In Eq. (2), P is the maximum pullout load, P_0 is the maximum pullout load at 0 degree orientation angle, f is snubbing coefficient, θ is orientation angle. In this study, snubbing coefficient is calculated by the first peak load per unit bond length obtained from Fig. 5. Fig. 6 shows the relationship between the first peak load per unit bond length and orientation angle. Exponential approximations are carried out by the least square method. The results are shown as the curves in Fig. 6, which indicate the snubbing coefficients of 1.2 and 0.98 in the case of set angle and embedded angle, respectively.

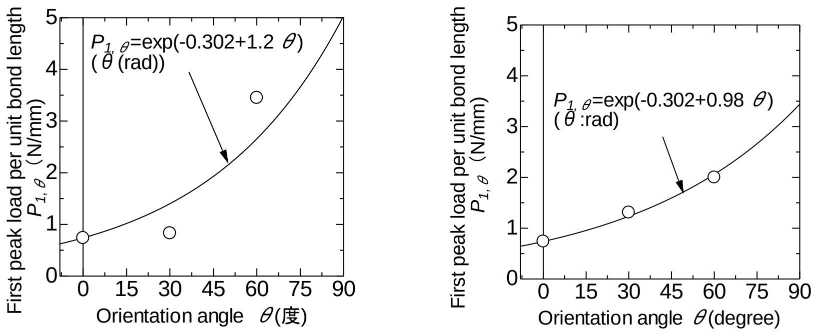

Fig. 6 Relationships between first peak load per unit bond length and orientation angle (left : set angle, right : embedded angle)

Fig. 7 shows the relationship of slip at first peak and bond length. The plots in Fig. 7 are approximated linearly by the least square method as expressed by Eq. (3).

$$s_{\max} = s_1 \cdot l_b \tag{3}$$

In Eq. (3), s_{max} is slip at first peak load, s_1 is slip at first peak load per unit bond length. The approximation result shows that s_1 is 0.19.

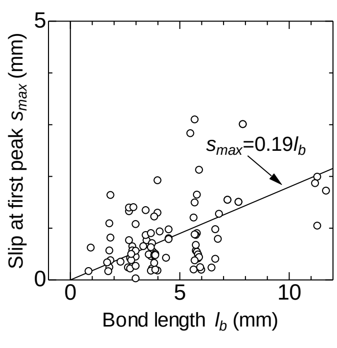

Fig. 7 Relationship between slip at first peak and bond length

To calculate the bridging law which is expressed by tensile stress-crack width relationship, the model should be given by crack width. Crack width is assumed to be two times slip by considering that the fiber is pulled out from the both ends in matrix, as given by Eq. (4).

$$\delta_{max} = 2s_{max} \tag{4}$$

By considering the influence of bond length and orientation angle, the single fiber pullout model is expressed by P_{max} and δ_{max} as Eq. (5).

$$P_{i,j}(\delta,\theta,l_b) = \frac{P_{\max}}{\delta_{\max}} \cdot \delta \quad (\delta \leqq \delta_{max})$$
$$P_{i,j}(\delta,\theta,l_b) = P_{\max} - \frac{P_{\max}}{l_b - \delta_{\max}} \cdot (\delta - \delta_{\max}) \quad (\delta_{max} < \delta) \tag{5}$$

4.3 Calculation of bridging law

In FRCC, fibers bridge across the crack as shown Fig. 8. The bridging law is calculated by applying the single fiber pullout model to each fibers. In this study, the bridging law is calculated by assuming the sectional area of 50mm x 50mm. Each fiber follows the elliptic distribution as same as previous researches (Kanakubo et al. 2016). The elliptic distribution is expressed by the orientation intensity *k* and the principal orientation angle *θr*. The principal orientation angle *θr* expresses principal orientation direction of fibers in crack plane. The orientation intensity *k* shows the fiber orientation tendency to principal orientation angle. In this study, *k* is set to 1 expressing the random orientation. The bridging law can be calculated by the sum of pullout load of each single fiber, as given by Eq. (6).

