Numerical Study of Blending Hybrid RANS/LES Method and $\gamma$-$Re_{\theta}$ Transition Model for Unsteady Turbulent Flow Analysis

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ABSTRACT

Hybrid RANS/LES methods were successfully applied to assess fully-turbulent external flows. The results provided by hybrid RANS/LES have generally shown excellent solutions for unsteady wake flows, but not for transitional flows because hybrid RANS/LES is fundamentally based on a fully turbulent assumption. The new blended model that combines the DDES-SST and $\gamma$-$Re_{\theta}$ transition models is hereby proposed to simulate both the unsteady wake and the laminar/turbulent transition inside of the boundary layer. The numerical results provided by the simulation are compared to experimental data obtained from an Eppler 387 airfoil. The results indicate that the hybrid RANS/LES-Transition model predicts both strong laminar/turbulent transition phenomena including the laminar separation bubble at low angle of attack and flow separation at high angles of attack.

1. INTRODUCTION

For several decades, aerodynamic problems have been analyzed by using a variety of turbulence models based on the Reynolds-Averaged Navier-Stokes (RANS) equations. Although RANS models provide reasonable accuracy in engineering applications, inaccurate solutions are often produced as a result of time-averaging with unsteady flows, such as in cases with a large separation or highly fluctuating perturbations. Hybrid RANS/Large Eddy Simulation (LES) methods have been uses as successful alternatives to solve unsteady flow problems with flow separation. These
methods treat the entire boundary layer with a RANS model and the unsteady separated regions with the LES in which the moderate number of grid points provide reasonable accuracy for the unsteady separation flows.

Spalart et al. (1997) developed the detached eddy simulation (DES) method. This is a hybrid RANS/LES method and the basic idea is to use the S-A RANS turbulence model in the near wall region and sub grid scale (SGS) modeling in the free shear region in order to yield results comparable to those of the LES approach. The inherently three-dimensional and time-dependent DES technique is greatly dependent on the grid densities and the first formulation of Spalart’s DES (DES97) (Spalart et al. (1997)) relies on the size of grid spacing to act as an LES model. However, the reliance on the grid spacing resulted in difficulties when grids with ambiguous sizes acted as LES in unwanted regions, causing the laminarization of the turbulent boundary layer (Spalart et al. (2006)).

The delayed DES (DDES) method (Spalart et al. (2006)) is an improved version of the original DES97 model. It ensures a proper transition from RANS to LES. A blending function, similar to the one used by Menter and Kuntz (2004) for the SST model, was introduced to limit the length scale of DES97. Im et al. (2011) carried out a DDES simulation of the aerodynamic stall phenomenon where DDES demonstrated an excellent capability to simulate the dynamic stall for a stalled NACA0012 airfoil flow. Gritskevich et al. (2012) proposed a coupling strategy for the DDES as well as $k-\omega$ SST turbulence models. The numerical results from SST-based DDES results were consistent with experimental data.

Transitional boundary layer flows are important in many CFD applications of engineering interest, such as for airfoils, airframes, rotor blades, and wind turbine blades, since their aerodynamic characteristics are affected by the transition process. RANS models that ignore the laminar/turbulent transitions can lead to over-prediction of the viscous drag force (Johansen (1997)). Furthermore, the turbulent transition is strongly related to the laminar separation bubble, especially at low-Reynolds numbers (Windte et al. (2006)).

Recently, several attempts were made to incorporate laminar/turbulent transition models into well-known turbulence models. Menter and Langtry (2009) developed the local correlation-based transition transport model (LCTM) using the $\gamma-Re_\theta$ transport equations. The concept of this transition model is to predict the transition using local variables, which allows this model to be implemented into a RANS framework while affording the same degree of parallelization as other transport equation models. The present transition model is built on a transport equation for intermittency and can be used to trigger a transition locally. In addition to the transport equation for the intermittency, the second transport equation is solved for the transition-onset momentum thickness Reynolds number. The $\gamma-Re_\theta$ transition model of Menter and Langtry produced results that were consistent with experimental data when estimating solutions to both two- and three-dimensional engineering problems.

Sorensen et al. (2011) suggested a hybrid method with SST-DES and a $\gamma-Re_\theta$ transition model in order to predict separated transitional flows in three dimensions. The hybridization was reasonable since DES exhibited good performance for fully turbulent flow analysis in unsteady wake, and the $\gamma-Re_\theta$ transition model had good performance for laminar/turbulent transition flow analysis on attached transitional flows. The method
seems to have disadvantages in that, if the turbulence has a high intensity, it is not possible to distinguish the laminar region from the LES regions. You and Kwon (2013) conducted a study to blend the scale-adaptive simulation (SAS) techniques and the $\gamma-Re_\theta$ transition model. The numerical results were promising, although their method was developed based on a heuristic blending function from specific experimental results.

