surroundings. (Shahinpoor 2001) introduced IPMC properties and its characteristics while performing as actuators and sensors which provide a wide understanding of the material. After this, as the material shows various aspects of applications review work of (Kim 2003) on IPMC manufacturing showed the techniques to produce the material using base Nafion polymer with platinum electrodes on both sides. Review work on modelling and simulation of IPMC as actuator and sensor is shown (Shahinpoor 2004) and work was also done to elaborate the applications of IPMC in the industrial and medical field as well by (Shahinpoor 2005). (Chen 2010) proposed a physics-based model for a biomimetic robot actuated by IPMC and the work is experimentally validated making the fish-shaped robot with a flexible IPMC tail oscillating in order to propel forward. (Aureli 2010) proposed a mathematical model which helps in understanding the condition of locomotion when the vibrating IPMC acting as the caudal fin propels the robotic fish like structure. They also suggested that the IPMC is very crucial as an actuator as it produces large deflection in very low power consumption. Work has also been done in mimicking the engagement of a robotic fish with the shoal of gregarious fish which completely follows the group as commanded through proper control of locomotion by (Aureli 2010). A PID closed loop position control model is also presented which will be helpful for mimicking the vibrating propulsor of a robotic fish-like motion more accurately by (Gupta 2018). It is well explained by (Lighthill 1960) that the efficiency of undulatory motion is much higher than the oscillatory motion in the biomimetic system. (Kim 2005) worked on biomimetic undulatory tadpole robot actuated by IPMC and suggested that the IPMC has a significant advantage for using as propulsion tail for underwater robots. Their proposed work is modelled according to the physical resemblance of bullfrog tadpole where it consists of the rigid head with a flexible body which oscillates and due to polydimethylsiloxane a polymer material which is fabricated over the IPMC to get an undulatory motion for locomotion. (Laurent 2001) compared the efficiencies of oscillatory and undulatory locomotion for a microrobot actuated by IPMC over a millimeter scale and suggested that the undulatory locomotion is more effective than oscillatory. (Guo 2003) illustrated an effective and new prototype of underwater micro robot actuated by two ionic conducting polymer film (ICPF) with the fin where the swimming speed and moving directions can be controlled by changing the applied frequency and voltage. They also explained the theory related to the undulatory motion of fish where carangiform mode involves tail portion and anguilliform mode of undulatory swimming involves most body part. (Zhang 2006) investigated computational fluid dynamics of undulatory fin actuated by shape memory alloy (SMA) for force calculations using unstructured mess to compute unsteady flow around the fin. Experimentation is done for complete swimming cycles calculating the velocity profile for the proposed mechanical fin. They concluded the importance of undulatory amplitude, wavelength and frequency for the thrust generation by the mechanical fin. (Liu 1996) studied the undulatory propulsion of tadpoles using computational fluid dynamics for their proposed 2D model. Investigations of thrust generation along the flow patterns also suggested that the Froude efficiency increases with an increment in Reynolds number. They also concluded that their CFD model enabled them to work on large size tadpoles producing effective jet stream canceling all previous thoughts about tadpoles. (Liu 1997) studied swimming of tadpoles using 3D modelling geometry. They confirmed their results with their 2D study and confirmed the kinematics of tadpole with a characteristic shape. Simulation work is done using 3D computational fluid dynamic methods for locomotion study by taking a virtual 3D tadpole and making it swim with lower amplitude propulsive waves using different grid topology for the finite volume method. The hydrodynamic model was prepared by (Shen 2013) considering the different fluid viscosities for locomotion of robotic fish and also considering various forms of locomotion like undulatory varying from thunniform to anguilliform type by varying input frequencies to IPMC. Velocity and thrust measurements are done analytically and experimentally for validation as well and suggested that the input voltage and IPMC size play important role in thrust efficiency. In the review paper by (Salazar 2018), where the various kinds of bioinspired aquatic systems are studied and suggested that the swimming efficiency of anguilliform mode of locomotion adopted by eels is high.

