Numerical simulation of dynamic fracture in functionally graded materials using peridynamic modeling with composite weighted bonds

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ABSTRACT

In this paper, the peridynamic (PD) method with the composite weighted bond for functionally graded materials (FGMs) was proposed. The cracks propagation in FGMs under dynamic loading was analyzed. Both uniform refinement grid (m-convergence) and decreasing radius of PD horizon (δ-convergence) of FGMs with double pre-crack were given. The effects of the micromodulus functions and material gradient form on the crack propagation were discussed, and it was found that the micromodulus function's effect on the crack propagation pattern was limited, while the effect of the gradient form of FGMs is existed. The numerical results show that there is almost no difference between the proposed composite weighted bond model and the previous model for FGMs with single pre-crack. By comparing the numerical results of the proposed model with the results from the experiment, the present PD model for FGMs is valid and effective.

1. INTRODUCTION

With the progress of science and technology in recent years, people's demands for material properties are getting higher. At present, composite materials have been widely used in engineering practice, and have achieved good results. However, traditional composite materials have obvious stress concentration due to the existence of material interface, which may not meet the need in some special practical environment. FGMs are designed by controlling the composition of the spatial distribution and the volume fraction of its components, so that the properties of the materials present a gradient change and eliminate the existing interface between different materials. The earliest proposal for the concept of FGMs was a program called "Basic Technology Research on the Development of FGMs for Reducing Thermal Stress" by the Japanese Science and Technology Agency in 1987, which was to improve the thermal insulation and engine
systems of spacecraft (Sobczak and Drenchev 2013).

FGMs was originally used to mitigate thermal stress. Applied to high temperature environment, especially the environment with large temperature difference on both sides of the material, its heat resistance, reusability and reliability is not comparable to the previous use of ceramic matrix composites. Now, FGMs has been used in metal, ceramic and organic composites to produce improved compositions and superior physical properties through the ingenious combination of inorganic and organic materials such as metals, ceramics, plastics (Kashtalyan and Menshykova 2009; Cheng et al. 2015). And FGMs have a wide range of applications in aerospace, biomedical, mechanical engineering, electromagnetic, nuclear engineering, civil and construction, energy engineering optics and other fields.

Due to the asymmetry of FGMs material properties, the crack propagation mechanism of numerical simulation FGMs under dynamic loading is still a great challenge to modern computational mechanics. At the same time, due to the complex fabrication process of FGMs and the harsh application environment of FGMs, fracture failure is one of the main destructive forms of FGMs (Kim and Paulino 2014). Delale and Erdogan proposed that the most prominent feature of FGMs is that the material parameter is a varies continuously function with spatial position, which is generally derivable, and they predicted that the problem of singularity for the FGMs whose material parameters with continuous derivability in the embedded crack tip is the same as that of homogeneous material (Delale and Erdogan 1983). With the deepening of the research, Erdogan proved that the singularity of the crack tip in FGMs is consistent with that of the homogeneous material (Erdogan 1985). Over the past several decades, many scholars have focused their research on the fracture problem associated with FGMs (Erdogan 1995; Rousseau et al. 2010; Jin and Batra 1996; Eischen 1987; Gu and Asaro 1997; Cheng et al. 2012; Kidane et al. 2010). The researchers studied the dynamic analysis of static cracks of FGMs under impact load (Guo et al. 2004) and crack propagation analysis of FGMs (Cheng et al. 2012; Itou 2010; Kidane et al. 2010; Parameswaran and Shukla 1999). Butcher et al. (Butcher et al. 1998) carried out the experimental studies about the material parameter measurement and dynamic fracture failure of the FGM specimen. The stress field at the crack tip of three-point bending specimen was measured by photoelastic technique and compared with the results of finite element method simulation.

Mathematical modeling and numerical simulation are very useful tools for the design and research of FGMs (Ma et al. 2009; Berezovski et al. 2003; Tilbrook et al. 2005; Kim and Paulino 2007; Doan et al. 2016; Parameswaran and Shukla 1998). The traditional research on FGMs is mainly based on the finite element method (FEM) and the boundary element method (BEM). Kim and Paulino established the orthogonal anisotropic FGMs model with continuous variation of the physical parameters using FEM, and analyzed the model of the internal arbitrary angle initial preset crack, and calculated the influence of the material heterogeneity on the stress intensity factors of
the crack tip (Kim and Paulino 2002). Yang et al. used FEM to simulate the crack bending and branching of brittle materials under dynamic biaxial loads (Yang et al. 2012). Dolbow and Gosz et al. studied the mixed stress intensity factors of FGMs using the extended finite element method (XFEM). FEM is a suitable method for studying fracture behavior. However, due to the changes in material properties of FGMs and remeshing, the FEM has many shortcomings in the study of dynamic failure processes of FGMs. Sladek et al. extended the meshless method based on the local Petrov-Galerkin method for stress analysis in two-dimensional (2D), anisotropic and linear elastic/viscoelastic solids with continuous material properties (Sladek et al. 2005). Chen et al. studied the FGM dynamic fracture problem in plane and three-dimensional (3D) cases using the meshless method (Chen et al. 2000). Although the meshless method has great flexibility in simulating the crack propagation in FGMs, the computational cost is still too high.