The present study proposes a strategy that blends the $\gamma-Re_\theta$ transition model and the DDES-SST method to simultaneously model the laminar/turbulent transition inside of the boundary layer and the unsteady fluctuating flow in the wake region. The proposed model was implemented into KFLOW, an in-house parallelized, multiblock-structured, Navier-Stokes solver program (Park et al. (2004, 2006)). Transitional flows over an Eppler 387 wing configuration were computed using the hybrid DES/Transition model, and the results were compared to experimental data (McGhee et al. (1988), Cole and Mueller (1990)) and MSES(Drela (2007)) results.

2. NUMERICAL METHOD

2.1 Governing equations and numerical methods

A Reynolds-Averaged Navier-Stokes code, KFLOW, is a parallelized multiblock structured, Navier-Stokes solver (Park et al. (2004, 2006)). It is capable of computing time-accurate moving body problems by using a parallelized CHIMERA overlapping grid system. Six turbulence models, including Spalart-Allmaras and $k-\omega$ Shear Stress Transport (SST) turbulent models, have been implemented. Low Mach-number preconditioning with a multigrid can be applied to low-speed compressible unsteady flows.

The Reynolds-Averaged Navier-Stokes equations and the $k-\omega$ SST turbulence equations are used as the baseline governing equations, and the time-derivative preconditioning techniques are used to reduce convergence problems that occur at low Mach number flows. A second-order-accurate dual-time-stepping scheme combined with a Diagonalized Alternating Directional Implicit (DADI) method is used to compute the unsteady flow-fields around 3D wing. The inviscid flux is calculated by Roe flux difference splitting scheme. Characteristic boundary conditions using the Riemann invariant are applied at the far-field boundary. At the solid wall, the no-slip condition is applied for velocities and the density and energy are extrapolated from the interior cells. For the purposes of the present study, DDES-SST model (Menter and Kuntz (2004)) and $\gamma-Re_\theta$ transition model (Menter and Langtry (2009)) is used for blending in the hybrid model.

2.2 The blending model with hybrid RANS/LES and Transition model

Because the DDES-SST model and $\gamma-Re_\theta$ transition model are based on the same $k-\omega$ SST turbulence model, the governing equations have a structural similarity to each other. The destruction term ($D_k$) from the kinetic energy equation in the original $k-\omega$ SST turbulence model is required some modifications in order to apply to the DDES-SST model. DDES-SST model is constructed by replacing the length scale ($l_{k-\omega}$) of the $k-\omega$
SST model to the length scale ($\tilde{l}$) of the DES model (Spalart et al. (2006)). Also, the present $\gamma$-$Re_\theta$ transition model has been calibrated for use with the $k$-$\omega$ SST turbulence model. The coupling between the transition model and the turbulence model is accomplished by modifying equations for the turbulent kinetic energy $k$ and the turbulence frequency $\omega$ in the turbulence model as follows:

$$\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_j}(\rho u_j k) = \overline{P}_k - \overline{D}_k + \frac{\partial}{\partial x_j}\left((\mu + \sigma_j \mu) \frac{\partial k}{\partial x_j}\right),$$  

$$\frac{\partial}{\partial t}(\rho \omega) + \frac{\partial}{\partial x_j}(\rho u_j \omega) = \alpha \frac{P}{v_j} - D_\omega + \frac{\partial}{\partial x_j}\left((\mu + \sigma_j \mu) \frac{\partial \omega}{\partial x_j}\right),$$

$$\overline{P}_k = \gamma_{eff} \overline{P}_k,$$

$$\overline{D}_k = \min\left\{\max\left(\gamma_{eff}, 0.1\right), 1.0\right\} D_k,$$

where $P_k$ and $D_k$ are the production and destruction terms from the turbulent kinetic energy equation in the original $k$-$\omega$ SST turbulence model. $\gamma_{eff}$ is the effective intermittency obtained from $\gamma$-$Re_\theta$ transition model. The blending model with DDES-SST and $\gamma$-$Re_\theta$ transition model (DDES-Transition) is constructed by replacing the length scale ($l_{k-\omega}$) of the $k$-$\omega$ SST model to the blended length scale ($\tilde{l}_{\text{Trans}}$) of the DDES-Transition model. The modified blending function is defined as follows:

$$\tilde{l}_{\text{Trans}} = l_{k-\omega} - \tilde{f}_{\text{Trans}} \max\left(0, l_{k-\omega} - C_{\text{DES}} \Delta \right),$$

$$f_{\text{Trans}} = \left(1 - F_{\alpha} \tilde{f}_{\text{DDES-Trans}}\right),$$

$$F_{\alpha} \tilde{f}_{\text{DDES-Trans}} = \left(\frac{\min\left(\max\left(F_{\alpha}, \chi\right), 1.0\right) - \chi}{1 - \chi}\right)^2, \quad \chi = 0.8,$$

The blending function $F_{\alpha}$ is used to turn off the source term in the boundary layer and allow the transported scalar $Re_\theta$ to diffuse in from the freestream. $F_{\alpha}$ is equal to zero in the freestream and one in the boundary layer (Menter and Langtry (2009)). The present blending function, $F_{\alpha} \tilde{f}_{\text{DDES-Trans}}$, with $\chi=0.8$ successfully separates the laminar boundary layer from the turbulent one as well as the RANS region from the LES one.

