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Free Vibration of a Very-Large-Sag Extensible Catenary Riser

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ABSTRACT

This paper presents a variational model formulation for free vibration of a very-large-sag extensible catenary riser. In the static equilibrium state, the catenary riser has a large-sag configuration. In the motion state, the catenary riser is assumed to displace with large amplitude from the static equilibrium position. The total virtual work of the catenary riser system involves the virtual strain energy due to bending, the virtual strain energy due to axial deformation, the virtual work done by the effective weight, and the inertia forces. The difference between the Euler's equations in the static state and the displaced state yields the nonlinear equations of motion for two-dimensional free vibration in the Cartesian coordinate system. The system is linearized to obtain the linear stiffness matrices and mass matrix of the catenary riser. The Galerkin finite element procedure is used to obtain the Eigenvalue problem. The natural frequencies and mode shapes are obtained. Interesting vibrational behaviors of the large-sag catenary riser are highlighted in this paper.

1. INTRODUCTION

A catenary riser is a long, slender pipe suspended in the catenary shape between the seafloor and fixed or floating platform on the water surface. It is commonly used in deep-water offshore engineering for conveying fluids such as oil, gas, or injection fluid. The catenary riser is designed to withstand the severe applied force and environmental loads, which induce severe internal stresses and motion, during the lifetime of an operation. The natural frequencies and mode shapes of the catenary riser are the important dynamic properties used to understand the vibrational behavior and the significant parameters used in the design stage. At present, most research on catenary risers is limited to large-sag catenary risers with small amplitude of vibration. To the authors' knowledge, there is no example in the literature of a very large sag with large-amplitude vibration. Therefore, the purpose of this research work is to focus on the development of a model formulation for a very-large-sag extensible catenary riser with large-amplitude free vibration. However, the numerical solution presented in this paper is limited to small-amplitude free vibration analysis of a very-large-sag extensible catenary riser in two dimensions. A literature review of research related to catenary risers is presented in this paper.

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Moe and Arntsen (2001) proposed an analytical solution for static analysis of a steel catenary riser under the influence of its own weight. Inextensible and extensible models were considered. Chucheepsakul, Monprapussorn, and Huang (2003) presented a mathematical formulation for large strain, extensible, flexible marine pipes transporting fluid. The large strain formulations were developed by using the principle of virtual work and the vectorial method considering both Cartesian and natural coordinates. Chatjigeorgiou (2008) proposed a finite difference solution method formulation for the simplified linear and nonlinear dynamic analysis of the catenary riser. The numerical results for static and dynamic responses were reported. Srinil and coworkers (2009a, 2009b) investigated vortex-induced vibration of a catenary riser subjected to ocean current forces. The riser is modeled using a pined-pined beam-cable model with bending and extensibility stiffness. Various behaviors of the response amplitude diagram, such as multi-mode lock-in, switching, sharing, and interaction features, are described.

Athisakul, Klaycham, and Chucheepsakul (2014) investigated static configurations of a steel catenary riser using the variational formulation and the finite element. The critical top tension and the possible configuration when the applied top tension was higher or lower than the critical top tension, providing unstable and stable configurations, respectively, were reported. Nonlinear free vibration analysis of a steel catenary riser transporting fluid was proposed by Klaycham, Athisakul, and Chucheepsakul (2014) based on Hamilton's principle. Instability analysis on the static state and vibration of a flexible riser conveying fluid were fully investigated by Kim and O'Reilly (2019). The riser, which can be either extensible or inextensible, was modeled in three-dimensional space and supported by the pinned-pinned ends. The static configuration of the riser was obtained numerically by using the finite difference scheme. A parametric study of the effects of the current and internal flow on the stability and dynamic behavior of the riser were carried out.

2. VARIATIONAL MODEL FORMULATION

Configuration of the large-sag extensible catenary riser in three states—static, dynamic, and undeformed—is illustrated in Fig. 1. A variational formulation of the mechanical behavior of the catenary riser is derived based on the work-energy principle in a two-dimensional Cartesian coordinate system. The catenary riser is modeled between the hinged support at one end and the free-sliding support at the other end. For analysis of the static equilibrium configuration, the bending strain energy is an internal strain energy while the external virtual work done is composed of the top horizontal tension force, effective weight, and the wave and current drag force. The virtual work for dynamic analysis involves the energy due to bending and axial strain while the external virtual work done is composed of the effective weight and inertia force. The arc-length coordinate is used as an independent variable.