In this paper, a kinematic and dynamic model for a biomimetic underwater robot actuated by ionic polymer metal composites (IPMC) is proposed. The Nafion polymer electroplated both sides by platinum form the structure of IPMC and this can be seen as a simple RC circuit. The relation between the charge stored and the applied voltage to actuate IPMC is explained by the ordinary differential equation for the RC circuit. In order to incorporate thrust introduced by electrically actuated IPMC, we have considered bending moment produced by the ionic movements. The base of the kinematic and dynamic model is elongated body theory. The application aspect is underwater propulsion so the hydrodynamic parameters are considered in the proposed analytical model. Hydrodynamic performance is evaluated through simulations considering the well-defined parameters. The biomimetic underwater is assumed to be an eel larvae with a rigid front body and flexible IPMC back in order to get oscillation and undulation movements. The velocity of the biomimetic underwater robot actuated by flexible IPMC is obtained using the proposed mathematical model.

## 2. Mathematical Modelling of robotic swimmer

In this section, the proposed mathematical model is illustrated for the biomimetic fish type prototype robotic model. The subsections were divided in kinematic, dynamic modelling and IPMC's RC circuit model where the kinematic part includes the undulatory motion and the dynamic part includes the hydrodynamic parameters and the description of Lighthill's elongated body theory. The IPMC's RC model provides the simplified ODE which relates the input voltage with the IPMC's resistance and capacitance.

### *2.1 Kinematic model*

The objective of our work is to produce a kinematic model for the robotic fish with a rigid head and a flexible back of IPMC with the fin. The kind of model which is required is the BCF fish for undulatory motion and the kinematics involved is well explained by Lighthill for the swimmer in this condition and the motion can be expressed as

$$h(x,t) = (Ax + Bx^2)\sin(\omega t + cx) \tag{1}$$

where the displacement of the undulatory motion is denoted by h(x,t) in the body-fixed coordinate.

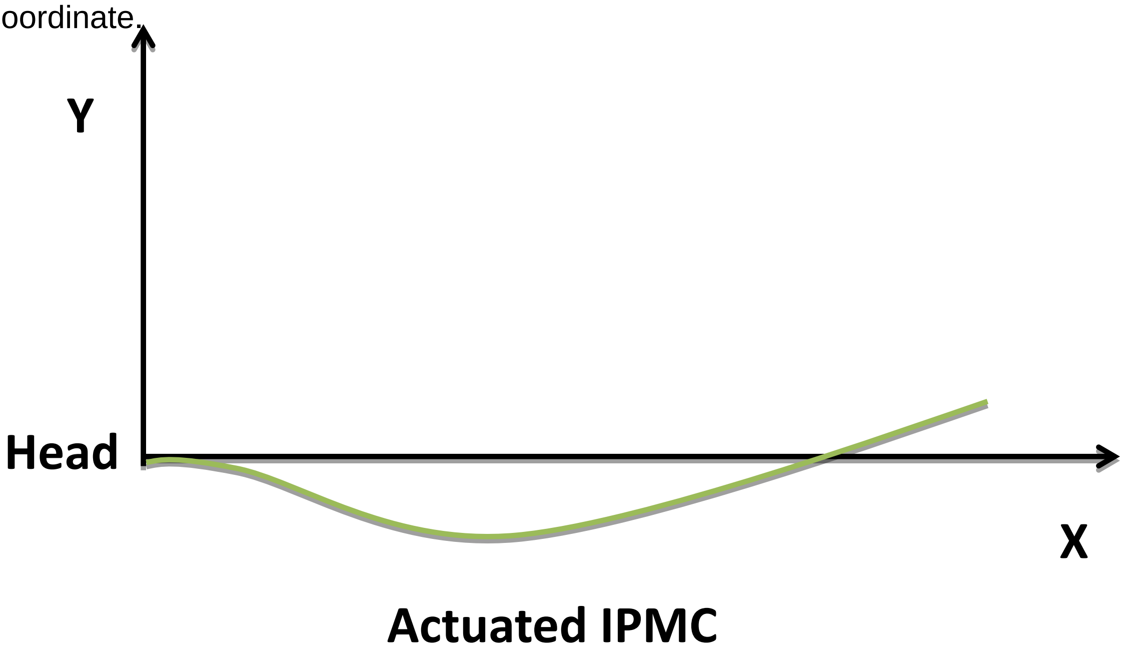

Fig. 1 Schematic kinematics of underwater propulsor

The coordinates and position are shown in the Fig. 1. The term ‘c’ which denotes the wavenumber is given as $c = 2\pi/\lambda$, where $\lambda$ denotes the wavelength. The equation can be adjusted by setting different values for A and B to reach a particular value for the body in an undulatory curve and also circular frequency $\omega = 2\pi f$ where f denotes flapping frequency, t is time.

