The 2019 World Congress on
Advances in Structural Engineering and Mechanics (ASEM19)
Jeju Island, Korea, September 17 - 21, 2019

Empirical model on thermal conductivity for various conditions of sands

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ABSTRACT

Recently, it is increased the demands to understanding thermal conductivity of soils as geothermal energy or underground facilities take center stage. There are many theoretical and empirical models to estimate the thermal conductivity of soils in certain conditions. However, there are too many the factors effect on thermal conductivity to see how to transfer heat energy. For more and intuitive understanding thermal conductivity, it was conducted that the experimental tests on various conditions of sands. Then, investigates the correlations between relative density of sand, effective stress, and degree of saturation. Moreover, with studying many models on thermal conductivity, find some common ground, then developed and verified the modified model on thermal conductivity of sands. As a results, the suggested model has an advantage to describe the thermal conduction as much simple way.

1. INTRODUCTION

Soil is three-phase material, which consisted of mineral soil particle (solid), pore water (liquid), and void (air). And the soil is particulate materials. As the soil is particulate materials, thermal conduction is dominant way to transfer heat rather than convection or radiation (Murashov and White, 2000). Thus, the thermal conductivity is very important value to see the heat transfer through the soils. Therefore, there has been huge demands to estimate the thermal conductivity for many areas such as estimating underground heat pumps efficiency, decision of allowable current of electrical power cables, efficiency of energy storage systems and stable usage of radioactive nuclear waste disposal.

The heat disposal ability of soils, which is thermal conductivity, is governed by not only the contact between particle-to-particle, but also that of void-to-pore water and void-to-particle (T.S. Yun and J.C. Santamarina, 2008; S.J. Lee et al., 2016). Therefore,

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thermal conductivity is affected by roughness of soil particles and effective contact area. The effective contact area is affected by dry unit density of soils, effective stress, particle size distribution. And the thermal conductivity of each phases consisting soils are affected by water contents, ice contents, quartz contents including mineral composition, etc. (J.C. Santamarina et al., 2019). As a results, the effective contact area of soils changed with those factors, the thermal conductivity of soils also changed. Empirically, the thermal conductivity of soils is linearly proportional to relative density or void ratio (T.S. Yun and J.C. Santamarina, 2008; S.J. Lee et al., 2016; S. Lu et al., 2007). Table 1 presents some models of thermal conductivity arranged with relative density for detailed analysis (Hashin and Shtrikman, 1963; O. Johansen, 1977; J. Cote and J. Konrad, 2005). Generally, those empirical models for thermal conductivity specify the range through values of dry (minimum) and saturated (maximum) thermal conductivity and follow the nonlinear function form with porosity and degree of saturation. However, the theoretical model from Hashin and Shtrikman, (1963) cannot describe actual soils' properties but only restrict possible thermal conductivity range. And empirical model from O. Johansen, (1977) cannot use general form at dry condition. Finally, empirical models from S. Lu et al., (2007) and S.J. Lee et al., (2016) are highly dependent on fitting parameters.

For much understanding of soils' thermal conductivity depending on some properties of soils, experimental tests to see the impact of relative density of soils, degree of saturation, and effective stress, are conducted in this study. While, there are significant difference between (quartz) sands and weathered silty soils, because the silt and clay particles adsorb water in less saturation, and expand their volume to decrease contact between particles (Tarnawski and Leong, 2000). Therefore, this study concentrates on sands to see thermal conduction through soils intuitively. Then, with the literature, follow the theoretical and empirical models on thermal conductivity of soils, and modify those models. Finally, based on the experiments and literatures, new thermal conductivity model would be suggested.