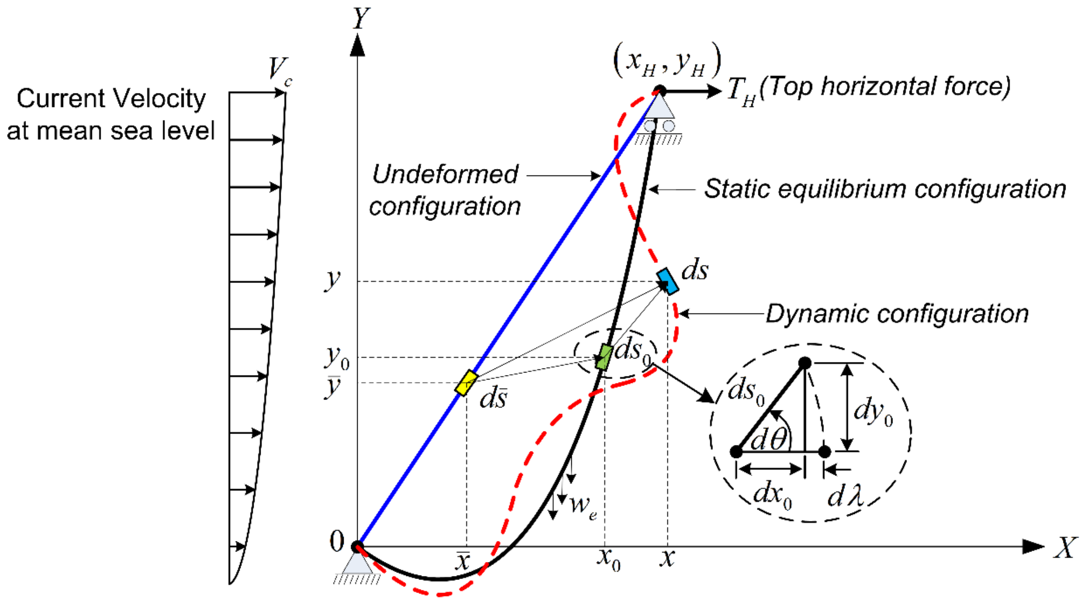

Fig. 1 Configuration of the catenary riser in three states.

The geometrical configuration of the catenary riser in three states, as shown in Fig. 1, provided the following relation:

$$sin\theta = \frac{dy_0}{ds_0} = \frac{y_0'}{s_0'} \tag{1a}$$

$$cos\theta = \frac{dx_0}{ds_0} = \frac{x_0'}{s_0'} \tag{1b}$$

$$s_0'^2 = x_0'^2 + y_0'^2 \tag{1c}$$

The prime symbol (′) denotes the derivative with respect to the unstrained arc-length $\bar{s}$, subscript ($_0$) defines the parameters in the equilibrium state, the angle (θ) is measured between the arc length of the catenary riser and the horizontal. The curvature (κ) of the catenary riser element can be obtained by differentiating Eq. (1a) with respect to the arc-length parameter s_0 and gives

$$\kappa = \frac{d\theta}{ds_0} = \frac{y_0''}{\left(1 - y_0'^2\right)^{\frac{1}{2}}} \tag{2}$$

The total axial strain (ε_0) of the extensible catenary riser in the equilibrium state can be defined by

$$\varepsilon_0 = \frac{ds_0 - d\bar{s}}{d\bar{s}} \tag{3}$$

The arc length of the catenary riser in the static equilibrium state ds_0 can be expressed in terms of the Cartesian coordinate components (x_0, y_0) by

$$ds_0 = (1+\varepsilon_0)d\bar{s} = \sqrt{{x_0'}^2 + {y_0'}^2}\,d\bar{s} \tag{4}$$

where $d\bar{s}$ is the infinitesimal arc length in the undeformed state. With Eqs. (1c) and (4), yield the following expression:

$$x_0' = \sqrt{(1+\varepsilon_0)^2 - {y_0'}^2} \tag{5}$$

The total virtual work of the catenary riser in the static equilibrium state is composed of the bending strain energy, the external virtual work done due to the top horizontal tension force (T_H), the effective weight (w_e), and the wave and current drag force, developed by Punjarat and Chucheepsakul (2019) as follows:

$$\delta\pi = \int_0^{s_t} \left\{ \begin{aligned} &\frac{EIy_0''}{1-{y_0'}^2}\delta y_0'' + \frac{EIy_0'{y_0''}^2}{\left(1-{y_0'}^2\right)^2}\delta y_0' + T_H \frac{y_0'}{\sqrt{(1+\varepsilon_0)^2 - {y_0'}^2}}\delta y_0' \\ &+ w_e \delta y_0 + \left(f_{Hny} - f_{Hty}\right)\delta y_0 \end{aligned} \right\} d\bar{s} \tag{6}$$

where w_e is the effective weight; $w_e = \left(\rho_p A_p - \rho_e A_e + \rho_i A_i\right)g$, ρ_p, ρ_e and ρ_i are the densities of the riser pipe, external fluid, and internal fluid, respectively. A_p, A_e, and A_i are the cross-sectional areas of the riser pipe, outside diameter, and inside diameter, respectively, and g is the gravitational acceleration; f_{Hny} and f_{Hty} are the current drag force in the normal and tangential directions.