### *2.2 Dynamic Model*

This part is required not only to get the equation for speed and thrust calculations but also to incorporate the hydrodynamic parameters in the equation so as to cover the application point of view of the robotic model. Since the robotic model is an underwater swimmer so there are forces over the rigid body part which required attention to mention.

The boundary layer pressure drag is the force acting on the rigid part of the robotic body due to underwater surrounding pressure. This force depends on the shape of the body and is directly proportional to the velocity of the robot and is expressed as

$$F_{PD} = \frac{1}{2}\rho V^2 C_{PD} A \tag{2}$$

$F_{PD}$ denotes force due to pressure drag, $C_{PD}$ denotes pressure drag force coefficient, $\rho$ is the density of the fluid and V is the velocity of the robot. The expression $C_{PD}$ can be written as

$$C_{PD} = \frac{6}{1+\sqrt{Re}} + 0.4 \tag{3}$$

Re is the Reynolds number and is computed by $Re = L_R V / \nu$ where $L_R$ is the length of the rigid part of the robot and $\nu$ is kinematic viscosity.
For thrust calculations by the robotic swimmer, the elongated body theory proposed by Lighthill is undertaken which will cover hydrodynamics part and the thrust produced at the tail can be expressed as

$$T(\mathrm{t}) = \frac{1}{2}\rho A\left[\left(\frac{\partial h}{\partial t}\right)^2 - V^2\left(\frac{\partial h}{\partial x}\right)^2\right]_{x=L_t} - \frac{d}{dt}\int_0^{L_t}\rho A\left[\frac{\partial h}{\partial t}\frac{\partial h}{\partial x} + V\left(\frac{\partial h}{\partial x}\right)^2\right]dx \tag{4}$$

Where $L_t$ denotes length at the end of the tail and A denotes the area at the tail is given by

$$A = \frac{1}{4\pi w^2 \beta} \tag{5}$$

Here, $w$ is the width of the tail, $\beta$ is a non-dimensional parameter close to 1.

### *2.3 IPMC simplified RC Model*

The structure of IPMC comprises of a polymer base ionomeric material with parallel platinum electrodes on both sides. The internal resistance with the capacitance formed between two electrodes is simplified as the RC circuit model as shown in Fig. 2 which will provide an ordinary differential equation relating the input voltage, the charge stored and the internal resistance.

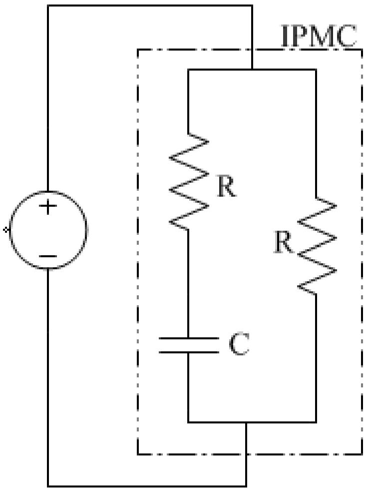

Fig. 2 Simplified RC model

The ordinary differential equation can be expressed as

$$R\dot{Q}(\mathrm{t})+\frac{1}{C}Q(\mathrm{t})=\mathrm{V}(\mathrm{t}) \tag{6}$$

Where R is the internal resistance, C is the capacitance between two electrodes and Q is the charge stored due to the input voltage V.

Since the charge stored is analog to the bending moment produced due to the ionic movements in the polymer region and can be expressed as

$$\mathrm{M}(\mathrm{t})=\mathrm{kQ}(\mathrm{t}) \tag{7}$$

Where, k is the actuation parameter, which relates the geometry and the constitutive behavior of IPMC material.

Now the balanced equation of force in the forward direction can be written using Eq. (2), (4) and (7) respectively and can be expressed as

$$T(\mathrm{t})+M^{'}(\mathrm{t})-\mathrm{F}_{PD}=m\frac{\partial U}{\partial t} \tag{8}$$

Where m is the mass at the end of the tail of the biomimetic propulsor. Solving Eq. (8) will evaluate the velocity of the robotic swimmer.

## 3. RESULTS AND DISCUSSIONS

The proposed mathematical equation Eq. (8) is solved with various parametric values provided in Table 1. The equation is solved evaluating the velocity of the undulatory underwater propulsor for different input frequencies.