The peridynamics (PD) theory proposed by Silling was originally designed to simulate crack propagation and branching during dynamic fracture (Silling 2000; Silling et al. 2007). PD is an emerging method to establish model based on nonlocal theory and to describe the failure behavior of materials by solving the spatial integral equation. PD combines the advantages of molecular dynamics and meshless methods, avoiding the singularity and complexity of traditional macroscopic methods based on continuity hypothesis modeling and solving spatial differential equations in the face of discontinuous problems. Bobaru and Ha have employed PD to study dynamic crack propagation and branching, and concluded that the PD model simulating dynamic crack propagation is a reliable method (Ha and Bobaru 2010). Madenci et al. compared the authenticity of PD theory in predicting cracks propagation and concluded that PD theory is an analytical method for dynamic fracture problems with complex cracks in complex branching modes (Agwai et al. 2011). Bobaru and his colleagues conduct several studies on brittleness and dynamic fracture problems in composites by using PD, including crack branching, impact loading and fragmentation (Ha and Bobaru 2012; Bobaru et al. 2009; Hu et al. 2012; Bobaru and Zhang 2015).

In this study, we proposed a PD method based on composite weighted bond to simulate the dynamic fracture behavior of FGMs. Cheng et al. have studied the crack propagation of FGMs under dynamic loading through PD method and proved that PD is valid and effective in simulating the crack propagation in FGMs (Cheng et al. 2018). We developed a PD model for FGM by defining composite weighted bonds with the consideration of non-homogeneity in elastic and fracture properties. The influences of material gradient, crack, and bond model on the dynamic fracture behavior of FGMs are investigated.

2 Brief review of bond-based PD theory

Silling proposed the basic idea of the PD method (Silling 2000), which is a
continuum theory that considers non-local force interactions. It is assumed that an object occupies a certain spatial domain $R$, as shown in Fig. 1. It is also assumed that there is an interaction $f$ between material point $x$ and any other material point $x' \in R : \|x' - x\| \leq \delta$ within a certain range $\delta$ (PD horizon) in the domain.

$$f = f\{x, x', u(x, t), u(x', t), t\}$$

(1)

According to Newton’s second law, the basic equation of the PD method can be obtained (Silling 2003):

$$\rho \ddot{u}(x, t) = \int_{H(x)} f(u(x', t) - u(x, t), x' - x) dV_x + b(x, t) \forall x \in R, t \geq 0$$

(2)

where $\ddot{u}$ is the acceleration vector field, $u$ is the displacement vector field, $b$ is a prescribed body force intensity, and $\rho$ is mass density. Also, $f$ is the pairwise force function in the PD bond that connects material points $x$ and $x'$. The internal sub-region is schematically depicted in Fig. 2 and is defined as

$$H_x = \{ x \in R : \| x' - x \| \leq \delta \}$$

(3)

where $\delta$ is the horizon, the ‘size’ of the nonlocal interaction. Note that no spatial derivatives appear in Eq. (2).

Fig. 1. Interaction between materials points

Fig. 2. The deformation of a PD bond
A micro-elastic material is defined when the pairwise force derives from a micro-elastic potential $\omega$:

$$ f(\eta, \xi) = \frac{\partial \omega(\eta, \xi)}{\partial \eta} $$

(4)

Where $\xi = x' - x$ is the relative position and $\eta = u(x';t) - u(x,t)$ is the relative displacement between points $x$ and $x'$. A linear micro-elastic material is obtained if we take

$$ \omega(\eta, \xi) = \frac{c(\xi)s^2 \|\xi\|}{2} $$

(5)

$$ s = \frac{\|\xi + \eta\| - \|\xi\|}{\|\xi\|} $$

(6)

where $c(\xi)$ is called the micro-modulus function and represents the elastic stiffness of a bond. $s$ is the relative elongation of a bond.

The corresponding pairwise force can be obtained from Eq. (4) and (5):

$$ f(\eta, \xi) = \begin{cases} \frac{c(\xi)s^2 \|\xi\|}{2}, & \|\xi\| \leq \delta \\ 0, & \|\xi\| > \delta \end{cases} $$

(7)

In this paper, we use the constant and conical 2D plane-stress micro-modulus functions (Ha and Bobaru 2010). Following the same procedure performed to calculate the micro-modulus function in 1D (Bobaru et al. 2009). Under the assumption of 2D plane stress, the value for the 2D constant micro-modulus is derived as:

$$ c(\xi) = \frac{6E}{\pi \delta^3 (1-\nu)} $$

(8a)

and the value for 2D conical micromodulus is derived as:

$$ c(\xi) = c_0 \left(1 - \frac{\xi}{\delta}\right) = \frac{24E}{\pi \delta^3 (1-\nu)} \left(1 - \frac{\xi}{\delta}\right) $$

(8b)

where $E$ is the elastic Young’s modulus and $\nu$ is the Poisson ratio. In the bond-based PD, any particle inside the horizon of another particle interacts only through a central potential. This assumption results in an effective Poisson ratio of $1/3$ in 2D and $1/4$ in 3D. In this paper, we utilize the bond-based PD, thus, in all the reported simulations here the effective Poisson ratio is $1/3$.