### 3. NUMERICAL RESULTS

In order to predict the Eppler 387 airfoil flow a C-type grid is generated. The height of the computational domain is set to $h/c=20$, also the length of wake region is same. The depth of the computational domain is set to the width $z/c=0.2$ as a compromise between span-wise extension and span-wise resolution. This choice is based on grids of other research groups (Schmidt and Breuer (2014)). The grid consists of 257 node point in the suction side of airfoil surface, 129 in the pressure side.
Furthermore, the grid consists of 129 node points in the stream-wise direction, 127 in the normal direction and 51 in the span-wise direction. The distance to the first cell centers from the wall is $1.0 \times 10^{-5}$, so that $y^+$ is set to below 1. The study concentrates on the prediction of the airfoil flow at the Reynolds number $Re = 1.0 \times 10^5$ and angle of attack $\alpha = 2$, 11 degrees. For the unsteady time-accurate calculations, a non-dimensional time step size, $\Delta t U_{\infty}/c$, of 0.002 was used. The unsteady calculations were made for 25 non-dimensional time steps, and the time averaged solution was obtained by taking the average over the last 10 time steps.

Fig. 1 compares the distributions of the time-averaged pressure coefficient at 2 degrees. MSES, a well-known airfoil prediction code, is also used for comparison. According to the experimental data, a large laminar separation bubble (LSB) is confined to the upper surface in the range from about 40% to 85% of the chord: the laminar separation at $x/c = 0.4$, the transition at $x/c = 0.75$, and the turbulent reattachment at $x/c = 0.85$. A laminar boundary layer separated from the surface exponentially increases the stream-wise instability under an adverse pressure gradient. The flow instability triggers a turbulent transition and finally makes an attached turbulent boundary layer of the laminar separation. LSB is the result of the transition process in low Reynolds number flow, and it results in a plateau in the pressure distribution. The hybrid DDES-transition model predicts the pressure distribution better than both the MSES and DDES-SST models near the leading edge on the upper surface. Numerical results show that the DDES-SST model with a fully turbulent assumption cannot predict the LSB and the corresponding pressure plateau together, as shown in Fig. 2. The laminar separation bubble is clearly seen in the time-averaged flow field obtained from the present hybrid model.

![Fig. 1 Distributions of the pressure coefficient $c_p$ of the time averaged flow field at the mid span ($\alpha=2.0$ deg.).](image1)

![Fig. 2 Time averaged streamlines and velocity contour near the laminar separation bubble ($\alpha=2.0$ deg.).](image2)
Figs. 3 and 4 show the pressure distributions and the streamlines around the leading edge at 11 degrees. The laminar separation bubble moves forward as the angle of attack increases. Both the DDES-SST and MSES models predict much smaller sizes for the separation bubble, but the present DDES-Tran model provides better agreement with the experimental data.

Pressure distribution of the experiment indicates that a strong leading-edge stall phenomenon occurred at this angle of attack. The unsteady RANS (URANS) with the transition model produces the LSB around the leading edge, but cannot predict strong flow separation. MSES predicts only a trailing-edge stall, whereas both the DDES-SST and DDES-Transition models predict the leading-edge stall that is consistent with experimental data. The hybrid DDES-Transition model shows better agreement in the pressure distribution since the laminar separation occurred earlier when the transition is considered. Fig. 5 illustrates distributions, which designate the vortex structure. The unsteady RANS model gives a small separation near the leading edge, while highly complex vortex structures are produced by the DDES models. These vortex structures are typical in the large eddy simulation and show that the present DDES-Transition model successfully blends the DDES and the transition model.
4. CONCLUSIONS

In the present study, by using the hybrid DDES/Transition model that combines $\gamma$-$Re_{\theta}$ Transition model and DDES-SST model, the unsteady transition flow analysis was performed. The $F_{\theta t}$ function is used for the blending of the $\gamma$-$Re_{\theta}$ Transition model and the DDES-SST model. The low Reynolds number flows around an Eppler 387 wing were investigated by using the hybrid DDES-Transition model that combines the $\gamma$-$Re_{\theta}$ Transition model and the DDES-SST model. A new hybrid function was proposed to separate the laminar boundary layer from the turbulent boundary layer as well as the wake region. Time-averaged results were compared with the baseline model and the experimental data. Models considering the transition process successfully predicted the laminar separation bubble at a low angle of attack, while only the DDES models predicted the steep leading-edge stall at higher angles of attack. Numerical results show that the hybrid DDES-Transition model simultaneously predicts the laminar separation and turbulent transition of low Reynolds number flows.

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