parameters by different single mode FRFs cannot be identical due to the noise and the modal node. Therefore, in order to identify more modal parameters (such as system poles, damping ratios and modal scaling factors) and more accurately, it is necessary to reduce the multi-mode FRF to single mode FRFs in the frequency domain. The complex problem of the multi-dimension modal coupling is transformed into a simple problem of a single mode FRF, i.e. the eFRF:

$$eH(\omega)_r = \{u_r\}^T \{\phi_r\} \frac{Q_r}{j\omega - \lambda_r} \{\phi_{r,drv}\}^T \{u_{r,drv}\} \quad (5)$$

where  $r$  represents the  $r$  th modal order,  $eH(\omega)_r$  is the eFRF,  $Q_r$  is the modal scaling factor,  $\lambda_r$  is the system pole,  $\{\phi_r\}$  is the displacement mode shape,  $\{\phi_{r,drv}\}$  is the vector of the displacement mode shape coefficients at all driving points (load points),  $\{u_r\}$  is the first left singular vector,  $\{u_{r,drv}\}$  is the vector of the first left singular vector coefficients at all driving points (load points).

The key issue to identify the flexibility matrix is to identify the modal scaling factors, defined by the linearly proportional factor of the displacement flexibility matrix identified by the impact test to the displacement flexibility matrix by the ambient vibration test. By using the polynomial fitting algorithm to fit the eFRF, the modal scaling factor  $Q_r$  can be calculated as

$$\frac{1}{Q_r} = \{u_r\}^T \{\phi_r\} \{\phi_{r,drv}\}^T \{u_{r,drv}\} \begin{Bmatrix} eH(\omega_1)_r \\ eH(\omega_2)_r \\ \vdots \\ eH(\omega_k)_r \end{Bmatrix}^+ \begin{Bmatrix} 1/(j\omega_1 - \lambda_r) \\ 1/(j\omega_2 - \lambda_r) \\ \vdots \\ 1/(j\omega_k - \lambda_r) \end{Bmatrix} \quad (6)$$

Now, with available displacement mode shapes, system poles and scaling factors from all necessary modal orders ( $r = 1, \dots, N$ ), the flexibility matrix of the structure can be identified by

$$\text{Flex}^d = \sum_{r=1}^N \left( \frac{Q_r \{\Phi_r\} \{\Phi_r\}^T}{-\lambda_r} + \frac{Q_r^* \{\Phi_r^*\} \{\Phi_r^*\}^T}{-\lambda_r^*} \right) \quad (7)$$

where \* represents the complex conjugate operation. It is observable that the displacement flexibility matrix is the superposition of the modal parameters in the complex mode.

The flexibility indicates the stiffness distribution of the structure. Assume that a static force vector acting on the structure is  $\{f_{\text{static}}\}$  and then the static deflection of the structure can be predicted by  $D = \text{Flex}^d \{f_{\text{static}}\}$ .

#### 4. LABORATORY EXPERIMENT

##### 4.1 Experiment design and monitoring strategy

In order to verify the proposed methodology, a laboratory experiment in which a loaded tire past through a simply supported beam was investigated. Real-time measurements included the tire pressure, vertical beam accelerations and vertical beam displacements. The tire pressure data was utilized to calculate the wheel force. Accelerations was utilized to identify the structural parameters and static displacements aimed to verify the predicted deflection by identified structural flexibility