The determination of failure in PD is very important. The material points are connected by a bond, and the destruction phenomenon occurs only when the material point bond breaks. At present, there are some methods to judge the breakage of point bond of substance according to the critical relative elongation $s_0$ and the critical point bond energy density $\omega_0$. In this paper, the critical relative elongation $s_0$ is used to judge
whether the material point bond is broken or not. We consider that the PD bonds can be broken when they are stretched beyond the "critical relative elongation, \( s_0 \). And \( s_0 \) is computed based on the fracture energy of a material. The energy per unit fracture length for complete separation of the two halves of the body is the fracture energy \( G_0 \), in plane stress conditions, the fracture energy can be derived as:

\[
G_0 = 2 \int_0^\delta \int_0^{s_0} \int_0^{\pi/2} \left( \frac{c(\xi)}{2} s_0^2 \xi^2 \right) \xi d\delta d\xi dz
\]  
\( \text{(9)} \)

Substituting the constant micro-modulus function and rearranging for \( s_0 \), we can rewrite this equation to obtain \( s_0 \):

\[
s_0 = \left( \frac{4\pi G_0}{9E\delta} \right)
\]  
\( \text{(10a)} \)

In similar way, the critical relative elongation for the conical micro-modulus function (Eq. 8b) is:

\[
s_0 = \left( \frac{5\pi G_0}{9E\delta} \right)
\]  
\( \text{(10b)} \)

The critical relative elongation depends on the material properties and the horizon \( \delta \). It is worth noting that when the horizon becomes zero, the critical relative elongation becomes infinite, so breaking such bonds requires more and more forces. This is consistent with the physical experience of atomic and subatomic scales. In order to split smaller and smaller sized bonds, one need more and more forces.

3 FGMs analysis based on PD model

PD is a new method, which avoids the singular problems faced by traditional numerical methods in discontinuous problems. Although more and more scholars have invested in it in recent years, the PD models for dynamic fracture problems of FGMs are also very limited. The whole scheme model is based on the bond PD model, and it is not necessary to predict the crack propagation path throughout the simulation process. Cheng et al. verified the feasibility of the scheme (Cheng et al. 2015), and it has certain advantages in dynamic crack propagation analysis.

3.1 Processing of FGMs parameters in PD

The main difference between FGMs and homogeneous materials, including composites, is the gradient distribution of material properties with respect to spatial coordinates. In the case of composite material composed of metals and ceramics, the properties of non-gradient composite material change abruptly, including elastic, thermal conductivity and coefficient of thermal expansion and there is a clear interface in the composite material where different materials are combined, which leads to stress concentration (see Fig.3a). The properties of gradient composite material composed of
metals and ceramics do not change abruptly, including elastic, thermal conductivity and coefficient of thermal expansion (see Fig. 3b). As shown in Figure 4, the performance of the FGMs varies along the thickness direction, including super resistance to effect of heat, mechanical property and thermal stress slope and mechanism. The FGMs transitions have no obvious material interface due to the change of its gradient form, which improves the stress concentration of the material.

![Fig. 3](image1.png)

(a) Material properties of metals and ceramics: (a) Non-gradient material properties; (b) Gradient material properties

![Fig. 4](image2.png)

Fig. 4. Material properties of FGMs

In PD, the fracture of the material is included in the constitutive function. There is no need to pre-set fracture criterion. For FGMs, it is considered as a locally homogeneous and overall non-homogeneous composite material. The local homogeneity here is the assumption that elastic parameters like elastic modulus, density, and fracture energy of a material at any point in the material and in a small enough area nearby are constant (Cheng et al. 2015). Due to this assumption, in the analysis of the isotropic linear elastic material, the Poisson's ratio of material is 1/3 for the plane problem, and the Poisson's ratio of the three-dimensional problem material is 1/4. For dynamic crack problems, the Poisson's ratio has no significant effect on the propagation velocity of crack or the shape of crack path (Delale and Erdogan 1983; Cheng et al. 2018). Therefore, in this paper,
the influence of Poisson's ratio is ignored, and the value is 1/3 directly.