Table 1. Thermal conductivity model with relative density

Model	Equation	Note
Semi-theoretical model		
Hashin-Shtrikman upper bound	$k_U = k_s\left(1+\frac{1}{\left(\frac{k_s}{k_{VU}-k_s}+\frac{1}{3}\right)e_{max}+\frac{k_s}{k_{VU}-k_s}-(e_{max}-e_{min})(\frac{k_s}{k_{VU}-k_s}+\frac{1}{3})D_r}\right)$	$k_{VU} = k_w\left(1+\frac{3(1-S)(k_a-k_w)}{3k_w+S(k_a-k_w)}\right)$
Hashin-Shtrikman lower bound	$k_L = k_{VL}\left(1+\frac{e_{max}-(e_{max}-e_{min})D_r}{\left(\frac{k_{VL}}{k_s-k_{VL}}+\frac{1}{3}\right)+\frac{k_{VL}}{k_s-k_{VL}}e_{max}-(e_{max}-e_{min})(\frac{k_{VL}}{k_s-k_{VL}})D_r}\right)$	$k_{VL} = k_a\left(1+\frac{3(S)(k_w-k_a)}{3k_a+(1-S)(k_w-k_a)}\right)$
Empirical model		
Johansen (1977)	$k = \left(k_{sat}-k_{dry}\right)(0.7logS+1.0)$	$k_{sat} = k_s^{\left\{1-\left(\frac{1}{1+e_{max}-(e_{max}-e_{min})D_r}\right)\right\}}k_w^{\left(\frac{1}{1+e_{max}-(e_{max}-e_{min})D_r}\right)}$ $k_{dry} = \frac{445.4(1+e_{max}-(e_{max}-e_{min})D_r)}{2700+2700e_{max}+0.97G_s\rho_w-2700\,(e_{max}-e_{min})D_r}-0.141$
Lu et al. (2007)	$k = \left(k_{sat}-k_{dry}\right)\exp\left[0.96\left(1-S^{-0.37}\right)\right]+k_{dry}$	$k_{dry} = -0.56\left(1-\frac{1}{1+e_{max}-(e_{max}-e_{min})D_r}\right)+0.51$
Lee et al. (2016)	$k = k_{dry} + (k_{sat}$	Fitting parameter β

Note. Thermal conductivity of soil (k_s), of water (k_w), of saturated soil (k_{sat}), of dry soil (k_{dry}) [W/mK]

2. EXPERIMENTAL STUDY

2.1 Thermal probe method

In this study, the line-source model is adopted to measure the thermal conductivity of sands simply and quickly. The line-source model is a method of deriving thermal conductivity by analysing temperature difference of voltage applied, thin, and linear metal wire over time. From this model, the temperature change at distance r from the heat source Q and after time t from applying voltage, can be expressed with thermal conductivity k and thermal diffusivity α, as shown in Eq. (1). The exponential integral function Ei(x) can be converted by Taylor expansion, thus the temperature change could be expressed also as Eq. (2) (Carslaw and Jaeger, 1959), because the series term $r^2/4\alpha t$ is small enough. Then, the temperature difference between temperature difference after time t1 and t2 from voltage applying is expressed as Eq. (3). Therefore, the gradient of linear range of temperature change with log time graph is inversely proportional to thermal conductivity.

$$\Delta T(r,t) = \frac{Q}{4\pi k} \times Ei\left(\frac{r^2}{4\alpha t}\right) \tag{1}$$

$$\Delta T = \frac{Q}{4\pi k}\left[-\gamma + \ln\left(\frac{4\alpha t}{r^2}\right)\right] \tag{2}$$

$$\Delta T_2 - \Delta T_1 = \frac{Q}{4\pi k}\ln\left(\frac{t_2}{t_1}\right) \tag{3}$$

To use line-source model for measuring thermal conductivity, the heat source should be very thin and linear (J.H. Blackwell, 1956). Thus in this study, the thermal probe which has diameter of 3.18mm and length of 150mm is used (Fig. 1). The resistance wire along probe can emit the heat by voltage applying, and in the middle of probe length, small sensor thermistor installed which can measure the temperature very sensitively. This thermal probe is very thin and purposed to apply on relatively shallow, particulate materials (sand), therefore it directly penetrated on target materials, do not need to use borehole or filling with grease. So in this case, the volumetric heat capacity and thermal diffusivity are not considered because the probe directly meets on soil particles.

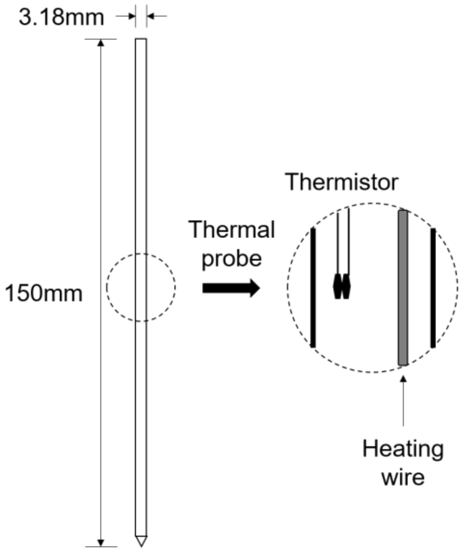

Fig. 1. Design of thermal probe

2.2 Experimental setup and design

The experiments were conducted to see and understand correlations between thermal conductivity (k) of sands, relative density (D_r) of sands, and overburden pressure (q_0) conditions. For this purpose, experimental k_0 chamber, which can control the stress conditions was manufactured (inner dia. = 478mm). For balanced loading on the sample surface, center-hole type hydraulic cylinder and load-cell installed on the middle of plate on the sample. And on these loading system, put the reaction system which can fixed at certain height from the sample. The overburden pressure is verified by value read from load-cell, and small metal plate and rubber ring makes precise loading and preventing deformation possible. Then, thermal probe was penetrated as field tests unlike ordinary lab scale tests makes the samples in probe pre-installed mould (Fig. 2).