Table 1 Parametric values used in the model

| Parameter | Symbol | Value |
|---|---|---|
| Internal resistance | R | 66 ohm |
| Viscosity | $\nu$ | 10^-6 $m^2$/s |
| Density | $\rho$ | 0.001 $g/mm^3$ |
| Rigid head length | $L_H$ | 20 mm |
| Rigid head width | $W_H$ | 15 mm |
| Rigid head thickness | $T_H$ | 1 mm |
| IPMC Length | $L_{IPMC}$ | 40 mm |
| IPMC Width | $W_{IPMC}$ | 10 mm |
| IPMC Thickness | $T_{IPMC}$ | 0.2 mm |
| Mass at tail | m | 5 g |
| Actuation Parameter | k | ~1 |

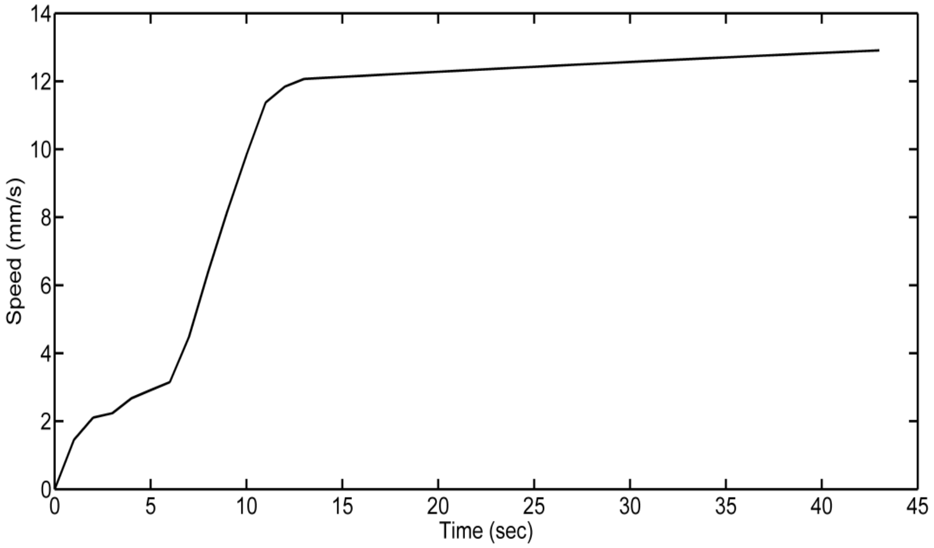

Fig. 3 Speed Time Graph

Fig. 3 Illustrates the velocity of the biomimetic underwater robot actuated by flexible IPMC obtained using the proposed mathematical model for respective frequency and input voltage. We observe that after attaining a particular speed the robotic swimmer tends to move forward with that velocity with a slight increment.

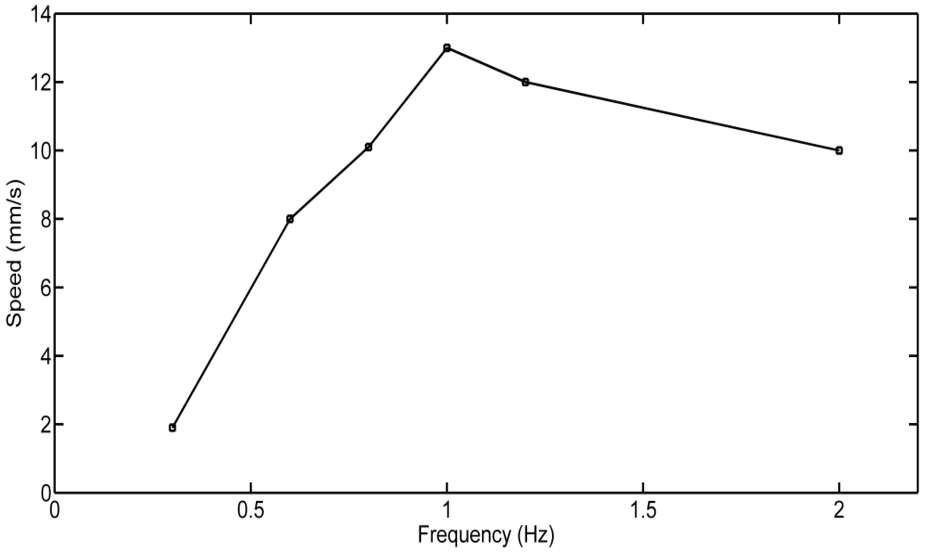

Fig. 4 Speed Vs Frequency

Fig. 4 Represents the values of speed for various input frequencies and also illustrates that the robotic swimmer achieves the peek at a particular frequency which will give the excitation frequency for which the velocity achieved is highest for the propulsion.