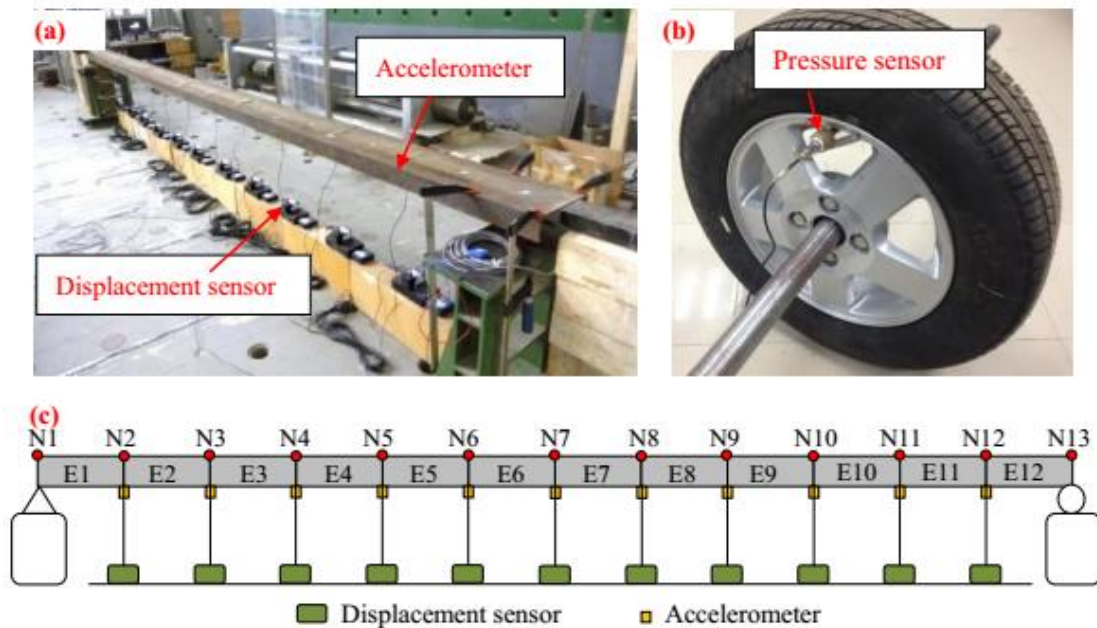


Fig. 3 Experiment design: (a) Beam; (b) Tire; (c) Monitoring strategy of the beam.

The structure to be studied was a simply-supported beam with a length of 5868mm as shown in Fig. 3. It is a hot-rolled channel-section steel beam with a Chinese standard steel material Q235. The beam is divided into 12 elements (E1~E12) and 13 nodes (N1~N13) as shown in Fig. 3. The monitoring system of the beam is constituted

by 11 accelerometers (type: ICP 393B04), 11 cable-extension position transducers (type: CELESCO PT1DC). Accelerometers and displacement transducers dynamically measured the vertical deformation of the beam's central axis from nodes N2 to N12. The single tire (type: Giti Wingro 165/70R13) was adopted to simulate the tire force on the beam structure as shown in Fig. 3. An axle penetrating through both sides of the tire served as a bar for the load control and additional masses were hung symmetrically. The monitoring system of the tire was constituted by a pressure sensor (type ICP 106B52), which was installed on the valve stem of the tire. All sensors were wired to the data acquisition system (type: NI PXIe-1082) and all data were collected simultaneously.

#### *4.2 Experiment case description*

Two kinds of tests were conducted. One test was a tire-beam interaction experiment and the other one was a static load experiment to verify the reliability of the identified flexibility.

##### *(1) Tire-beam interaction experiment*

The experiment was conducted by using the load tire to roll on the beam with the speed of 0.88m/s, and the total mass of the tire was 120kg. Strip-shaped magnets were placed on the steel beam to become surface obstacles. The tire pressure data were utilized to calculate the wheel force. And vertical beam accelerations were combined with the wheel force to calculate the structural parameters.

##### *(2) Static load experiment*

In order to verify the reliability of the identified parameters, namely the flexibility and basic modal parameters constituting flexibility, static load tests were conducted by applying mass blocks at specified nodes on the simply supported beam and its static deflections were measured by displacement transducers. Two static load cases were conducted: CASE 1, where 3 mass blocks weighing 30kg were placed on nodes N5 and N9 respectively; CASE 2, where 2 mass blocks weighing 30kg were placed on nodes N3, N4, N6, N8 and N11 respectively.

#### *4.3 Parameter identification and verification*

The equivalent nodal loads of the simply supported beam were calculated firstly. Since the beam was divided into 12 elements, i.e. the length of the element  $l$  is 489mm. Based on the aforementioned equations, equivalent loads of each node were obtained.

The simply supported beam was regarded as a MIMO system, consisting of 26 inputs (13 vertical forces and 13 bending moments) and 11 outputs (vertical accelerations). The FRF representing the relationship between vertical forces and vertical accelerations was a  $11 \times 11$  matrix. By performing the SVD of the FRF, a CMIF plot was drawn. It should be noted that the number of curves in the CMIF plot was determined by the reference nodes. For instance, there are 13 curves because there were 13 equivalent impacting nodes in the test. Natural frequencies were identified from CMIF peaks. The natural frequencies of the first six modes were 4.78, 18.89, 41.49, 71.52, 106.19, 142.18 (Hz), and the damping ratios 4.44, 0.05, 0.82, 0.33, 0.34,

0.71(%). The mode shapes and the modal participation factor matrices were identified to construct the eFRF, which was further used to identify the mode shapes.