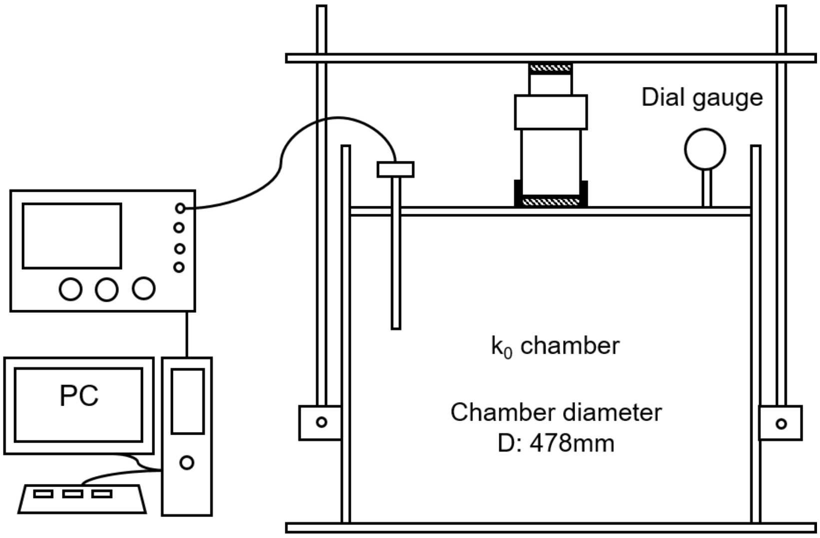

Fig. 2. Experimental setup

Table 2 presents tests conditions, all tests were designed to be conducted with drainage permitted. And Table 3 presents geotechnical properties of sand specimen (Jumunjin Sand) used in this study. The particle size distribution of Jumunjin Sand refers to J.T. Han, (2010), and the maximum and minimum void ratio refers to K.K. Kim, (2013).

Table 2. Tests conditions

	Specific gravity (G_s)	D10 [mm]	D50 [mm]	Uniformity coefficient (C_u)	Maximum void ratio (e_{max})	Minimum void ratio (e_{min})
Jumunjin Sand	2.65	0.38	0.57	1.58	0.98	0.61

Table 3. Geotechnical properties of Jumunjn Sand

Degree of saturation S [%]	Relative denstiy D_r [%]	Overburden pressure q_0 [kPa]
0 (Dry condition)	35, 51, 78	50, 75, 100
100 (Saturated condition)	35, 55, 81	25, 50, 75

2.3 Experimental results and analysis

The temperature change over time with log scale was presented as S-shape, which can be divided as drastic increasing range with S-shape, linearly increasing range following line-source model, and gradient change occurring range by boundary condition. The boundary condition in this case means thermal gradient changing occurred by heat insulation at the wall of chamber, or by cooling effect from relatively infinite fields. As the line-source model, the thermal conductivity is inversely proportional to temperature gradient at linear range of graph. The raw data of certain cases is shown as Fig. 3. From the Fig. 3, at same conditions of relative density and overburden pressure, the thermal conductivity was dramatically increased by increasing degree of saturation.