## 4. Conclusions

The proposed mathematical model can be used to evaluate speed and thrust of a biomimetic robotic swimmer actuated by IPMC with different RC characteristics because of the inclusion of bending moment caused by charge stored. The swimmer speed is calculated using the proposed model for various voltage and frequency inputs and it was seen that at a particular frequency the swimming speed achieved is highest than other frequency ranges showing material's resonant frequency. The mathematical model can also be used to evaluate thrust of the robotic swimmer. Further the biomimetic robotic swimmer assumed to be like eel larvae will be used for experimental validation of this work.

## REFERENCES

Shen, Q., Wang, T., Liang, J. and Wen, L.(2013), "Hydrodynamic performance of a biomimetic robotic swimmer actuated by ionic polymer – metal composite", Smart Mater. Struct., **22**.

Kim, B., Kim, D., Jung, J. and Park, J. (2005), "A biomimetic undulatory tadpole robot using ionic polymer-metal composite actuators", Smart Mater. Struct., **14**, 1579-1585.

Guo, S., Fukuda, T. and Asaka, K. (2003), "A New Type of Fish-Like Underwater Microrobot", *IEEE/ASME Trans. Mechatronics*, **8**(1), 136–141.

Liu, H., Wassersug, R., Kawachi, K. (1996), "A computational fluid dynamics study of tadpole swimming", J. Exp. Biol., **199**, 1245-1260.

Liu, H., Wassersug, R., Kawachi, K. (1997), "The three-dimensional hydrodynamics of tadpole locomotion", J. Exp. Biol., **200**, 2807–2819.

Laurent, G. and Piat, E. (2001), "Efficiency of swimming microrobots using ionic polymer metal composite actuators", In Proc. of the IEEE Conf. on Robotics and Automation, Seoul, Korea, **4**, 3914–3919.

Zhang, Y. H., He, J. H., Yang, J., Zhang, S. W. and Low, K. H. (2006), "A Computational Fluid Dynamics ( CFD ) Analysis of an Undulatory Mechanical Fin Driven by Shape Memory Alloy", Int. J. Autom. Comput., **4**, 374–381.

Lighthill, M. J. (1960), "Note on the swimming of slender fish", J. Fluid. Mech., **9**, 305-317.

Chen, Z., Shatara, S. and Tan, X. (2010), "Modeling of Biomimetic Robotic Fish Propelled by An Ionic Polymer-Metal Composite Caudal Fin", IEEE/ASME Trans. Mechatronics, **15(**3), 448–459.

Aureli, M., Kopman, V. and Porfiri, M. (2010), "Control-oriented modeling of ionic polymer metal composites for biomimetic underwater propulsion", Proc. Am. Control Conf. (ACC), 6016–6021.

Shahinpoor, M. and Kim, K. J. (2001), "Ionic polymer-metal composites: I. Fundamentals", Smart Mater. Struct., **10**, 819–833.

Kim, K. J. and Shahinpoor, M. (2003), "Ionic polymer-metal composites: II. Manufacturing techniques", Smart Mater. Struct., **12**, 65–79.

Shahinpoor, M. and Kim, K. J. (2004), “Ionic polymer-metal composites: III. Modeling and simulation as biomimetic sensors, actuators, transducers, and artificial muscles”, Smart Mater. Struct., **13**, 1362–1388.

Shahinpoor, M. and Kim, K. J. (2005), “Ionic polymer-metal composites: IV. Industrial and medical applications”, Smart Mater. Struct., **14**, 197–214.

Shahinpoor, M. (1992), “Conceptual design, kinematics and dynamics of swimmina robotic structures using ionic polymeric gel muscles”, Smart Mater. Struct., **1**, 91–94.

Aureli, M., Kopman, V. and Porfiri, M. (2010), “Free-locomotion of underwater vehicles actuated by ionic polymer metal composites,” IEEE/ASME Trans. Mechatronics, **15**(4), 603–614.

Gupta, A. and Mukherjee, S. (2018), “Position Control of a Biomimetic IPMC Underwater Propulsor”, ASME International Conference on Smart Materials, Adaptive Structures and Intelligent Systems (SMASIS 2018), Paper No. 7901.

Salazar, R., Fuentes, V. and Abdelkefi A. (2018), “Clasiification of biological and bioinspired aquatic systems: A review”, Ocean Eng., **148**, 75–114.