

# Surface crack detection in a thin plate using time reversal analysis of SH guided waves

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## 1. ABSTRACT

Ultrasonic nondestructive testing with guided waves is applied for the detection of defects in a thin plate. The most important advantage of the application of guided waves is the efficient inspection of structures, such as thin plates and pipes. However, owing to the wave dispersion and wave mode excitability in plates, the wave propagation inside them becomes more complex. Thus, in this study, we developed defect detection for thin plate inspection using guided waves by using the time reversal method<sup>3)</sup>, which takes advantage of the reciprocity and reversibility characteristics of waves. However, it is difficult to determine the convergence point of time reversal waves, whose point corresponds to the location of the defect. In this study, the topological sensitivity<sup>4),5)</sup> is utilized as the indicator of defect detection for the time-reversal method. In this paper, we first explain the problem to be solved for a scattering problem of 2-D SH guided waves. Next, the forward analysis method for demonstrating scattered wave fields is discussed. Afterwards, a brief description of the time reversal method and the topological sensitivity is introduced. Finally, the topological sensitivity obtained from the forward and time reversal analysis results is calculated to detect surface cracks in thin plates.

Objective: **Development of a surface crack detection method using the time reversal method with the topological sensitivity.**

## 2. PROBLEMS

### Forward problems

The problem is determining the position and length of a surface breaking crack using SH guided waves transmitted from ultrasonic probes. Let the infinite plate be an isotropic homogeneous elastic solid with a shear modulus of  $\mu$ , and the mass density of  $\rho$ . The antiplane displacement,  $u(x, t)$ , in a thin plate  $\Omega$  satisfies the following equations:

### Basic equation

$$\Delta u(x, t) = \frac{1}{c^2} \ddot{u}(x, t) \quad (x \in \Omega, 0 < t)$$

### Boundary conditions

$$u(x, 0) = \dot{u}(x, 0) = 0 \quad (x \in \Omega)$$

$$t(x, t) = \mu \frac{\partial u}{\partial n}(x, t) = 0 \quad (x \in \partial\Omega, 0 < t)$$

$u(x, t)$ : displacement  
 $c$ : wave velocity  
 $n$ : outward normal vector  
 $h$ : thickness  
 $\Omega$ : the reference domain  
 $l$ : length  
 $u^{in}$ : incident wave  
 $u^{sc}$ : scattered wave

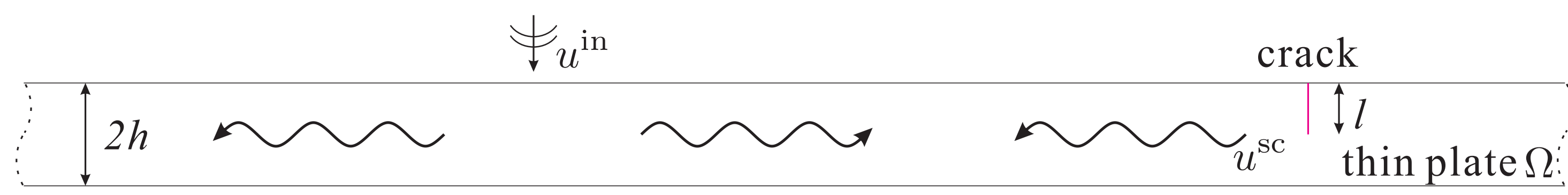


Fig.1 SH waves scattering in a thin plate.

We consider scalar wave scattering and transmission in infinite regions  $\Omega$  as shown in Fig. 1.

➔ A forward analysis of wave scattering by the surface breaking crack is solved by using the **Finite Element Method (FEM)**.

### Time reversal method

Scattered waves from a defect are recorded with receivers, and the time reversed waves are emitted into the numerical model. The time-reversed waves propagate in the model and focus on the defect location.