In the local region, the material properties of material points A and B are different, as shown in Fig. 5. The micro-modulus of the bond between them is specified by using the average bond model (Cheng et al. 2015):

\[
G_0 = \frac{G_A + G_B}{2}; \quad E = \frac{E_A + E_B}{2}
\]

(11)

\(s_0\) is obtained as a critical relative elongation of the bond between two material points. However, the material parameters of the two points in FGM are obviously different, and using this average method will inevitably produce a small range of errors. With the accumulation of time steps, the error should not be ignored. Here, we introduce an optimization method to deal with the local material property mutation problem of PD model in FGMs. A new composite weighted bond is proposed to avoid the effect of local material properties mutation on crack propagation and reducing the error with considering the effect of material properties on crack propagation synthetically, such as Young's modulus and fracture toughness. Moreover, it is closer to the physical truth of material from microcosmic perspective.

3.2 Composite weighted bonds model in FGMs

Fig. 5. Material points in FGM: (a) bonds between different material points; (b) energy-bond between points A and B

It is seen from the constitutive function of the PD that the essence of the PD bond is the energy bond. When discriminating failure, there are methods such as critical relative elongation \(s_0\) and critical energy density \(G_0\), but the essence is based on energy bonds. In this paper, the critical relative elongation \(s_0\) is defined. The "contribution" of fracture energy \(G_A\) and \(G_B\) is different when the bond between point A and B is broken (see Fig. 5b). In this paper, the fracture energy \(G_A\) and \(G_B\) of the two material points A and B are weighted in the case of bond failure to describe the effect of different \(G_A\) and \(G_B\) on the bond:
\[ G_0 = \alpha G_A + \beta G_B; \quad E = mE_A + nE_B \]  

(12)

The fracture energy and Young's modulus of the two material points is proportional to the effect of the bond, so the weight functions \( \alpha \) and \( \beta \), \( m \) and \( n \) have the following forms:

\[ \alpha = \frac{G_A}{G_A + G_B}; \quad \beta = \frac{G_B}{G_A + G_B}; \quad m = \frac{E_A}{E_A + E_B}; \quad n = \frac{E_B}{E_A + E_B} \]  

(13)

4. Convergence studies in dynamic crack branching of FGMs

In this section, composite weighted bonds modeling is used to study the convergence of double pre-cracks. We consider a thin FGM rectangular plate with 152 mm × 37 mm and two parallel cracks with length 9 mm and the distance between the two cracks is 30 mm. The geometric size and boundary conditions are shown in Figure 6. The load \( \sigma = 45 \) MPa impact on upper end of the specimen (see. Fig.6) and the curve of the load versus time is plotted in Fig. 7. The conical micro-modulus function (eq. 10b) is used in this section. The mechanical properties of material parameters are shown in Table 1. In this section, PD convergence of FGM rectangular plates with linear gradient form is investigated to further prove the feasibility of the PD method in studying crack bending of gradient materials under dynamic loading conditions.

![Fig. 6. Specimen geometry and boundary condition](image)

![Fig. 7. Applied load versus time](image)

<table>
<thead>
<tr>
<th>TABLE 1. Mechanical properties of FGM specimen</th>
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<tbody>
<tr>
<td>Position</td>
</tr>
<tr>
<td>stiff edge (E₂)</td>
</tr>
<tr>
<td>compliant edge (E₁)</td>
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</tbody>
</table>
4.1. δ-convergence of FGM specimens

For the δ-convergence, we select a fixed value \( m = 4 \), the horizon size \( \delta = 1.6 \text{ mm} \) (\( \Delta x = 0.4 \text{ mm} \)), \( \delta = 2.0 \text{ mm} \) (\( \Delta x = 0.5 \text{ mm} \)), \( \delta = 3 \text{ mm} \) (\( \Delta x = 0.75 \text{ mm} \)), \( \delta = 4 \text{ mm} \) (\( \Delta x = 1 \text{ mm} \)), respectively. A uniform time step size \( \Delta t = 0.04 \mu s \) is used, and impact load \( \sigma = 45 \text{ MPa} \) is applied to the FGM models in the section. When the grid density \( \Delta x = 0.4 \text{ mm} \), the total number of the discrete material points is 35908, at this time \( \delta = 1.6 \text{ mm} \). When the grid density is \( \Delta x = 0.5 \text{ mm} \), the total dispersion is 23256 material points, \( \delta = 2 \text{ mm} \). When the grid density \( \Delta x = 0.75 \text{ mm} \), it is divided into 10455 material points, \( \delta = 3 \text{ mm} \). When the grid density \( \Delta x = 1 \text{ mm} \), it is divided into 6006 material points, \( \delta = 4 \text{ mm} \). From the δ-convergence of Fig. 8a, b c and d, it can be found that the path trend of cracks is same. When \( \Delta x = 1 \text{ mm} \) and \( \Delta x = 0.75 \text{ mm} \), the mesh is too rough. The computational time of \( \Delta x = 0.4 \text{ mm} \) is much more than \( \Delta x = 0.5 \text{ mm} \), so considering the computational efficiency and accuracy, \( \Delta x = 0.5 \text{ mm} \) is relatively appropriate. In conclusion, considering the mesh density and accuracy of the model, the effect is best when \( \Delta x = 0.5 \text{ mm} \), \( m = 4 \) (\( \delta = 4 \times \Delta x = 2 \text{ mm} \)).