$$P(\delta) = \sum_h \sum_i \sum_j \{N_f \cdot P_{ij}(\delta,\psi,l_b) \cdot p(\theta_i) \cdot p(\phi_i) \cdot p_d(y_h, z_h) \cdot \Delta\theta \cdot \Delta\phi \cdot \Delta A\} \tag{6}$$

$P(\delta)$ = tensile force
$P_{ij}(\delta,\psi)$ = tensile force of single fiber
$P(\theta_i)$, $p(\varphi_j)$ = probability based on elliptic distribution for x-y plane and z-x plane
$P_d(y_h,z_h)$ = probability of fiber distribution along x axis (constant)
θ, φ = refer to Fig.8
N_f = number of fibers in crack plane

N_f can be expressed by Eq. (7).

$$N_f = V_f \cdot A_m / A_f \tag{7}$$

V_f = volume fraction of fiber
A_m = cross sectional area of matrix
A_f = cross sectional area of fiber

Table. 3 shows the parameters for calculation of bridging law.

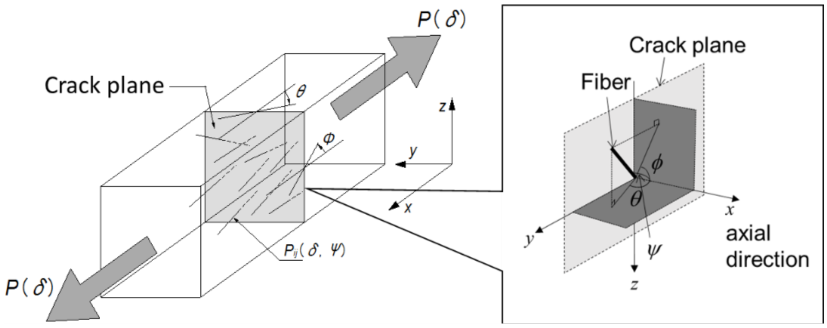

Fig. 8 Bridging fibers in crack plane

Table 3 Input parameters to calculation of bridging law

Input item	Input number	
	Embedded	Set
Cross sectional area of matrix A_m (mm^2)	50 x 50	
Fiber length l_f (mm)	13	
Snubbing coefficient f	0.98	1.2
Peak load per unit bond length at orientation angle 0 degree $P_{1,\theta}$ (N/mm)	0.739	
Crack width at first peak load per unit bond length of orientation angle 0 degree δ_1 (mm)	0.38 (0.19 x 2)	
Fiber volume fraction V_f (%)	1.0	
Orientation intensity k	1	

4.3 Result of calculation

The calculation results of tensile stress-crack width relationships are shown in Fig. 9. Tensile stress in the case of set angle is larger than that of embedded angle.

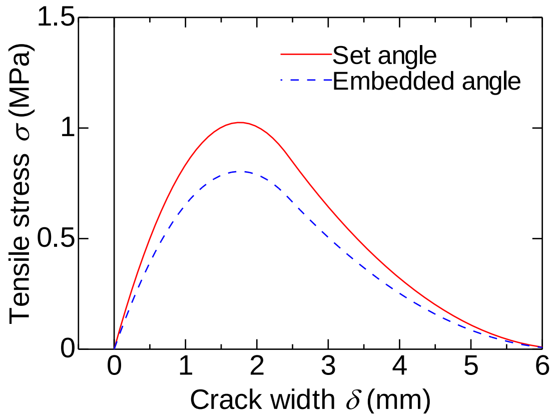

Fig. 9 Result of calculation

5. CONCLUSIONS

The result of single steel fiber pullout test shows that the snubbing coefficient in the case of set angle is larger than that of embedded angle. The results of calculation of bridging law using those snubbing coefficients show that the difference of snubbing effect causes the difference in tensile stress.

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