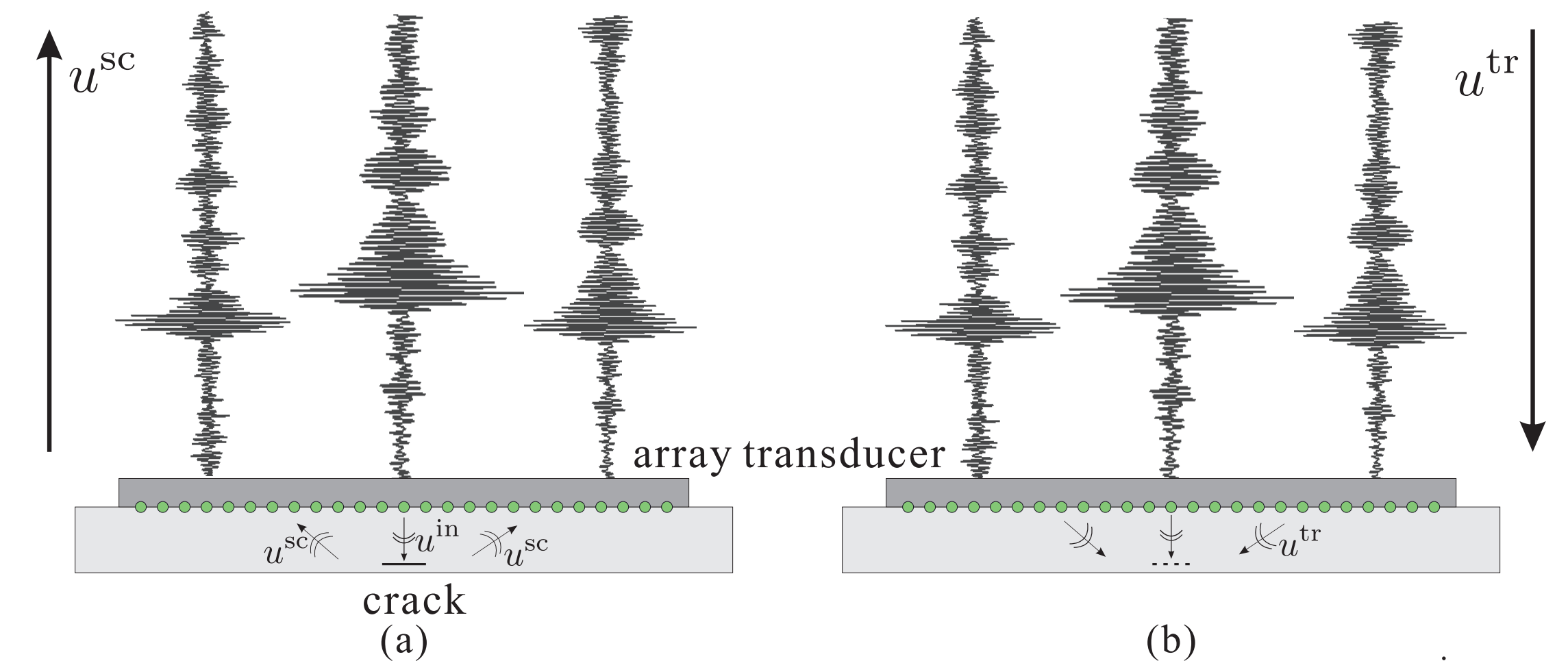


Fig.2 Time-reversal method (a) The first step involves target defect by transmitting wave  $u^{in}$ . The scattered waves are recorded as  $u^{sc}$  by receivers. (b) Then the receivers emit the time-reversed waves. These waves propagate through the medium and comes to the shape of wave .

### Major advantages of time reversal method

The ray tracing and signal conditioning before back-propagation **are not the requisites** for the time reversal method. The range of application of time reversal method is hence broader **than the conventional SAFT**.

➔ In this research, by calculating **the topological sensitivity** with forward analysis and time-reversal analysis, the defect shape can be reconstructed.

## 3. FORWARD ANALYSIS

In general, the BEM is good at treating cracks, However, the FEM cannot strictly deal with cracks without any modifications. Thus, for simplicity, **the surface breaking cracks are modeled as slits with a thickness  $\Delta x = 1.0 \times 10^{-4}$  [m]** in this study.

As an numerical example, the reconstruction of a surface crack in the thin plate using some transmission and receiver points is demonstrated. Forward analysis was performed using the analysis parameters in Table 1. Fig. 3 shows the analysis model and the length of the vertical surface breaking crack,  $l$ , as  $l = 2.5$  [mm]. Fig. 4 shows the numerical results for SH guided wave propagation and scattering by the surface breaking crack, as shown in Fig. 3, obtained by the FEM at times  $t = 1.2, 1.6, 2.0$  and  $2.4$  [ $\mu$ s].