(a)

(b)

(c)
Fig. 8. δ-Convergence in terms of crack path for an FGM plate computed with different horizon sizes (and grids corresponding to $m = \delta / \Delta x = 4$): (a) $\Delta x = 1$ mm, $\delta = 4$ mm, (b) $\Delta x = 0.75$ mm, $\delta = 3$ mm, (c) $\Delta x = 0.5$ mm, $\delta = 2$ mm, and (d) $\Delta x = 0.4$ mm, $\delta = 1.6$ mm.

4.2. $m$-convergence of FGM specimens

For the $m$-convergence, the selection of FGM model and boundary conditions are same as the δ-convergence, we select a fixed horizon size $\delta = 2$ mm, respectively take $m = 2$ ($\Delta x = 1$ mm), $m = 3$ ($\Delta x = 0.67$ mm), $m = 4$ ($\Delta x = 0.5$ mm), $m = 5$ ($\Delta x = 0.4$ mm). A uniform time step size $\Delta t = 0.04$ μs is used. When the grid density $\Delta x = 0.4$ mm, the total number of the discrete material points is 35908, at this time $m = 5$. When the grid density is $\Delta x = 0.5$ mm, the total dispersion is 23256 material points, $m = 4$. When the grid density $\Delta x = 0.67$ mm, it is divided into 13340 material points, $m = 3$. When the grid density $\Delta x = 1$ mm, it is divided into 6006 material points, $m = 2$. The crack paths of PD simulation are shown in Figure 9. Moreover, the crack paths have not different with the $m$ increase, but become clearer and clearer. A large $m$ value requires a higher calculation cost, but the result is not affected. In the later simulation study, we will choose $m = 4$, which is a good choice.
4.3. $\Delta t$-convergence of FGM specimens

For time integration we use an explicit method, the Velocity-Verlet algorithm. The Velocity-Verlet algorithm (Ha and Bobaru 2010) is:

$$u_{n+\frac{1}{2}} = u_{n} + \frac{\Delta t}{2} u'_{n}$$  \hspace{1cm} (14)

$$u_{n+1} = u_{n} + \Delta t u'_{n+\frac{1}{2}}$$ \hspace{1cm} (15)

$$u'_{n+1} = u'_{n+\frac{1}{2}} + \frac{\Delta t}{2} u''_{n+1}$$  \hspace{1cm} (16)

Where $u$, $u'$ and $u''$ donate displacement, velocity and acceleration vectors, respectively, $\Delta t$ is the time step size per iteration. Silling has pointed out that a computationally stable time step size $\Delta t$ should satisfy (Silling and Askari 2005):

$$\Delta t < \frac{2\rho}{\sum V_c}$$ \hspace{1cm} (17)

Bobrau has studied the brittle fracture on the convergence of the PD model using the time step size for $\Delta t = 25$ ns, which was considered as the stable time step for the finest model (Ha and Bobaru 2010). Here, we study the convergence of the time step size for FGM samples to find the optimal stable time step of the FGM model.

For convergence study of time step, we select a horizon size $\delta = 2$ mm, the grid spacing for $\Delta x = 0.5$ mm ($m = 4$), and the time step size of comparison is $\Delta t = 0.02$ $\mu$s, $0.04$ $\mu$s, $0.06$ $\mu$s. The crack paths of FGM samples in various $\Delta t$ are compared, as shown in Figure 10. Because the meshing of the model is the same here, the models with various time step size $\Delta t$ has the same crack width. The crack paths with $\Delta t = 0.02$ $\mu$s, $0.04$ $\mu$s and $0.06$ $\mu$s are very similar, the crack paths expanded faster as the $\Delta t$
changed (see Figure10). Considering the calculation cost, the time step size $\Delta t = 0.04$ μs is a good choice, which is a precise and stable explicit central difference method.

Based on the PD convergence research of FGM material, we adopt these parameters with $\delta = 2$ mm, $m = 4$, $\Delta t = 0.04$ μs, to balance the computational cost and accuracy requirements after the numerical simulation of the FGM specimen.

Fig. 10. Crack path computed with different time step size for $\delta = 2$ mm, $\Delta x = 0.5$ mm, $m = 4$: (a) $\Delta t = 0.02$ μs; (b) $\Delta t = 0.04$ μs, (c) $\Delta t = 0.06$ μs

5 FGMs model

5.1 Problem setting

Fig. 11. Specimen geometry and boundary condition Fig. 12. Applied load versus time
As shown in Fig. 11, the calculation model is a 152 mm×37 mm rectangular FGM sheet with an edge crack of length a = 9 mm. And the load σ = 33 MPa impact on upper end of the specimen. The load acts only on the boundary of the specimen without any other boundary conditions. In this paper, $E_1$ represents the elastic modulus at the bottom of the specimen, and $E_2$ represents the elastic modulus of the upper part of the specimen. In the original paper experiment, the upper part was impacted by a pneumatic impact hammer, and the direct stress application method was used instead here. Since the rebound time of the air hammer is not known here, it is assumed here that the air hammer rebounds at 100 μs, that is, the time of the applied impact force $T = 100$ μs. The physical parameters of the FGM sheets used in this paper are shown in Table 1. Young's modulus, density, and fracture toughness are same with the FGM parameters used in the experiment of Roussesu and Tippur (Roussesu and Tippur 2001). The impact load is applied to upper ends of the specimen, and the magnitude of the force is shown in Fig. 12.

