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Forced frequency-multiplication vibration of bridge with resilient supports under moving loads

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Abstract: Based on the modeling of the moving-loads-induced inhomogeneous excitation on the modal coordinates as multiple frequency-multiplication harmonics, this study investigates the forced multi-harmonic modal oscillation and resonance behaviors and characteristics of the single-span beam bridges with resilient supports under moving loads. The forced closed-form modal responses are derived to be composed of multiple frequency-multiplication steady-state harmonics and one damped single-frequency complementary harmonics. The analysis for the studied single-span beam bridges with resilient end rotation elastic restraints shows that moving vehicle load may induce super-harmonic forced resonance amplification when the moving speed is far below the critical speed. FEM structure modeling based numerical case studies on a single-span beam bridge verify the derived oscillation pattern and the resonance occurrence. Considering the variation of the end rotation elastic stiffness, the related multi-harmonic excitation amplitude, phase angle, and forced resonance amplification characteristics of the first two modes for beam bridge response estimation and analysis are presented.

Keywords: forced vibration; beam bridge; end rotation elastic restraints; moving load; multiple harmonics; frequency-multiplication

1. Introduction

The dynamic behavior of bridge structures with elastic or inelastic end restraints is a recent research focus under the framework of resilient bridge engineering. Most of the current researches on this topic focus mainly on the free vibration behavior analysis of the related system. For beams or plates with different boundary restraints, the modal characteristics equations are formulated and the modal frequency parameter distribution is computed (Kang and Kim 1996, Wang and Lin 1996, Naguleswaran 2002, Li *et al.* 2009, Hozhabrossadati *et al.* 2015). Considering different action and interaction mechanisms of the operation and environment loads with the bridges, structure forced vibration will present various patterns and characteristics. The forced vibration characteristics are very important for bridge resistance design and behavior interference.

Moving vehicle loads is a category of the most general loads for bridge structures. The excitation of the moving vehicle loads on the supporting bridge structures may induce various

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resonances (Rao 2000, Ju and Lin 2003, Xia *et al.* 2006, Sun 2016, Jin *et al.* 2016, Yang and Yau 2017) and diverse spatial-temporal shapes and patterns (Siddiqui *et al.* 1998, Yang *et al.* 2004, Gentile and Saisi 2015, Sun 2018) under different conditions. If the moving vehicle loads are simplified as a concentrated force, the governing equations in the modal domain are uncoupled and the analytical solutions of modal response were derived. The most classical analytical solution is derived for the single-span simply-supported uniform beam (Li *et al.* 1992). According to this derivation, the critical moving speed when the forced resonance occurs equals to the twice of beam spanning length divided by the period of the fundamental mode. Pesterev *et al.* (2003) analytically derived the maximum response function for a simply supported beam and numerically constructed this function for a clamped-clamped beam based on the assumption that the beam maximum response can be approximated using the fundamental mode. Mamandi *et al.* (2010) investigated the longitudinal-transversal nonlinear coupled dynamic behavior of an inclined uniform pinned-pinned beam under the simultaneous action of an axial force and transverse moving force considering the mid-plane stretching. Piccardo and Tubino (2012) analyzed the amplitude modulation characteristics and the maximum response of the EB beams under resonant harmonic moving loads. Johansson *et al.* (2013) derived the closed-form solution to the vibration of multi-span beam under moving loads by solving the governing equation of motion in the frequency domain using a Laplace transformation. Gašić *et al.* (2014) presented a study based on the assumed-modes with polynomial shape functions to approximately determine the dynamic response of a flexible L-shaped structure with lumped tip mass subjected to a moving load. Sudheesh Kumar et al. (2015) studied the free responses cancellation occurrence condition for the simply-supported uniform beams subjected to a single moving point load based on the closed-form expression. Maximov and Dunchev (2018) analytically investigated the dynamic response of bridge crane due to the telpher moving load based on a multi-harmonic interpolation of structure deflection shape.

In this study, the moving-load-induced multiple frequency-multiplication modal forced oscillation (MFMFO) characteristics of the supporting single-span Euler-Bernoulli beam bridges with resilient end rotation elastic restraints (ERERs) are analytically investigated. A unified load model to represent the moving-load-induced inhomogeneous excitation on the modal coordinates as multiple frequency-multiplication harmonics is proposed. The forced resonance occurrence condition is discussed and the closed-form modal non-resonance and resonance responses are derived. FEM structure modeling based numerical integration case studies are conducted to verify the analytical pattern and accuracy of the derived solution. Based on the analytical solutions, the modal oscillation amplitude magnification and phase angle characteristics for the single-span beam bridges with resilient ERERs are computed.

2. Formulation

2.1 System modeling

The studied structure is a single-span beam bridge of length L with a uniform cross-section and resilient ERERs as shown in Fig. 1. The vehicle loads are simplified as concentrated force acting on the gravity center of each vehicle of the magnitude F in a series with uniform spacing distance of l_o travelling in a constant speed of s. The governing equation of motion (EOM) for the vertical