**Incident wave**

$$u^{in}(x^m, t) = \begin{cases} 1 - \cos(2\pi t/T_0) & (t \leq T_0) \\ 0 & (t > T_0) \end{cases}$$

$T_0$ : Period of incident wave

Number of elements	178,551
Number of incident points	7 [points]
Number of observation points	41 [points]
Time step size	$\Delta t = 1.0 \times 10^{-8}$ [s]
Density	$\rho = 2700$ [kg/m <sup>3</sup> ]
Mesh unit	$\Delta x = 1.0 \times 10^{-4}$ [m]
Period	$T_0 = 2.0 \times 10^{-6}$ [s]
wave speed	$c_T = 3130$ [m/s]

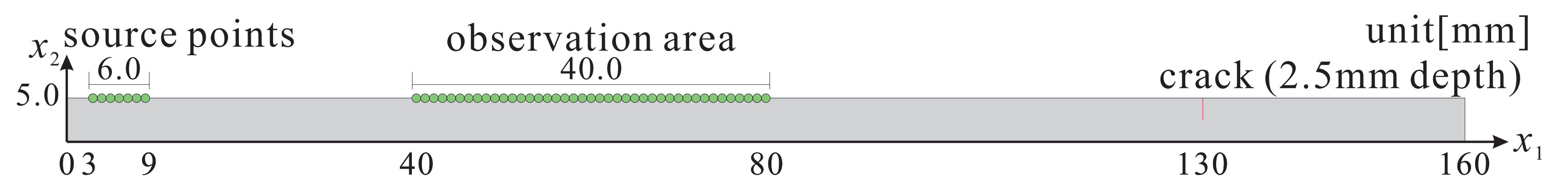


Fig.3 Analysis model for the reconstruction of a surface breaking crack from the upper side of a plate using some transmission and receiver points.

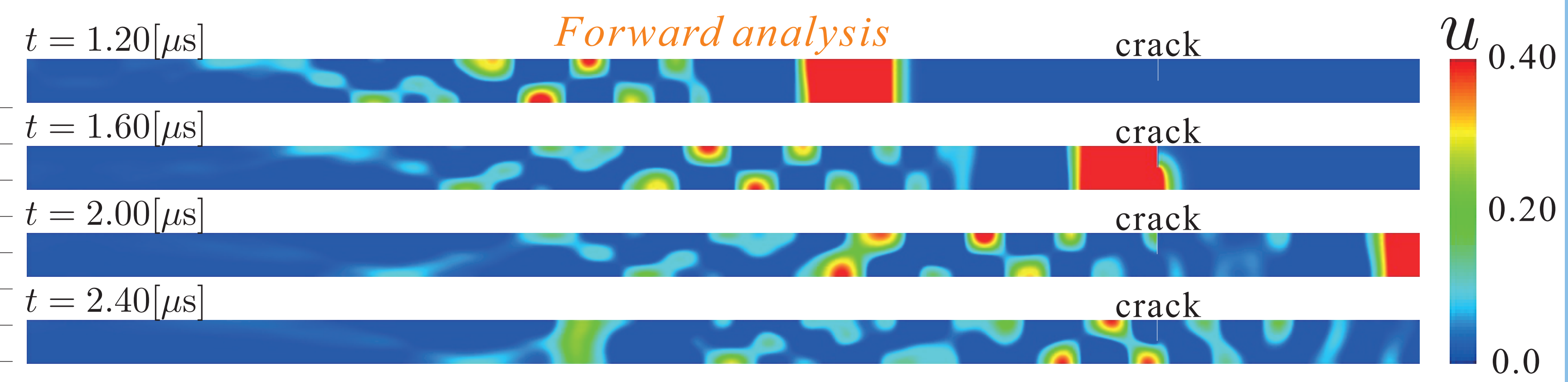
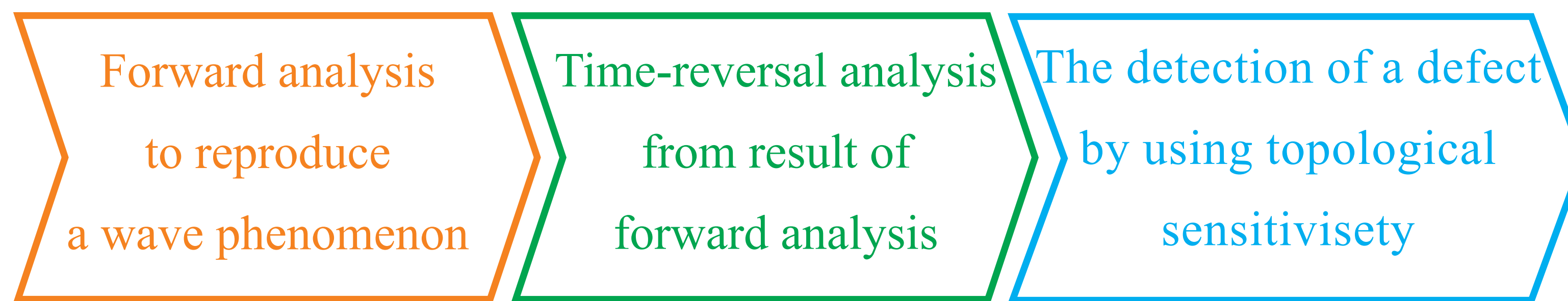


Fig.4 SH guided wave propagation and scattering by the surface breaking crack.