5.2 FGM material parameters in the experiment

The material parameters were derived from paper (Roussesu and Tippur 2001), and the samples tested in this study were made from epoxy systems in different amounts with solid soda lime glass beads (average diameter is 42 μm). The slow curing and low shrinkage properties of the epoxy resin ensure the minimum residual stress in the granular composite. Microscopic examination of the composite showed that the inclusions were uniformly dispersed in the matrix. In addition to the Poisson's ratio reduction in the test of the sample, other properties monotonously increase as the volume $V$ of the mixed glass beads increases. When the volume fraction $V$ varies between 0 and 0.5, the Young's modulus increases nearly threefold. With the corresponding volume fraction, the Poisson's ratio decreases from 0.37 to 0.32. For the dynamic crack problem, the Poisson ratio has no significant effect on the propagation velocity or the shape of the crack path. Therefore, the effect of Poisson's ratio is ignored in this article, assuming that it is constant 1/3 in FGM. And the mechanical properties of material parameters are shown in Table 1.

The parameters of the material increases monotonically as the filling volume increases, and in the variation from the bottom to the top of the specimen, the density and elastic modulus are approximately assumed to vary linearly. Studies have shown that no matter what form of the gradient of volume fraction (exponential form and trigonometric form, etc), the fracture toughness gradient always varies linearly in dynamic fracture tests (Kirugulige and Tippur 2006). Therefore, it is assumed that the parameters of the material vary linearly, and the elastic modulus, density and fracture toughness functions are as follows:
The 2019 World Congress on
Advances in Structural Engineering and Mechanics (ASEM19)
Jeju Island, Korea, September 17 - 21, 2019

\begin{align*}
E &= 3.811 \times 10^9 + \frac{(11.13 - 3.811) \times 10^3}{W} \times y \\
p &= 0.948 \times 10^9 + \frac{(1.812 - 0.948) \times 10^3}{W} \times y \\
K_{ICR} &= 2.1 \times 10^6 + \frac{(3.6 - 2.1) \times 10^6}{W} \times y \\
G_{ICR} &= \frac{K_{ICR}^2}{E}
\end{align*}

(18) \quad (19) \quad (20) \quad (21)

The maps with the linear curve fit for density and elastic modulus, as well as the variation of fracture energy and fracture toughness along the width of the specimen, are shown in Fig. 13.

![Fig. 13](image)

(a) Linear curve fits for elastic modulus and density across the width of the specimen. (b) Fracture energy and fracture toughness variation across the specimen's width.

6 Numerical simulation results
6.1 FGM plate with Single pre-crack under impact load

The single pre-crack specimen model and its loading conditions are shown in Fig. 11 and 12, respectively. The mechanical parameters of the FGM specimen are shown in Table 1.

![Diagram](image1.png)

Fig. 14. Two forms of crack initial time under the constant micro-modulus function modeling: (a1), (a2), (a3) for average bonds; (b1), (b2), (b3) for composite weighted bonds

(a1) $t = 70 \mu s$  
(b1) $t = 70 \mu s$  

(a2) $t = 70 \mu s$  
(b2) $t = 70 \mu s$  

(a3) $t = 76 \mu s$  
(b3) $t = 76 \mu s$
Fig. 15. Strain energy density at different times under constant micro-modulus modeling: (a1), (a2), (a3) and (a4) for average bond model; (b1), (b2), (b3) and (b4) for composite weighted bond model

The specific parameters of the two-dimensional model of PD constructed in this paper are: uniform discretization according to the lattice constant $\Delta x = 0.5$ mm. According to the research results and suggestions in paper (Ha and Bobaru 2010), the lattice constants of 4 times in the PD horizon is taken ($\delta = 2$ mm). Using Velocity-Velert time integrals, the displacement vectors and the corresponding velocities of each material point are obtained as the solutions of the control Eq. (2), in order to guarantee the stability and convergence of the calculation results, the time iteration step takes $\Delta t = 0.04$ $\mu$s.

The crack initiation time of the average bond model and the composite weighted
bond model for the constant micro-modulus function under impact loading was shown in Fig. 14. It can be found that the crack initiation time of the two models are both at about 76 μs. Some snapshots of the strain energy density for the two bond models are shown in Fig. 15. According to the strain energy density scale DAM map, the strain energy density region of the composite weighted bonds model is smaller than that of average bonds, that is, the strain energy density of the composite weighted bonds is smaller. It can be observed that there is few difference between the crack paths of the two models. It means that both the two bond models have little influence on the numerical results of FGM plate with edge crack under impact loading. The composite weighted bond model can be used to investigate the dynamic fracture behavior of FGM.