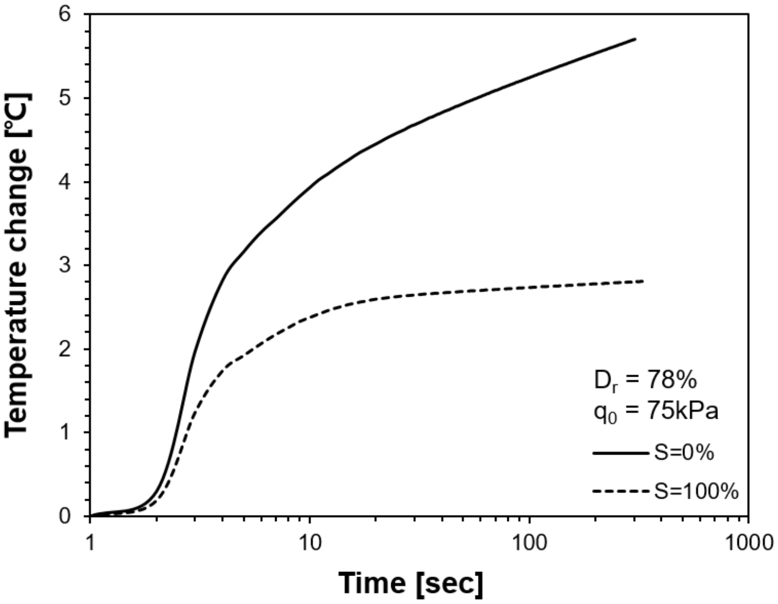

Fig. 3. Temperature change with log scaled time by degree of saturation

All of experimental results are shown as Fig. 4. The thermal conductivity of sands were much higher in saturated condition than dry condition, and also higher at condition that higher relative density and overburden pressure.

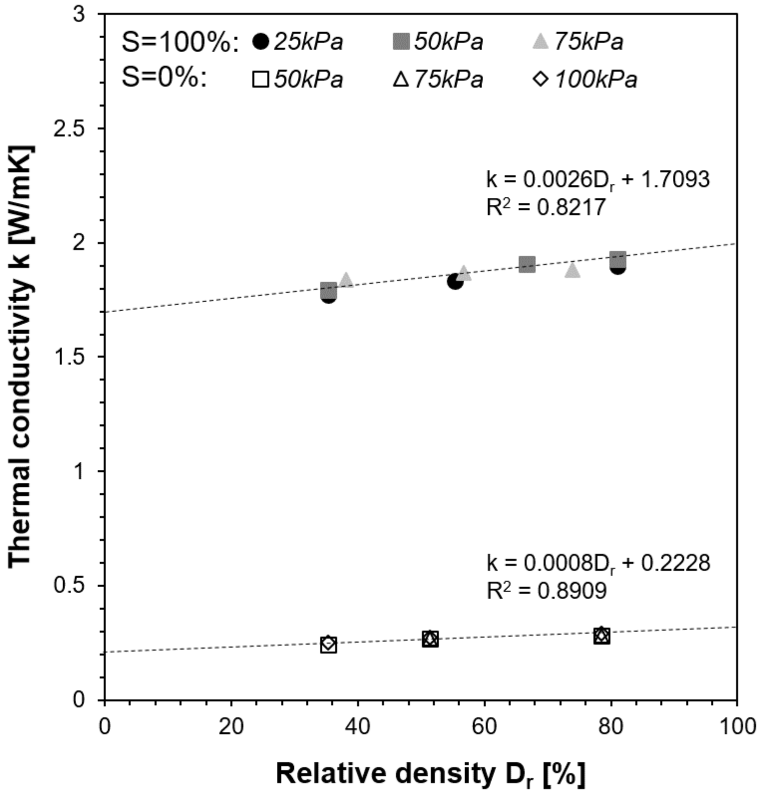

Fig. 4. Thermal conductivity with relative density and overburden pressure

Because the sand is particulate materials, degree of saturation is the most powerful factor on thermal conductivity, and relative density, overburden pressure followed in order. The thermal conductivity of air which is known as 0.026W/mK (Andersland and Ladanyi, 2004) is much lower than that of water 0.598W/mK (Cortes et al., 2009), and the thermal conductivity of water is also known as much lower than that of soil mineral particle 2~8W/mK (Horai and Simmons, 1969). Based on the order of each elements' thermal conductivity, dry sand's thermal conductivity is high at higher relative density and smaller volume of voids, because the soil particles (the highest k) meets with voids filled by air (the lowest k). In the same manner, as degree of saturation increases, the thermal conductivity also increases because water (higher k than that of air) substitute air voids. The experimental results showed the increment of thermal conductivity by increasing relative density was significant at saturated condition than dry condition because pore water increases heat conduction. Also, experimental results showed that thermal conductivity increases slightly with overburden pressure acting as confinement and cause of deformation.

3. DISCUSSIONS

The experimental results on this study agreed with tendency of literatures. From those thermal conductivity models (Table 1), new empirical thermal conductivity model is suggested in this study. The model shown as Eq. (4) from S. Lu et al., (2007) used soil texture parameter α, and shape parameter 1.33. And they empirically described thermal conductivity at dry condition as linearly proportional to porosity.

$$k = \left(k_{sat} - k_{dry}\right) \cdot \exp\left[\alpha\left(1 - S^{(\alpha - 1.33)}\right)\right], k_{dry} = -0.56n + 0.51 \tag{4}$$

To generalize soil texture parameter and shape parameter, parameter C and D applied to original model. Then, for describing thermal conductivity of soil particle itself, independent to saturation, parameter E added on as Eq. (5).