## 4. THE DETECTION OF A CRACK

As an numerical example, the surface breaking crack, as shown in Fig.3, is reconstructed by using **the time-reversal method with the topological sensitivity**.

### Flow of the numerical analysis



### Time reversal analysis

The time reversal waves  $u^{tr}(x^m, t)$  are injected from the source points  $x^m$ , and the location of the defect is identified by converging the time reversal wave. Time-reversed waves use the scattered waves  $u^{sc}(x^m, t)$  obtained by forward analysis.

$$u^{tr}(x^m, t) = u^{sc}(x^m, T - t)$$

### Topological sensitivity $\mathcal{T}(x)$

Topological sensitivity is a mathematical tool aimed at quantifying sensitivity of objective function to the appearance of a very small cavity centered at a specified location  $x$  in the reference medium  $\Omega_0$ .

$$\mathcal{T}(x) = \lim_{\epsilon \rightarrow 0} \frac{J(\Omega_\epsilon) - J(\Omega_0)}{\pi \epsilon^2}$$

### Objective function $J(\Omega_0)$

Misfit function which evaluates the adequation between the measurements obtained for the inspected medium  $\Omega_\epsilon$  and measurements performed on a reference medium  $\Omega_0$  known as safe.

$$J(\Omega_0) = \int_0^T \int_{S^{obs}} \frac{1}{2} |u_0(x, t) - u^{obs}(x, t)|^2 dS_x dt$$

$$\mathcal{T}(x) = \left( \nabla u_0 * (A \nabla \dot{u}_0) + \frac{1}{c^2} \dot{u}_0 * \dot{u}_0 \right) (x, T), \quad A_{ik} = 2\delta_{ik}$$

$u_0$ : displacement of the forward analysis  $\dot{u}_0$ : displacement of the time-reversal analysis  $S^{obs}$ : observation area

## 5. CONCLUSION

- In this study, the shape reconstruction of surface breaking cracks in a 2-D thin homogeneous isotropic plate was considered using the time reversal method in which the topological sensitivity was employed to identify cracks.
- Although the time-reversal wave did not converge, **the topological sensitivity used as the indicator to identify the convergence points of the time-reversal waves allows us to determine the surface breaking crack position and size.**

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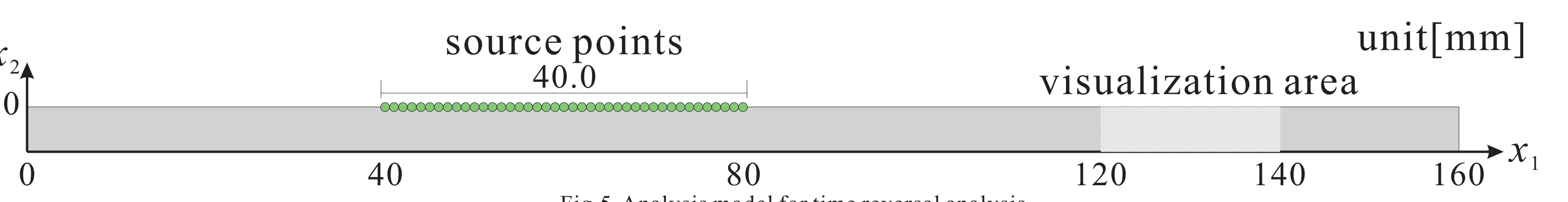


Fig.5 Analysis model for time reversal analysis.