![Fig. 15. Two forms of crack initial time under conical micro-modulus function modeling: (a1), (a2), (a3) for average bonds; (b1), (b2), (b3) for composite weighted bonds](image-url)
Fig. 17. Strain energy density at different times under conical micro-modulus modeling: (a1), (a2), (a3) and (a4) for average bond model; (b1), (b2), (b3) and (b4) for composite weighted bond model
The crack initiation time of the average bond model and the composite weighted bond model for the conical micro-modulus function under impact loading was shown in Fig. 16. It can be found that the crack initiation time of the two models is almost the same, but the crack initiation time $t = 83 \mu s$, which is later than the constant micro-modulus. Comparing Fig. 17, the strain energy density of two models are almost the same in a short period of time, and there are little differences between $t = 100 \mu s$ and $t = 125 \mu s$, but the crack paths in the two models are basically the same.

Fig. 19 shows local enlargement diagram of the paper experiment. Compared with the experiment, the simulation has certain error, but the overall crack path is quite same. The time required for the first shock wave measured in the experiment to reach the bottom from the top is 12.5 $\mu s$. From the Fig. 18, it can be found that the time required for the first shock wave to reach the bottom is about 12 $\mu s$, which is closer to the experimental results. In the experiment, the crack initiation time is about 85 $\mu s$, which is closer to the result of the selection of the conical micro-modulus modeling. In addition, the crack path is closer to the experimental results at $t = 100 \mu s$ and $t = 125 \mu s$.

6.2 FGM plate with double cracks under impact load

In this section, we show two sets of numerical samples for FGM plates with two...
parallel edge cracks under impact loading. Two parallel cracks with length 9 mm and the distance between the two cracks is 30 mm. The configuration of the samples is shown in Fig. 6. The impact load $\sigma = 45$ MPa was applied on the upper surface and the curve of the load versus time is plotted in Fig. 7. The loading time is 100 $\mu$s. The conical micro-modulus function and composite weighted bonds are used in this section. The discrete model of PD $\delta = 2$ mm, $m = 4$, a uniform time step size $\Delta t = 0.04 \mu$s, the grid density $\Delta x = 0.5$ mm and the total dispersion is 23256 material points.

Fig. 20 Strain energy density and crack paths of FGM plate at $t = 178 \mu$s under PD modeling with composite weighted bonds: (a) crack paths, (b) strain energy density

Fig. 21 Strain energy density and crack paths of FGM plate at $t = 240 \mu$s under PD modeling with composite weighted bonds: (a) crack paths, (b) Strain energy density

In Fig. 20 and 21, some snapshots taken at 178 $\mu$s and 240 $\mu$s from the initial impact were shown. It can be observed that the initial propagation of the two cracks is
close to straight, but soon after, it begins to deviate from the straight line and it propagates to the center line of the samples. Before the branches reach the free edge, the elastic waves mainly propagate along and ahead of the branching directions. After the crack paths reach the free edge, the elastic waves are reflected and concentrate around the faces of the crack paths (Ha and Bobaru 2011). It can be explained that the Rayleigh wave generate during the unloading phase, then attract the cracks and the cracks are sensitive to the strain energy density concentration created by the moving Rayleigh wave on the top surface.

6.3 Study of FGM specimen with different gradient forms for double pre-crack under tensile loading

The purpose of this section is to investigate the effect of gradient form of FGM parameters on crack growth under double pre-crack condition. We consider a thin FGM rectangular plate with 50 mm × 25 mm and two cracks satisfies \( a_1 + a_2 = a \) / \( W = 0.5 \), \( a_1 = a_2 = 6.25\text{mm} \), distance of two cracks is \( s = 0.5W \). The initial cracks are the bottom cracks \( a_1 \) and \( a_2 \), the geometric size and boundary conditions are shown in Fig. 22. The conical micro-modulus function (eq. 10b) and composite weighted bonds are used in this section. The material composition of FGM rectangular plate is epoxy mixed with a certain amount of glass to make its mechanical properties change step by step. The tensile loading is applied to both ends of the specimen, and the magnitude of the force is shown in Fig. 23. In this case, the double pre-crack is chosen to be parallel and located on the side of larger elastic modulus. The discrete model of PD \( \Delta x = 0.25 \text{mm} \), \( \delta = 1 \text{mm} \), \( m = 4 \), and the total dispersion is 20604 material points. In order to ensure the stability and convergence of the calculation results, the uniform time step size \( \Delta t = 0.05 \text{μs} \). The mechanical properties of material parameters are shown in Table 2. The values of density, elastic modulus and fracture toughness of the FGM specimen were taken from the results of Kirugulige and Tippur (Kirugulige and Tippur 2008). The gradient form of material is also divided into an exponential function and a trigonometric function.