A variational formulation based on the work-energy principle for dynamic analysis is developed for a catenary riser composed of an axial deformation, strain energy due to bending, and virtual work done due to effective weight and inertia force. The work-energy functional is expressed in terms of the deformed arc-length coordinate of the catenary riser. The strain energy due to shear is neglected.

A schematic of the static equilibrium and dynamic configurations of the large-sag extensible catenary riser and dynamic displacement from the static equilibrium position to the dynamic displaced position in u and v of the Cartesian coordinate system is illustrated in Fig. 2.

The arc length at the stretched state, ds, can be defined by

$$ds = \sqrt{(x_0' + u')^2 + (y_0' + v')^2}\,d\bar{s} \tag{7}$$

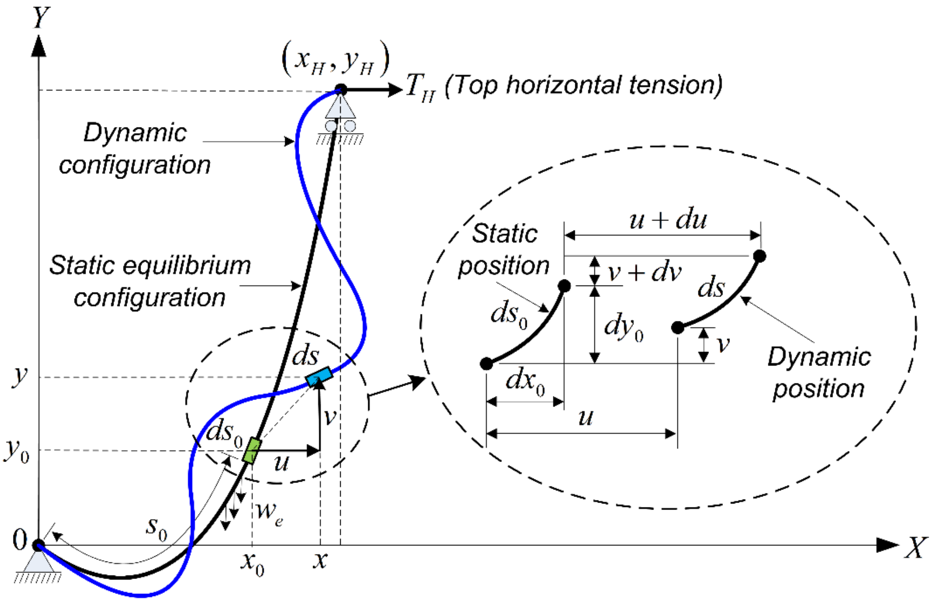

Fig. 2 Schematic of static and dynamic configurations of the catenary riser.

The strain in the displaced state, ε, followed by the total Lagrangian description can be expressed as

$$\varepsilon = \frac{ds - d\bar{s}}{d\bar{s}} = \frac{ds}{d\bar{s}} - 1 = \sqrt{(x_0' + u')^2 + (y_0' + v')^2} - 1 \tag{8}$$

and its derivative is

$$\delta\varepsilon = \frac{(x_0' + u')\delta u' + (y_0' + v')\delta v'}{\sqrt{(x_0' + u')^2 + (y_0' + v')^2}} \tag{9}$$

2.1 Virtual Strain Energy Due to Axial Deformation

The variation of the axial strain energy due to axial deformation can be expressed by

$$\delta U_a = \int_o^{S_t} EA\varepsilon(\delta\varepsilon) d\bar{s} \tag{10}$$

where E is the elastic modulus, A is the catenary riser cross section.
Substitution of Eqs. (8) and (9) into Eq. (10) gives

$$\delta U_a = \int_o^{S_t} EA\left(\sqrt{(x_0' + u')^2 + (y_0' + v')^2} - 1\right)\frac{(x_0' + u')\delta u' + (y_0' + v')\delta v'}{\sqrt{(x_0' + u')^2 + (y_0' + v')^2}} d\bar{s} \tag{11}$$