The modal scaling factor was identified via Eq. (6). Then, the structural flexibility matrix of the beam was identified via Eq. (7) and structural deflections of the structure under any static load could be predicted. Static loads Case 1 and Case 2 were used to verify the effectiveness of the identified flexibility. For instance, the predicted deflection under the static load Case 1, i.e. 882N, 882N at the node N5, N9 was shown in Fig. 4(a) and the predicted deflection under Case 2, i.e. 588N, 588N, 588N, 588N, 588N at the node N3, N4, N6, N8, N11 was shown in Fig. 4(b). In both predictions, curves denoted by '3 modes' means that the flexibility was calculated from the modal parameters of the first three modes. It is seen from Eq. (7) that the flexibility is estimated as the sum of the residuals normalized by the eigenvalues and its conjugate in all identified structural modes. Due to the simplicity of this structure, Fig. 4 illustrates that the flexibility calculated only using the first mode could accurately capture the real structural characteristics.

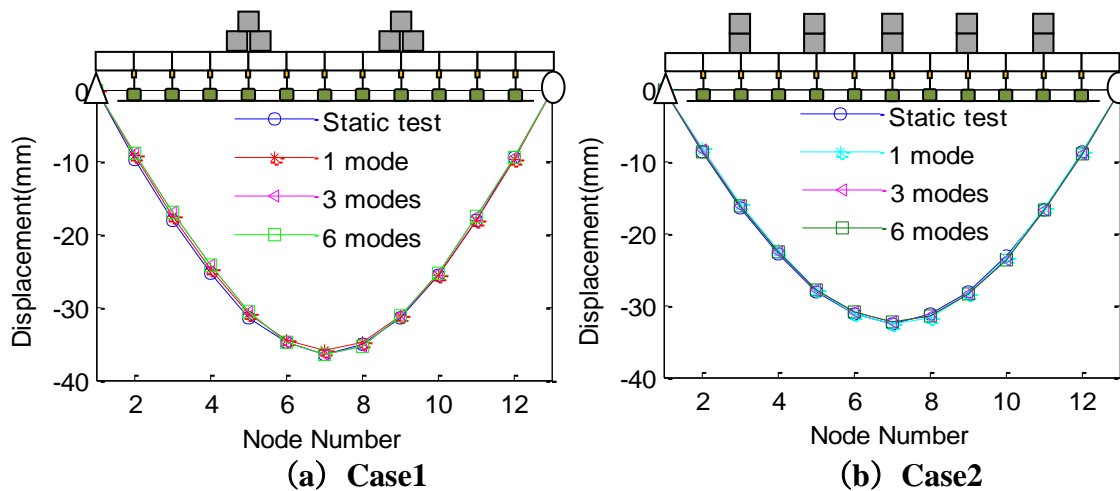


Fig. 4 Deflections prediction: (a) Case 1; (b) Case 2.

## 5. CONCLUSIONS

To overcome the limitations of the traditional impact test, a moving-vehicle based rapid impact test and its related structural identification theory are proposed in this paper. It uses the moving vehicle to impact the bridge rapidly, and real-time monitoring of the wheel forces and the bridge accelerations is utilized to identify basic modal parameters as well as the flexibility. The advantages of the proposed methodology are as follows:

(1) Convenient and efficient. Different to the intermittent point impact tests, it uses a moving vehicle to excite the bridge continuously and monitors the wheel force and bridge accelerations in real time.

(2) Deep-level parameter identification. Due to the characteristics of the spatially continuous excitation, traditional impact testing theories are no longer applicable. Therefore, a theory supporting the new method is developed. Firstly, equivalent nodal loads can be calculated using the wheel force and the load distribution function.



Secondly, an MIMO FRF is estimated by using nodal loads and accelerations. Finally, deep-level parameters are identified by reconstructing the eFRF.

(3) Results prove the theory reliable. Laboratory examples show that the proposed methodology is reliable and can be applied to practical engineering structures. The identified deep-level parameters can show the structure stiffness distribution, and can be used for damage identification and structural bearing capacity evaluation, so as to realize the reliable diagnosis of the bridge.

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