![Fig. 22. Specimen geometry and boundary condition](image-url)
When the material gradient form is exponential function, we use the following exponential function curve fitting:

\[ E = E_i \times e^{\alpha y} \]

\[ \alpha = \frac{\ln(E_i / E_0)}{W} \]  

\[ \rho = \rho_0 \times e^{\beta y} \]

\[ \beta = \frac{\ln(\rho_0 / \rho_0)}{W} \]  

\[ K_{\text{IC}} = 2.2 \times 10^6 \times e^{\gamma y} \]

\[ \gamma = \frac{\ln(1 / 4 \pi \cdot 2)}{W} \]  

**TABLE 2. Mechanical properties of FGM specimen**

<table>
<thead>
<tr>
<th>Position</th>
<th>Elastic modulus (E) (MPa)</th>
<th>Density (\rho) (kg/m(^3))</th>
<th>Poisson ratio (\nu)</th>
<th>Fracture toughness (K_{\text{IC}}) (MPa (\cdot) m(^{1/2}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compliant edge ((E_2))</td>
<td>4000</td>
<td>1175</td>
<td>0.33</td>
<td>1.4</td>
</tr>
<tr>
<td>Stiff edge ((E_1))</td>
<td>10000</td>
<td>1750</td>
<td>0.33</td>
<td>2.2</td>
</tr>
</tbody>
</table>

When the material gradient form is exponential function, we use the following exponential function curve fitting:

\[ E = E_i \times e^{\alpha y} \]

\[ \alpha = \frac{\ln(E_i / E_0)}{W} \]  

\[ \rho = \rho_0 \times e^{\beta y} \]

\[ \beta = \frac{\ln(\rho_0 / \rho_0)}{W} \]  

\[ K_{\text{IC}} = 2.2 \times 10^6 \times e^{\gamma y} \]

\[ \gamma = \frac{\ln(1 / 4 \pi \cdot 2)}{W} \]  

(a\(_1\)) \(t = 25 \mu s\) 
(b\(_1\)) \(t = 25 \mu s\) 
(a\(_2\)) \(t = 57 \mu s\) 
(b\(_2\)) \(t = 57 \mu s\)
Fig. 24. Crack propagation path and strain energy density in the FGM specimen at various moments for different tensile loading times: (b1), (b2), (b3), (b4), (b5) strain energy density; (a1), (a2), (a3), (a4), (a5) crack propagation path.

When the material gradient form is trigonometric function, we use the following trigonometric function curve fitting:

\[ E = (E_1 + E_2) - (E_1 - E_2) \sin \left( \frac{\pi W}{2} \right) \times \gamma \]  
(25)

\[ \rho = \left( \frac{\rho_1 + \rho_2}{2} \right) - (\rho_1 - \rho_2) \sin \left( \frac{\pi W}{2} \times \gamma \right) \]  
(26)

\[ K_{ICR} = 1.8 \times 10^5 - 0.4 \sin \left( \frac{\pi W}{2} \times \gamma \right) \]  
(27)

Note: Regardless of exponential function or trigonometric function, the relationship between fracture energy and fracture toughness and elastic modulus is Eq. (17).
Fig. 25. Crack propagation path and strain energy density in the FGM specimen at various moments for different tensile loading times: (b_1), (b_2), (b_3), (b_4) strain energy density; (a_1), (a_2), (a_3), (a_4) crack propagation path.

It can be seen from Figs. 24(a_1) and (b_1) that the crack initiation time $t = 25 \, \mu s$ when the gradient form is exponential function. When the gradient form is trigonometric function, the crack initiation time is $t = 62 \, \mu s$ (see Figs. 25(a_1) and (b_1)). The crack initiation time of exponential function in gradient form is earlier than that of trigonometric function. When the gradient form is exponential or trigonometric function, the crack penetrates through the specimen completely at $t = 92 \, \mu s$ (see Figs. 24(a_5) and (b_5)), $t = 189 \, \mu s$ (see Figs. 25(a_4) and (b_4)), respectively. It is earlier than when the gradient form is trigonometric, the crack propagation speed is faster when the gradient form is an exponential function (see Figs. 24 and 25). When the gradient form is trigonometric function, it can be seen that cracks are more difficult to crack and propagate than when the gradient form is exponential function. On the whole, the crack propagation path is the same, and the deflection angle increases gradually on the side with smaller material parameters, but the crack is broken in a straight along the gradient direction before it is fully penetrated.

7 Conclusions
In this paper, a composite weighted bond PD model is built to study the crack propagation in FGM under impact loading. By comparing with the existed PD model for FGM, the numerical results of the crack initiation time and the crack path from the composite weighted bond PD model showed that the present model is effective and valid to studying the dynamic fracture of FGM. The FGM plate with two parallel edge cracks under impact loading was further studied. The results showed that both the cracks propagate toward the center of the sample.

REFERENCES