$$k = \left(k_{sat} - k_{dry}\right) \cdot C \cdot \exp\left(1 - S^{D}\right) + E \tag{5}$$

The parameter C and D are shape parameters that describe changing thermal conductivity values with saturation, and the parameter E is soil texture parameter that reflects particle size distribution and thermal conductivity of particle itself. Then, as the boundary condition of dry (S = 0) and saturated (S = 1) is applied to Eq. (5), parameter C and E have specific values as shown as Eq. (6) with the base of natural logarithm *e*.

$$k = \left(k_{sat} - k_{dry}\right) \cdot \frac{1}{1-e} \cdot \exp\left(1 - S^{D}\right) + \frac{1}{1-e} k_{dry} - \frac{e}{1-e} k_{sat} \tag{6}$$

Using the thermal conductivity data of Ottawa 20/30 from S.J. Lee et al., (2016), deducted value of parameter D by regression with least square method as shown as Fig. 5. As a result, the value of parameter D is 0.632, which is 1/(1-C) as Eq. (7).

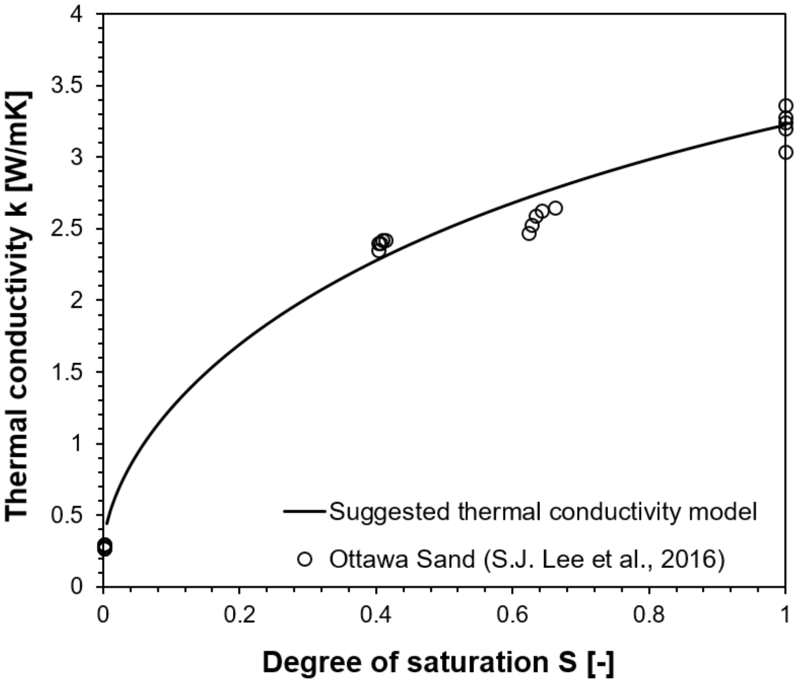

Fig. 5. Construction of thermal conductivity model by regression with Ottawa Sand, data from S.J. Lee et al., 2016

$$\begin{aligned} k &= \left(k_{sat} - k_{dry}\right) \cdot \frac{1}{1-e} \cdot \exp\left(1 - S^{\left(\frac{e-1}{e}\right)}\right) + \frac{1}{1-e} k_{dry} - \frac{e}{1-e} k_{sat} \\ k &\approx -0.582\left(k_{sat} - k_{dry}\right) \cdot \exp\left(1 - S^{0.632}\right) - 0.582 k_{dry} + 1.582 k_{sat} \end{aligned} \tag{7}$$

Suggested thermal conductivity model has the general form that came into existence on whole range of saturation. And, only the minimum and maximum values of specific sands makes it possible that predicts thermal conductivity value at certain degree of saturation.

4. CONCLUSIONS

Experimental tests were conducted to understand the sands' heat transfer mechanism by measuring thermal conductivity varying relative density of sands, overburden stress on sands, and degree of saturation, using thermal probe method. As the experiments results, thermal conductivity on sands linearly increased with relative density increased. And thermal conductivity on saturated condition was much higher than dry condition because the pore water substitute with air voids. Also, it was suggested that generalized thermal conductivity model with modifying models from literatures, and verified with thermal conductivity values along degree of saturation from literature. Newly developed model can estimate the thermal conductivity with the values of minimum (dry condition) thermal conductivity, maximum (saturated condition) thermal conductivity, and degree of saturation. This model have an advantage of generalized

Horai, K. I., & Simmons, G. (1969). Thermal conductivity of rock-forming minerals. *Earth and Planetary Science Letters*, **6**(5), 359-368.