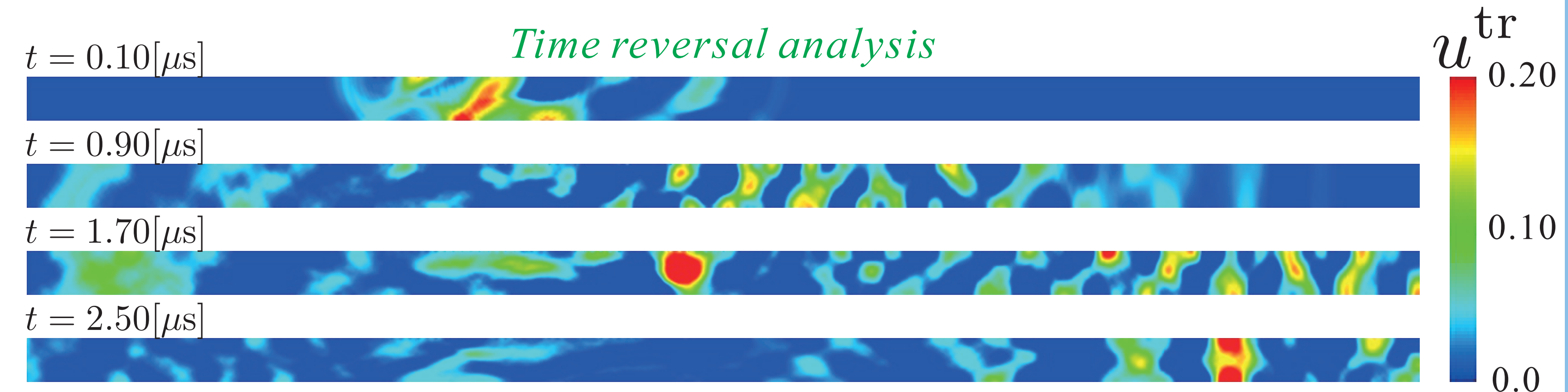


Fig.6 Time reversal of the received waves and their propagation inside the plate.

Number of elements	178,551
Number of incident points	41 [points]
Time step size	$\Delta t = 1.0 \times 10^{-8}$ [s]
Density	$\rho = 2700$ [kg/m <sup>3</sup> ]
Mesh unit	$\Delta x = 1.0 \times 10^{-4}$ [m]
Period	$T_0 = 2.0 \times 10^{-6}$ [s]
wave speed	$c_T = 3130$ [m/s]

### Indicates the topological sensitivity $\mathcal{T}(x)$

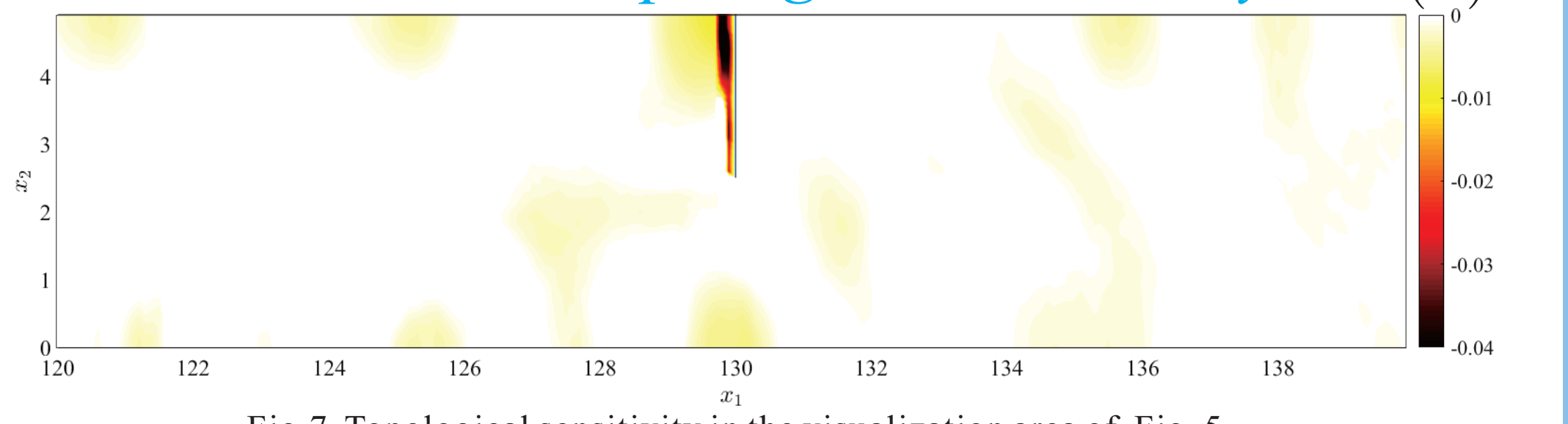


Fig.7 Topological sensitivity in the visualization area of Fig. 5.

- Fig.5 shows time reversal analysis model without crack.
- Fig.6 shows time reversal analysis results obtained by the FEM at times  $t = 0.1, 0.9, 1.7$  and  $2.5$  [ $\mu$ s].
- Fig.7 shows the result of deriving the topological sensitivity in the visualization area of Fig. 3. (The blue straight line shows the real surface breaking crack.)

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